University of Zagreb
Faculty of Electrical Engineering and Computing
Department of Applied Mathematics

Workshop

Modern challenges in continuum mechanics

sponsored by

Zagreb, April 3 – 6, 2017

Introduction letter

Dear speakers and participants,

welcome to the workshop "Modern challenges in continuum mechanics". Organizing this workshop our aim was to bring together researchers working in related fields of continuum mechanics: fluid mechanics, fluid-structure interaction problems, elasticity, plasticity, multi-component mixtures, scaling laws, multi-scale problems, homogenization etc., and moreover, to foster new ideas and establish collaborations among participants.

Program of the workshop consists of 4 mini-courses, 11 invited talks and 12 short oral poster presentations. The topics are closely related to the topics of the research project "Mathematical Analysis of Multi-Physics Problems Involving Thin and Composite Structures and Fluids" (grant no. 9477), funded by the Croatian Science Foundation (HRZZ), under whose auspices this workshop is organized. Financial support of the Foundation of the Croatian Academy of Sciences and Arts (Zaklada HAZU) is also greatly acknowledged.

Events like this are expected to contribute in the development of the applied mathematics in Croatia, and we believe that this event will make it internationally more visible and recognizable and will be especially useful for young researchers.

We wish you all a pleasant stay in Zagreb and fruitful discussions during the workshop.

Organizers,
Mario Bukal
Boris Muha
Igor Velčić

Speakers

Mini-course lecturers

Peter Bella (University of Leipzig)
Ansgar Jüngel (Vienna University of Technology)
Šárka Nečasová (Czech Academy of Sciences, Prague)
Annie Raoult (Paris Descartes University)

Invited speakers

Nenad Antonić (University of Zagreb)
Tomasz Cieślak (Polish Academy of Sciences, Warsaw)
Elisa Davoli (University of Vienna)
Antonio Gaudiello (University of Cassino and Southern Lazio)
Matthieu Hillairet (University of Montpellier)
Peter Hornung (TU Dresden)
Mladen Jurak (University of Zagreb)
Darko Mitrović (University of Montenegro)
Paolo Piovano (University of Vienna)
Siniša Slijepčević (University of Zagreb)
Barbara Zwicknagl (University of Würzburg)

Poster presenters

Matteo Caggio (Czech Academy of Sciences, Prague)
Andrijana Ćurković (University of Split)
Ivan Dražić (University of Rijeka)
Marija Galić (University of Zagreb)
Petar Kunštek (University of Zagreb)
Erika Maringová (Charles University in Prague)
Marin Mišur (University of Zagreb)
Marija Prša (University of Zagreb)
Marko Radulović (University of Zagreb)
Loredana Simčić (University of Rijeka)
Michail Turbin (Voronezh State University)
Andrey Zvyagin (Voronezh State University)
Victor Zvyagin (Voronezh State University)

Mini-courses

Lecturer | Peter Bella (peter.bella@math.uni-leipzig.de),
University of Leipzig, Germany

Subject 1 | **Variational viewpoint on wrinkling of thin elastic sheets**

Abstract | Wrinkling of thin elastic sheets can be viewed as a way how they avoid compressive stresses. While the question of where the wrinkles appear is well-understood, to understand the length scale of wrinkling require some work and is problem dependent. Considering a variational viewpoint, the problem amounts to minimization of an elastic energy, which can be viewed as a non-convex membrane energy singularly perturbed by a higher-order bending term. To understand the global minimizer (ground state), the first step is to identify its energy, in particular its dependence on the small physical parameter (thickness). I will discuss several problems where optimal scaling law was identified.

Subject 2 | **Quantitative theory of homogenization for elliptic operators with random coefficients**

Abstract | It is a classical result that elliptic equations with periodic coefficients behave on large scale like equations with constant coefficient. While it is known for a long time that for a large class of elliptic equations with *random* coefficients similar statement holds (on large scales they "become close" to equations with constant deterministic coefficient), the question how fast the homogenization happens is much more recent. I will first explain some of the result on the error estimate, and will later focus on connection with regularity theory for heterogeneous equations.

Lecturer | Ansgar Jüngel (juengel@asc.tuwien.ac.at),
Vienna University of Technology, Austria

Subject | **Entropy methods for cross-diffusion systems from biology and physics**

Abstract | Cross-diffusion equations describe the diffusive interaction in multicomponent systems. Examples include population dynamics, cell biology, and nonequilibrium thermodynamics. Mathematically, they consist of strongly coupled parabolic equations with a full diffusion matrix. The major challenge is that the diffusion matrix is generally neither symmetric nor positive definite. A common feature of the models is that they possess a formal gradient-flow or entropy structure. This structure is exploited to analyze the cross-diffusion systems mathematically. In this lecture series, we introduce the key ideas leading to the so-called boundedness-by-entropy method. A general global existence result is stated, and the key ideas of its proof are given. The theorem is applied to several cross-diffusion systems from biology and physics. The formal derivation of these systems from lattice, kinetic, and fluid models is explained. The relation between principles from nonequilibrium thermodynamics and the entropy structure of cross-diffusion systems is detailed. Surprisingly, this structure can be also found in nonstandard systems describing van-der-Waals fluids and exotic financial derivatives.

Lecturer | Šárka Nečasová (matus@math.cas.cz),
Czech Academy of Sciences, Prague, Czech Republic

Subject 1 | **Viscous compressible Navier-Stokes-(Fourier) system coupled to the radiative transfer equation**

Abstract | We consider relativistic and "semi-relativistic" models of radiative viscous compressible Navier-Stokes-(Fourier) system coupled to the radiative transfer equation extending the classical model introduced in [1]. We concentrate on the problem of existence of weak solution of the problem. Secondly we will study some of its singular limits (low Mach and diffusion) in the case of well-prepared initial data and Dirichlet boundary condition for the velocity field. In the low Mach number case we prove the convergence toward the incompressible Navier-Stokes system coupled to a system of two stationary transport equations see [2]. Moreover, in the diffusion case we prove the convergence toward the compressible Navier-Stokes with modified state functions (equilibrium case) or toward the compressible Navier-Stokes coupled to a diffusion equation (non equilibrium case), see [3, 4].

Subject 2 | **Motion of viscous compressible fluids in time dependent domains**

Abstract We consider the problem of the motion of compressible fluids in domain with varying boundary. We focus on the existence of weak solution and the singular limit in the low Mach number regime [5, 6].

References:

[1] B. Ducomet, E. Feireisl, Š. Nečasová. On a model of radiation hydrodynamics. Ann. I. H. Poincaré-AN 28 (2011), 797–812.

[2] B. Ducomet, Š. Nečasová. Low Mach number limit in a model of radiative flow, Journal of Evolution equations 14 (2014), 357–385.

[3] B. Ducomet, Š. Nečasová. Diffusion limits in a model of radiative flow. Ann. Univ. Ferrara Sez. VII Sci. Mat. 61 (2015), 17–59.

[4] B. Ducomet, Š. Nečasová. Singular limits in a model of radiative flow. J. Math. Fluid Mech. 17 (2015), 341–380.

[5] E. Feireisl, O. Kreml, Š. Nečasová, J. Neustupa, J. Stebel. Weak solutions to the barotropic Navier-Stokes system with slip boundary conditions in time dependent domains. J. Differential Equations 254 (2013), 125–140.

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Lecturer | Annie Raoult (annie.raoult@parisdescartes.fr),
Université Paris Descartes, Paris, France

Subject | **Hierarchical models of prestrained thin materials**

Abstract | In the course of two lectures, we will describe the asymptotic derivation of thin nonlinear elastic models. A hierarchy of the four main models was first obtained by means of a systematic approach in [6]. Namely, the nonlinear membrane model, the nonlinear bending model (also known as Kirchhoff model), the von Kàrmàn model and the linear model were recovered. What distinguishes the models is the loading order of magnitude (or equivalently the order of the energy) and the induced magnitude of the deformations. A rigorous variational derivation of the nonlinear membrane model was then given in [9]. A similar approach had been previously used in a 1d-setting, where there is no need of quasiconvexification, in [1]. Then, [7], [8] provided a rigidity result that allowed to rigorously justify the other models. Some recent papers [2], [5] suggest models where thin bodies (leaves, gel disks) try to reach an elastic equilibrium state under some given non Euclidean metrics, see also [4]. When considered as 3d bodies, they may fail (case when the 3d metric tensor has no realization). We will see the role played by separate entries of the 3d Riemann curvature tensor in selecting the appropriate thin model, see [3], [10], [11].

References:

[1] E. Acerbi, G. Buttazzo, D. Percivale. A variational derivation for the strain energy of an elastic string. J. Elasticity 25 (1991), 137–148.

[2] P. Bella, R. V. Kohn. Metric-induced wrinkling of a thin elastic sheet. J. Nonlinear Sci. 24, (2014), 1147–1176.

[3] K. Bhattacharya, M. Lewicka, M. Schaffner. Plates with incompatible prestrain. Arch. Rational Mech. Anal. 221 (2016), 143–181.

[4] J. Dervaux, M. Ben Amar. Morphogenesis of growing soft tissues. Phys. Rev. Lett. 101 (2008), 068101.

[5] E. Efrati, E. Sharon, R. Kupferman. Elastic theory of unconstrained non-Euclidean plates. J. Mech. Phys. Solids 57 (2009), 762–775.

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[7] G. Friesecke, R. James, S. Müller. A theorem on geometric rigidity and the derivation of nonlinear plate theory from three dimensional elasticity. Comm. Pure Appl. Math. 55 (2002), 1461–1506.

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Invited talks

Speaker | Nenad Antonić (nenad@math.hr),
University of Zagreb, Croatia

Title | **H-measures, H-distributions and applications**

Abstract | H-measures, including their variants (such as semiclassical/Wigner measures) and H-distributions are microlocal defect functionals developed to provide better understanding of limits in various asymptotic problems. We shall try to briefly present these objects, stressing their possible applications in problems appearing in continuum mechanics, quantum theory and conservation laws.

Speaker | Tomasz Cieślak (t.cieslak@impan.pl),
Polish Academy of Sciences, Warsaw, Poland

Title | **Representation of weak solutions to Hunter-Saxton and Camassa-Holm equations along characteristics and consequences**

Abstract | I will review the recent results yielding the solutions of open problems in Hunter-Saxton and Camassa-Holm equations. They are based on Dafermos' idea of generalized characteristics being very useful in the context of continuous weak solutions, which is the case of H-s and C-H systems. In a common paper with G. Jamroz we utilized it to show the Zhang/Zheng conjecture stating that dissipative (unique) solutions of H-S are the only ones to satisfy the maximal rate of dissipation of energy. I will try to explain the main ideas of our proof. I will also mention about the recent result of Jamroz showing that dissipative solutions of C-H are indeed unique (this was an open problem for several years.

References:

[1] T. Cieslak, G. Jamroz. Maximal dissipation in Hunter-Saxton equation for bounded energy initial data. Adv. Math. 290 (2016), 590–613.

[2] G. Jamroz. On uniqueness of dissipative solutions of the Camassa-Holm equation. *arXiv:1611.00333* (2016).

Speaker | Elisa Davoli (elisa.davoli@univie.ac.at),
University of Vienna, Vienna, Austria

Title | **Dynamic perfect plasticity as a convex minimization**

Abstract We present a novel approximation of solutions to the equations of dynamic linearized perfect plasticity, based on a global variational formulation of the problem by means of the Weighted-Inertia-Dissipation-Energy (WIDE) approach. Solutions to the system of dynamic Prandtl-Reuss perfect plasticity are identified as limit of minimizers of parameter-dependent energy functionals evaluated on trajectories (the WIDE functionals). Compactness is achieved by means of time-discretization, uniform energy estimate on minimizers of discretized WIDE-functionals, and passage to the limit in a parameter-dependent energy inequality. This is a joint work with Ulisse Stefanelli.

Speaker Antonio Gaudiello (gaudiell@unina.it),
Universitá degli Studi di Cassino e del Lazio Meridionale, Cassino, Italy

Title **Homogenization of the brush problem with a source term in L^1**

Abstract We consider a domain which has the form of a brush in 3D or the form of a comb in 2D, i.e. an open set which is composed of cylindrical vertical teeth distributed over a fixed basis. All the teeth have a similar fixed height; their cross sections can vary from one teeth to another one and are not supposed to be smooth; moreover the teeth can be adjacent, i.e. they can share parts of their boundaries. The diameter of every tooth is supposed to be less than or equal to epsilon, and the asymptotic volume fraction of the teeth (as epsilon tends to zero) is supposed to be bounded from below away from zero, but no periodicity is assumed on the distribution of the teeth.
In this domain we study the asymptotic behavior, as epsilon tends to zero, of the solution of a second order elliptic equation with a zeroth order term which is bounded from below away from zero, when the homogeneous Neumann boundary condition is imposed on the whole of the boundary. First, we revisit the problem where the source term belongs to L^2. This is a classical problem, but our homogenization result takes place in a geometry which is more general that the ones which have been considered before. Moreover we prove a corrector result which is new.
Then, we study the case where the source term belongs to L^1. Working in the framework of renormalized solutions and introducing a definition of renormalized solutions for degenerate elliptic equations where only the vertical derivative is involved (such a definition is new), we identify the limit problem and prove a corrector result.
This is joint work with Olivier Guibé (Université de Rouen, France) and Francois Murat (CNRS, Université Pierre et Marie Curie, Paris VI, France).

Speaker | Matthieu Hillairet (matthieu.hillairet@univ-montp2.fr),
University of Montpellier, France

Title | **Regularity issues in fluid-structure interactions problems**

Abstract | In order to construct a mathematical existence theory for system of PDEs (or PDE/ODEs) modeling fluid-structure problems, a classical difficulty is that the fluid domain may become singular. For instance, in the case of rigid bodies moving in a fluid, contacts may occur between the bodies, or, in the case of a flow inside an elastic channel, the shape of the channel may not remain smooth or may collapse. In this talk, I will focus on these two examples and discuss the possible loss of regularity of the fluid domain in presence of an incompressible viscous fluid.

Speaker | Peter Hornung (peter.hornung@tu-dresden.de),
Technische Universität Dresden, Dresden, Germany

Title | **Regularity of intrinsically convex $W^{2,2}$ surfaces and a derivation of a homogenized bending theory of convex shells**

Abstract | We show that $W^{2,2}$ isometric immersions of a smooth $2d$ Riemannian manifold with positive Gauss curvature into three dimensional Euclidean space are locally smooth. In particular, the deformed configuration of a convex shell by a $W^{2,2}$ isometric immersion remains convex. Regularity is also useful in the derivation of a model for homogenized convex shells (this includes layered materials) from $3d$ non-linear elasticity. This work is a collaboration with Igor Velčić.

Speaker | Mladen Jurak (jurak@math.hr),
University of Zagreb, Croatia

Title | **Non-isothermal immiscible incompressible two-phase flow in porous media**

Abstract | We present an existence result for a model of non-isothermal immiscible incompressible two-phase flow in porous media. The model consists of the two-phase flow equations coupled with the energy conservation equation with simple thermodynamic assumptions. The existence theorem is proved under standard assumptions for the two-phase flow by use of the non-isothermal global pressure variable.

Speaker | Darko Mitrović (darkom@ac.me),
University of Montenegro, Podgorica, Montenegro

Title | **Existence of solution to dynamics capillarity-diffusion equation**

Abstract In this talk, we prove the existence of a solution to the initial-boundary value problem

$$u_t + \operatorname{div}(f(u)) = A\triangle u + B\partial_t\triangle u, \quad (t,x) \in [0,T) \times \mathbb{R}^d\,, \tag{1}$$

$$u|_{t=0} = u_0(x) \in H_0^3(\mathbb{R}^d)\,, \tag{2}$$

$$u|_{|x|\to\infty} = 0\,. \tag{3}$$

for a C^1-function f, where $A, B, T > 0$ are constants. Such a problem appears in investigations concerning flow in porous media [1], where justification of certain experimental results [2] were obtained by sending the diffusion parameter A and the dynamic capillarity parameter B simultaneously to zero. We stress that problem (1) is not solved except in special situations when the solution is assumed to be of the traveling wave type.

References:

[1] C. J. van Duijn, L. A. Peletier, I. S. Pop. A New Class of Entropy Solutions of the BuckleyLeverett Equation. SIAM J. Math. Anal. 39 (2007), 507–536.

[2] D. A. DiCarlo. Experimental measurements of saturation overshoot on infiltration. Water Resources Research 40 (2004), W04215 (9 pages).

Speaker Paolo Piovano (paolo.piovano@univie.ac.at),
University of Vienna, Austria

Title **Analytical validation of the Young-Dupré law for epitaxially-strained thin films on arbitrarily elastic substrates**

Abstract In this talk the problem of characterizing the equilibrium shape of multilayer islands of thin films deposited on deformable substrates is considered both in the Stranski-Krastanow (SK) and Volmer-Weber (VW) modes. SK differs from VW for the presence of a layer of film atoms that always wets the substrate. We refer to the model introduced in [2] for which the island shape is the results of two competing mechanisms. On one hand, the lattice mismatch between the film and the substrate generate large stresses, and corrugations are created because film atoms move to release the elastic energy. On the other hand, the flat profile is preferable to minimize the surface energy. Regularity results for the film profile are established by extending ideas developed in [1] to include the VW mode and the case of (possibly) different elastic properties for the film and the substrate, and by employing regularity results introduced for transmission problems. As a byproduct of the analysis, geometrical conditions on the wetting angle, i.e., the angle formed by the film profile and the substrate surface, are provided. In particular, the validity of the Young-Dupré equation is assessed for solid islands in the context of linear elasticity.

References:

[1] I. Fonseca, N. Fusco, G. Leoni, M. Morini. Equilibrium configurations of epitaxially strained crystalline films: existence and regularity results. Arch. Ration. Mech. Anal. 186 (2007), 477–537.

[2] B. J. Spencer. Asymptotic derivation of the glued-wetting-layer model and the contact-angle condition for Stranski-Krastanow islands. Phys. Rev. B 59 (1999), 2011–2017.

Speaker | Siniša Slijepčević (slijepce@math.hr),
University of Zagreb, Croatia

Title | **Energy methods in fluid mechanics and new a-priori bounds for the Navier-Stokes equation on a strip**

Abstract | The energy method is a standard tool in obtaining a-priori bounds for evolutionary PDE. In this talk we outline some new techniques for obtaining uniformly local bounds and stability results, by understanding local relationship of energy, energy flux and energy dissipation. As an example, we study the incompressible Navier-Stokes equations in the two-dimensional strip $\mathbb{R} \times [0, L]$, with periodic boundary conditions and no exterior forcing. If the initial velocity is bounded, we prove that the solution remains uniformly bounded for all times, and that the vorticity distribution converges to zero. We deduce that, after a transient period, a laminar regime emerges in which the solution rapidly converges to a shear flow governed by the one-dimensional heat equation. The approach yields explicit estimates on the size of the solution and the lifetime of the turbulent period in terms of the initial Reynolds number. This is a joint work with Thierry Gallay.

Speaker | Barbara Zwicknagl (barbara.zwicknagl@mathematik.uni-wuerzburg.de),
University of Würzburg, Germany

Title | **Low-energy martensitic inclusions**

Abstract | In this talk, I will discuss recent analytical results on pattern formation in shape memory alloys. We will consider the minimal energy that is necessary to build a martensitic nucleus in an austenitic matrix. This energy barrier is typically modeled by (possibly singularly perturbed) nonconvex elasticity functionals. In this talk, I will discuss recent results on the resulting variational problems, including microstructures in the low volume fraction limit and stress-free inclusions. This talk is partly based on joint works with S. Conti, J. Diermeier, D. Melching, A. Rueland and C. Zillinger.

Posters

Authors Matteo Caggio (caggio@math.cas.cz),
Czech Academy of Sciences, Prague, Czech Republic

Title **Regularity criteria for the Navier-Stokes equations based on one component of velocity**

Abstract We study the regularity criteria for the incompressible Navier-Stokes equations in the whole space $\mathbb{R}^3$ based on one velocity component, namely u_3, ∇u_3 and $\nabla^2 u_3$. We use a generalization of the Troisi inequality and anisotropic Lebesgue spaces and prove, for example, that the condition $\nabla u_3 \in L^\beta(0,T;L^p)$, where $2/\beta + 3/p = 7/4 + 1/(2p)$ and $p \in (2,\infty]$, yields the regularity of u on $(0,T]$.

Authors Andrijana Ćurković (andrijana@pmfst.hr),
University of Split, Croatia,
Eduard Marušić-Paloka (emarusic@math.hr),
University of Zagreb, Croatia

Title **Asymptotic analysis of the interaction between a thin fluid layer and an elastic plate**

Abstract A non stationary flow in a thin channel with an elastic wall is considered. The flow is modelled by Stokes equation for the incompressible fluid. Due to the small displacement od the plate, deformation of the fluid domain is neglected. The fluid-plate interaction problem is studied in the limit when the thickness of the channel tends to zero. Non-standard sixth order equation for the plate displacement is obtained. Results on existence, uniqueness and regularity of the solution of the effective equation are given.
Acknowledgments. Research supported by the Croatian Science Foundation under the project no. 3955.

Authors Ivan Dražić (idrazic@riteh.hr) ,
Loredana Simčić (lsimcic@riteh.hr),
University of Rijeka, Croatia

Title **Compressible viscous and heat-conducting micropolar fluid model with spherical and cylindrical symmetry**

Abstract We consider nonstationary 3-D flow of a compressible viscous and heat-conducting micropolar fluid which is in the thermodynamical sense perfect and polytropic. We analyze the problem on two domains. The first domain is bounded with two concentric spheres and the second one with two coaxial cylinders that present solid thermo-insulated walls. Therefore we assume the spherical symmetry of the solution in the first case, and cylindrical symmetry of the solution in the second case. In this work we present the existence and uniqueness theorems for corresponding problems with homogeneous boundary data for velocity, microrotation and heat flux.

Authors Marija Galić (marijag5@math.hr),
University of Zagreb, Croatia

Title **Analysis of the nonlinear 3D fluid-stent-shell interaction problem**

Abstract We consider a nonlinear moving boundary fluid-structure interaction problem in which the fluid is modeled with 3D incompressible Navier-Stokes equations. The structure is modeled as a 2D shell coupled with 1D hyperbolic net. The motivation comes from studying blood flow through a compliant vessel treated with vascular stents. We formulate this 3D FSI problem and give the main steps in proving the existence of the weak solution by using an operator splitting approach in combination with Arbitrary Lagrangian Eulerian mapping. This is a joint work with Boris Muha.

Authors Petar Kunštek (petar@math.hr),
Marko Vrdoljak (marko@math.hr),
University of Zagreb, Croatia

Title **Classical optimal design on annuli**

Abstract We consider the conductivity problem in an annulus $\Omega \subseteq \mathbf{R}^d$:

$$-\mathrm{div}(\mathbf{A}\nabla u) = 1\,,$$
$$u \in \mathrm{H}^1_0(\Omega)\,,$$

where the conductivity matrix $\mathbf{A}$ is of the form $\mathbf{A} = \chi\alpha\mathbf{I} + (1-\chi)\beta\mathbf{I}$, with a characteristic function χ representing the region occupied by the first phase. The optimal design problem deals with maximization of the energy functional $I(\chi) = \int_\Omega fu\,d\mathbf{x}$, over the set of all measurable characteristic functions χ satisfying the condition $\int_\Omega \chi\,d\mathbf{x} = q_\alpha$. In elasticity, the problem models the maximization of the torsional rigidity of a cylindrical rod with annular cross section made of two homogeneously distributed isotropic elastic materials. By analysing the optimality conditions, we are able to show that in the case of annulus, the solution is unique, classical and radial. Depending on the amounts of given materials, we find two possible optimal configurations. If the amount of the first phase is less than some critical value, then the better conductor should be placed in an outer annulus. Otherwise, the optimal configuration consists of an annulus with the better conductor, surrounded by two annuli of the worse conductor.

The second author was supported by the Croatian Science Foundation under the project 9780 WeConMApp.

Authors Erika Maringová (maringova@karlin.mff.cuni.cz),
Charles University in Prague, Czech Republic

Title **On a Navier-Stokes-Fourier-like system capturing a transition between viscid and inviscid fluid regimes and between no slip and perfect slip boundary conditions**

Abstract A Navier-Stokes-Fourier system for an incompressible fluid is considered. We relate the deviatoric part of the Cauchy stress tensor with the symmetric gradient via a maximal monotone 2-graph that is continuously parametrized by the temperature. As such, the inspected fluid may exhibit simultaneously Bingham plasticity for one value of the temperature and the Newtonian nature for another. At the same time, we regard a generalized threshold slip on the boundary that also may change continuously with the temperature. All material coefficients like dynamic viscosity, friction or activation coefficients are assumed to be temperature-dependent. We develop the large-data and long time existence analysis for the weak solutions to the problem. This is a joint work with J. Žabenský .

Authors | Marin Mišur (mmisur@math.hr),
University of Zagreb, Croatia

Title | **Some new applications of microlocal defect functionals**

Abstract | We introduce defect functionals adapted to equations which change type. We apply developed tools to prove a velocity averaging result for degenerate parabolic transport equation and to obtain existence of solutions to a Cauchy problem for nonlinear degenerate parabolic equation with discontinuous flux.
This is a work in progress with Marko Erceg and Darko Mitrović.

Authors | Marija Prša (marija.prsa@grf.hr),
University of Zagreb, Croatia

Title | **Heat conduction problem in a dilated pipe – uniqueness and existence result**

Abstract | We study the heat conduction through a pipe filled with incompressible viscous fluid. The goal of this work is to take into account the effects of the pipe's dilatation due to the heating. In view of that, we assume that the longitudinal dilatation of the pipe is described by a linear heat expansion law. We prove the existence and uniqueness theorems for the corresponding boundary-value problem. The main difficulty comes from the fact that the flow domain changes depending on the solution of the heat equation leading to a non-standard coupled governing problem. This is a joint work with Eduard Marušić-Paloka and Igor Pažanin.

Authors | Marko Radulović (mradul@math.hr),
University of Zagreb, Croatia

Title | **Flow of a micropolar fluid through a channel with small boundary perturbation**

Abstract | This is a joint work with Prof. Igor Pažanin and Prof. Eduard Marušić-Paloka. We investigate the effects of small boundary perturbations on the flow of an incompressible micropolar fluid. Using formal asymptotic analysis with respect to ϵ, we derive the effective model in the form of explicit formulae for the velocity, pressure and microroration. The asymptotic solution acknowledges the effects of the boundary perturbation and the micropolar nature of the fluid which is illustrated with some numerical examples.

Authors | Michail Turbin (mrmike@mail.ru),
Voronezh State University, Russia

Title | **Existence of trajectory and global attractors for Bingham fluid on the 3D torus**

Abstract | We consider the qualitative dynamics of weak solutions in the model of Bingham fluid motion on 3D torus using the theory of trajectory and global attractors of trajectory spaces. We establish the existence of weak solutions for this model. For this purpose we demonstrate the solvability of an approximating problem, using some a priori estimates and the topological degree theory. Then the convergence (in some generalized sense) of solutions of approximating problems to a solution of the given problem is proved. Then we define a family of trajectory spaces, introduce the notions of a trajectory and global attractors and prove the existence of these attractors.

Authors | Andrey Zvyagin (zvyagin.a@mail.ru),
Voronezh State University, Russia

Title | **Thermoviscoelasticity problem for Voigt model**

Abstract | Let $\Omega \subset \mathbb{R}^n$, $n = 2, 3$. We consider the following initial boundary-value problem

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - \mu_0 \Delta v - 2\mathrm{Div}\ (\mu(\theta)\mathcal{E}(v)) - \varkappa \frac{\partial \Delta v}{\partial t} + \mathrm{grad}\ p = f; \tag{1}$$

$$\mathrm{div}\ v = 0; \tag{2}$$

$$\frac{\partial \theta}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial \theta}{\partial x_i} - \chi \Delta\, \theta = 2\widetilde{\mu}(\theta)\mathcal{E}(v) : \mathcal{E}(v) + 2\varkappa \frac{\partial \mathcal{E}(v)}{\partial t} : \mathcal{E}(v) + g; \tag{3}$$

$$v|_{t=0} = v_0; \qquad v|_{[0,T]\times\partial\Omega} = 0; \tag{4}$$

$$\theta|_{t=0} = \theta_0; \qquad \theta|_{[0,T]\times\partial\Omega} = 0. \tag{5}$$

Here $v(x,t) = (v_1, ..., v_n)$ is the velocity vector-function, $\theta(t,x)$ is the temperature function, $p(x,t)$ is the fluid pressure, $f(x,t)$ is the density of external forces, g is the external heat source, $\mathcal{E}(v) = (\mathcal{E}_{ij}(v))_{j=1,...,n}^{i=1,...,n}$, $\mathcal{E}_{ij}(v) = \frac{1}{2}(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})$ is the strain velocity tensor, $\chi > 0$ is the coefficient of thermal conductivity, $\varkappa > 0$ is the time of retardation (delay), $\mu_0 > 0$ is the initial viscosity of a fluid, $\mu(s)$ is the viscosity of a fluid and $\widetilde{\mu}(s) = \mu_0(s) + \mu(s)$.
This model of fluid motion (1)–(5) describes the motion of a linearly elastic-delayed Voigt fluid with the viscosity depending on a temperature. The temperature addition results to the appearance of energy balance equation (3). This significantly complicates the investigated problem. Namely, we obtain a parabolic equation with coefficients from Sobolev spaces and right-hand side from $L_1(0,T;L_1(\Omega))$.
In the present talk we establish the existence of weak solutions of the initial boundary-value problem (1)–(5).

Authors | Victor Zvyagin, (zvg_vsu@mail.ru),
Voronezh State University, Russia

Title | **Solvability of non-Newtonian hydrodynamics model with memory**

Abstract Consider the motion of a fluid that occupies a bounded domain $\Omega \subset \mathbb{R}^N$, $N = 2, 3$, with locally Lipschitz boundary $\partial\Omega$ on the time interval $[0, T]$, $T > 0$. The equation of motion in the Cauchy form is

$$\rho(\partial v/\partial t + \sum_{i=1}^{N} v_i\, \partial v/\partial x_i) = -\nabla\, p + \mathrm{Div}\, \sigma + \rho f, \quad (t, x) \in Q_T\,, \tag{1}$$

where $Q_T = [0, T] \times \Omega$, $v = (v_1(t, x), \ldots, v_N(t, x))$ is the velocity at the point $x \in \Omega$ at time t; ρ is the fluid density; $p = p(t, x)$ is the pressure of the fluid; $\sigma = \{\sigma_{ij}(t, x)\}_{i,j=1}^{N}$ is deviator of the stress tensor; $f = f(t, x)$ is the density of the external forces acting upon the fluid; $\mathrm{Div}\, \sigma$ is a vector function, the coordinates of which are divergences of lines of the matrix σ. The number of unknowns in the equation (1) exceeds the number of equations and the system is supplemented by the constitutive law (rheological relation) which determines the type of a fluid under consideration.
One of the most well-known non-Newtonian fluids is determined by the rheological relation

$$(1 + \lambda\, d/dt)\sigma = 2\nu(1 + \varkappa\nu^{-1} d/dt)\mathcal{E}(v)$$

where $d/dt = \partial/\partial t + \sum_{i=1}^{N} v_i \partial/\partial x_i$, $\mathcal{E}(v) = \{\mathcal{E}_{ij}\}_{i,j=1}^{N}$, $\mathcal{E}_{ij} = \frac{1}{2}(\partial v_i/\partial x_j + \partial v_j/\partial x_i)$ is the strain velocity tensor and $\lambda, \varkappa, \nu$ are positive constants. Fluids with such rheological relations have been extensively studied in the mid-twentieth century by Jeffreys and Oldroyd.
Integrating the Jeffreys-Oldroyd rheological relation along velocity field v, expressing σ from this relatinship and substituting it in the equation (1) we get for describing of the motion of a fluid the following initial boundary-value problem

$$\partial v/\partial t + \sum_{i=1}^{N} v_i \partial v/\partial x_i - \mu_0 \Delta v - \mu_1 \mathrm{Div} \int_0^t \exp\left((s - t)/\lambda\right) \mathcal{E}(v)(s, z(s; t, x)) ds + \nabla p = f; \tag{2}$$

$$\mathrm{div}\, v(t, x) = 0, \quad (t, x) \in Q_T; \tag{3}$$

$$z(\tau; t, x) = x + \int_t^\tau v(s, z(s; t, x))\, ds, \quad 0 \leqslant t, \tau \leqslant T, \quad x \in \overline{\Omega}; \tag{4}$$

$$v(0, x) = v^0(x),\ x \in \Omega; \quad v(t, x) = 0, \quad (t, x) \in [0, T] \times \partial\Omega \tag{5}$$

with integral term in (2) which takes into account the memory of the system. Here, for simplicity, we assume $\rho = 1$ in rheological relation and $\mu_0 = 2\varkappa$, $\mu_1 = 2(\nu - \varkappa)$.
The system (2)-(5) contains not only the unknown velocity v and pressure p, but also the trajectory $z(\tau; t, x)$, defined by velocity field v.
In the present talk we establish the existence of weak solutions of the initial boundary-value problem for system (2)-(5) with memory, taking into account the background of the motion along the trajectories $z(\tau; t, x)$ of the velocity field v.

List of participants

Ibrahim Aganović (University of Zagreb)
Nenad Antonić (University of Zagreb)
Peter Bella (University of Leipzig)
Adara-Monica Blaga (West University of Timisoara)
Mario Bukal (University of Zagreb)
Marin Bužančić (University of Zagreb)
Matteo Caggio (Czech Academy of Sciences, Prague)
Tomasz Cieślak (Polish Academy of Sciences, Warsaw)
Igor Ciganović (University of Zagreb)
Andrijana Ćurković (University of Split)
Elisa Davoli (University of Vienna)
Ivan Dražić (University of Rijeka)
Marko Erceg (SISSA Trieste)
Tomislav Fratrović (University of Zagreb)
Marija Galić (University of Zagreb)
Antonio Gaudiello (University of Cassino and Southern Lazio)
Gordan Gledec (University of Zagreb)
Matthieu Hillairet (University of Montpellier)
Peter Hornung (TU Dresden)
Ivan Ivec (University of Zagreb)
Mia Jukić (University of Zagreb)
Ansgar Jüngel (Vienna University of Technology)
Mladen Jurak (University of Zagreb)
Mate Kosor (University of Zadar)
Jadranka Kraljević (University of Zagreb)
Petar Kunštek (University of Zagreb)
Martin Lazar (University of Dubrovnik)
Matko Ljulj (University of Zagreb)
Erika Maringová (Charles University in Prague)
Josipa Pina Milišić (University of Zagreb)
Marin Mišur (University of Zagreb)
Darko Mitrović (University of Montenegro)
Boris Muha (University of Zagreb)
Šárka Nečasová (Czech Academy of Sciences, Prague)
Ljudevit Palle (University of Zagreb)
Matthäus Pawelczyk (TU Dresden)
Paolo Piovano (University of Vienna)
Marija Prša (University of Zagreb)
Braslav Rabar (University of Zagreb)
Ivana Radišić (University of Zagreb)
Ana Radošević (University of Zagreb)
Marko Radulović (University of Zagreb)
Annie Raoult (Paris Descartes University)

Petar Rudan (University of Zagreb)
Loredana Simčić (University of Rijeka)
Siniša Slijepčević (University of Zagreb)
Josip Tambača (University of Zagreb)
Michail Turbin (Voronezh State University)
Zvonimir Tutek (University of Zagreb)
Igor Velčić (University of Zagreb)
Anja Vrbaški (University of Zagreb)
Marko Vrdoljak (University of Zagreb)
Andrey Zvyagin (Voronezh State University)
Victor Zvyagin (Voronezh State University)
Barbara Zwicknagl (University of Würzburg)
Darko Žubrinić (University of Zagreb)
Josip Žubrinić (University of Zagreb)

Notes


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