

## Matematička analiza 1 - 1. auditorna vježba - 2.10.2023.

**Zadatak 1** Neka su  $A, B$  i  $C$  logički sudovi. Dokažite ili opovrgnite istinitost sljedećih sudova:

- (a)  $(A \implies B) \iff (\neg A \vee B)$ ,
- (b)  $((A \implies B) \wedge (B \implies C)) \implies (A \implies C)$ ,
- (c)  $((A \implies B) \wedge (A \implies C)) \implies (B \implies C)$ ,
- (d)  $(A \implies (B \wedge C)) \iff ((A \implies B) \wedge (A \implies C))$ .

**Zadatak 2** Napišite negacije sljedećih sudova te ispitajte njihovu istinitost:

- (a)  $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x^2 + y < 0)$ ,
- (b)  $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(x^2 + y < 0)$ ,
- (c)  $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(xy > 0 \implies y < 0)$ ,
- (d)  $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(xy > 0 \implies y < 0)$ .

**Zadatak 3** Dokažite da za svaki prirodan broj  $n$ , 5 dijeli  $7^{n+2} + 2 \cdot 12^{n+1} + 2 \cdot (-3)^n$ .

**Zadatak 4** Matematičkom indukcijom dokažite:

- (a)  $\sum_{i=1}^n \ln\left(1 + \frac{1}{i}\right) = \ln(1 + n), \forall n \in \mathbb{N}$ ,
- (b)  $\sum_{i=1}^n \frac{1}{\sqrt{i}} \leq 2\sqrt{n} - 1, \forall n \in \mathbb{N}$ .

**Zadatak 5** Niz  $(a_n)_{n \in \mathbb{N}}$  realnih brojeva zadan je s

$$a_1 = 1, a_2 = 5$$

i

$$(\forall n \in \mathbb{N})(a_{n+2} = a_{n+1} + 2a_n).$$

Dokažite da za svaki prirodan broj  $n$  vrijedi

$$a_n = 2^n + (-1)^n.$$