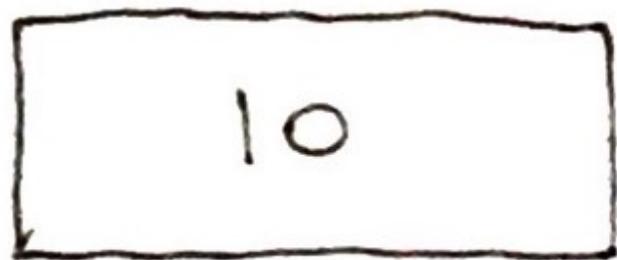
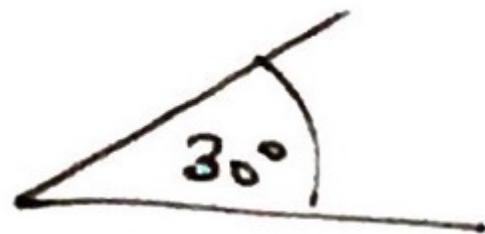
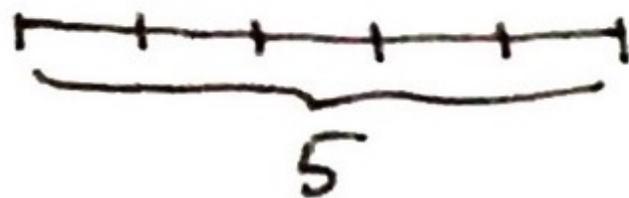


# TOPOLOGIA

by Dario Bojanjac

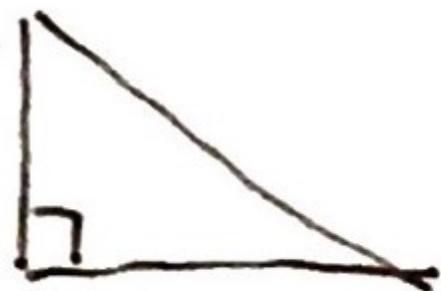
GEOMETRIJA proučava oblike  
ali pazi na **EGZAKTNE** mjere  
udaljenosti, kutove, površine, volumene..

GEOMETRIJA proučava oblike  
ali pazi na **EGZAKTNE** mjere  
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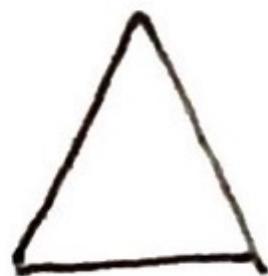


geometrija razlikuje pravokutni  
i jedna kotacni trokut

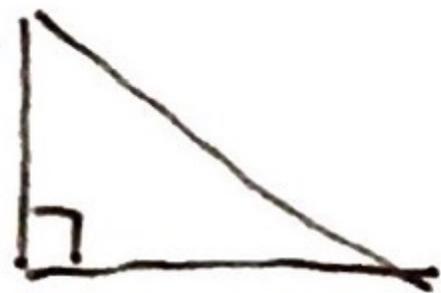
geometrija razlikuje pravokutni  
i jedna kolica ni trokut



$\neq$



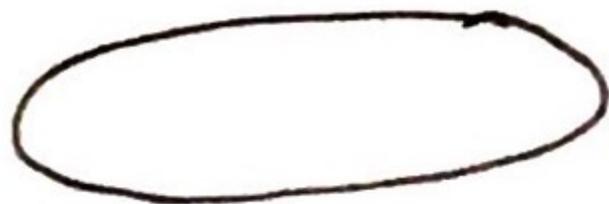
geometrija razlikuje pravokutni  
i jedna kolica ni trokut



$\neq$

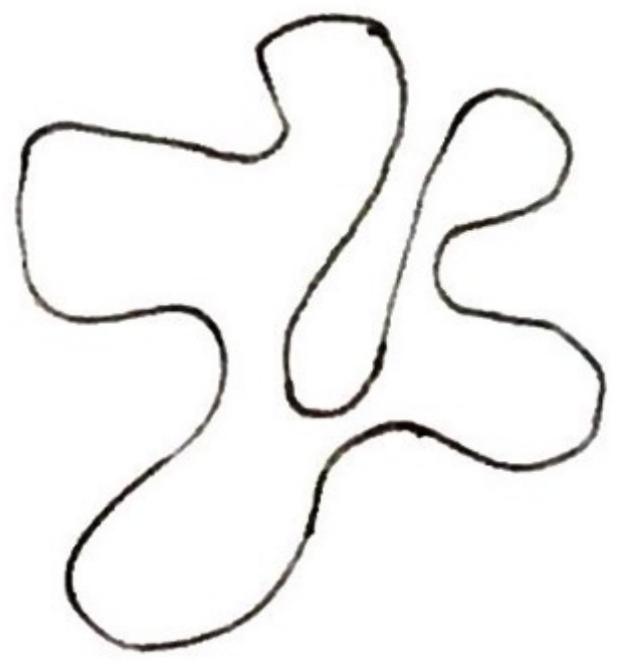
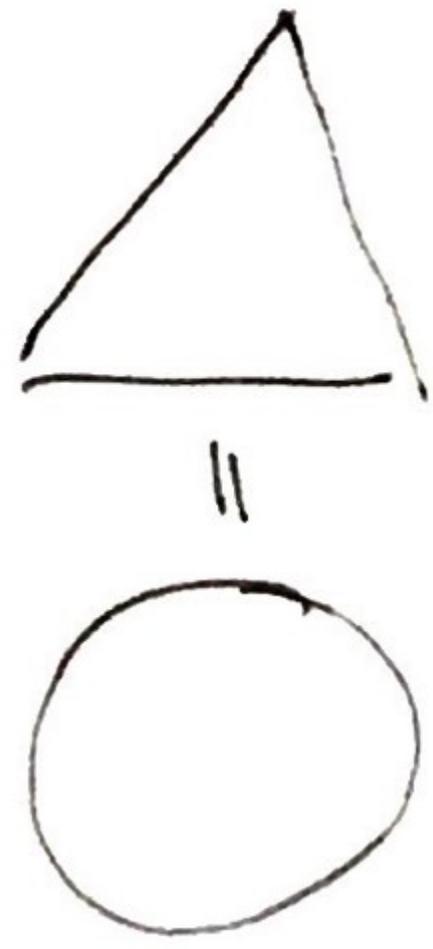
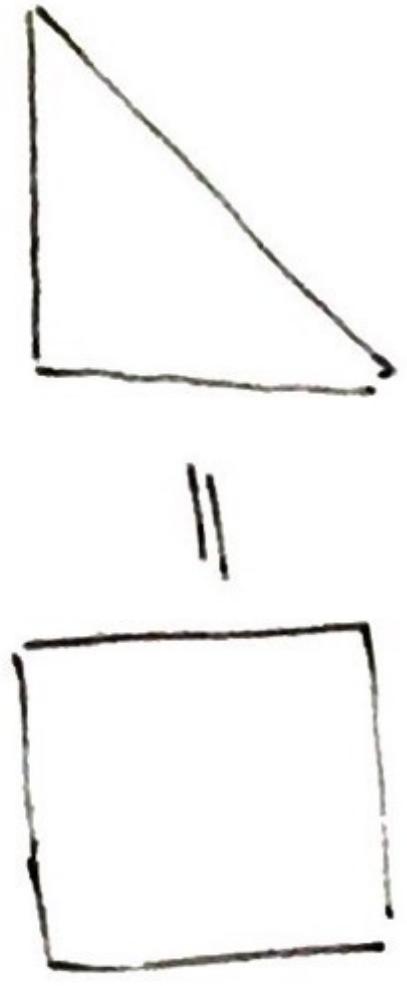


$\neq$



TOPOLOGIJA proučava oblike, ali kao  
da su tijela nastajena od materijala  
koji se može oblikovati

lagano, bez destruktivnih  
metoda (bez trganja rezanja)

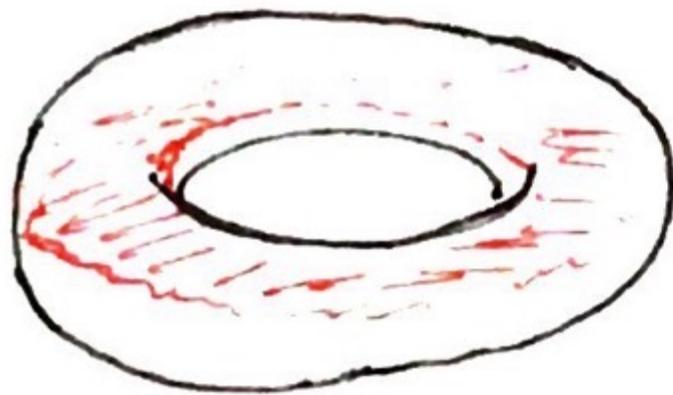


" Topolog ne razlikuje šalicu  
od krafne "

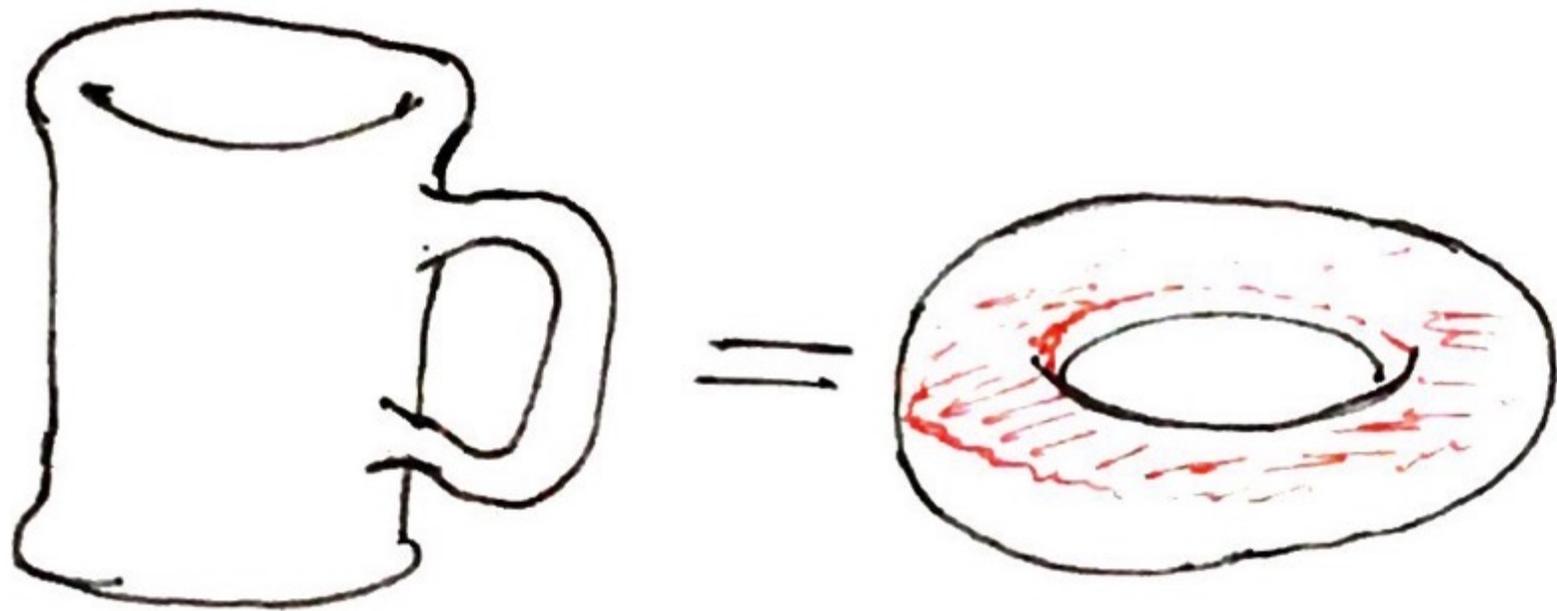
" Topolog ne razlikuje šalicu  
od kvačice "



" Topolog ne razlikuje šalicu  
od krafne "



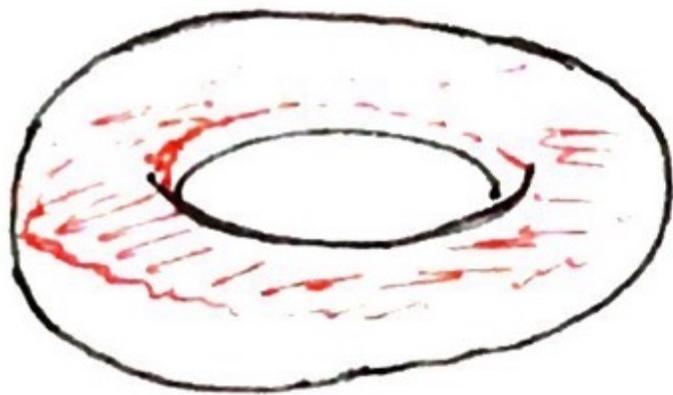
" Topolog ne razlikuje šalicu  
od krafne "



" Topolog ne razlikuje šalicu  
od krafne "



=



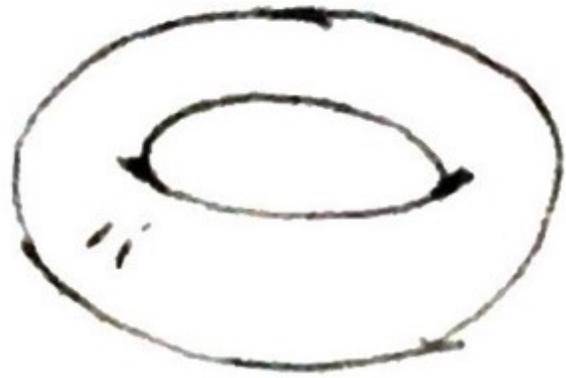
TORUS

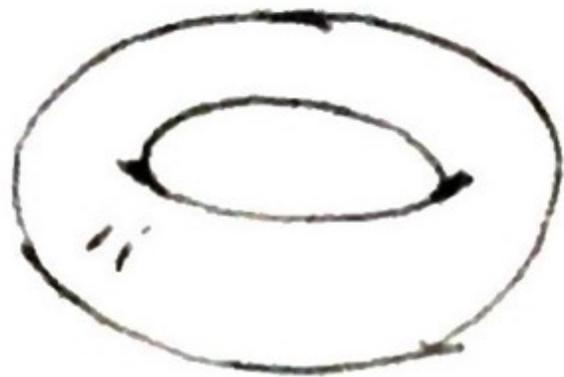
Sve osim ruke možemo stisnuti  
Samo u sebe

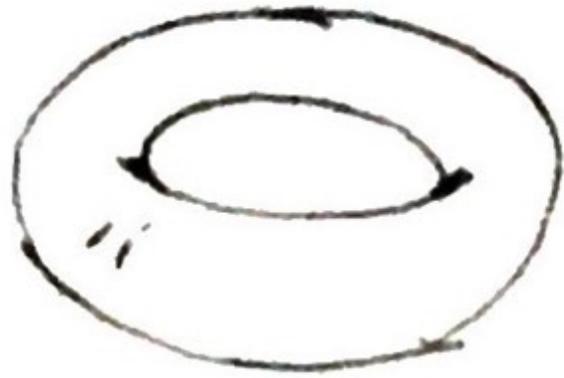
RUČKA JE BITNA!!!

ZASTO?

Topologija promatra "svojstva"  
tijela koja su sačuvana prilikom  
nedestruktivnih "transformacija"

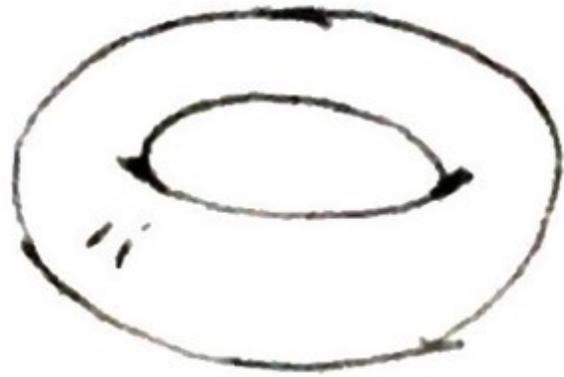




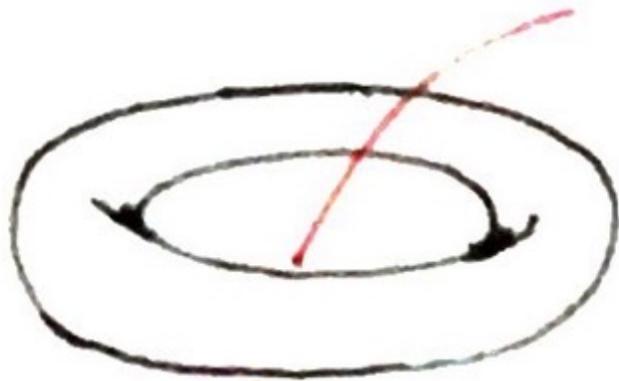


#

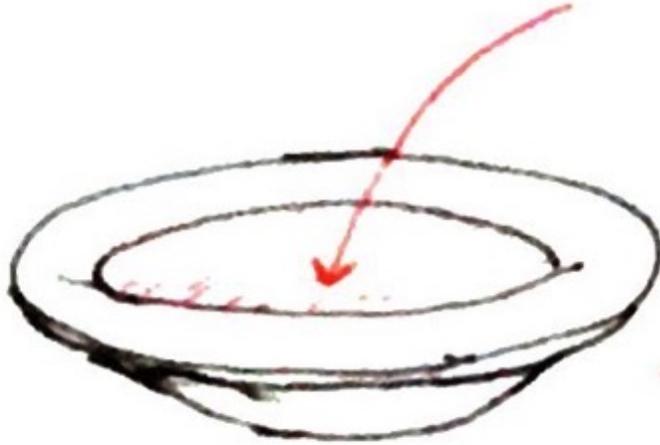




$\neq$



možeme  
proci

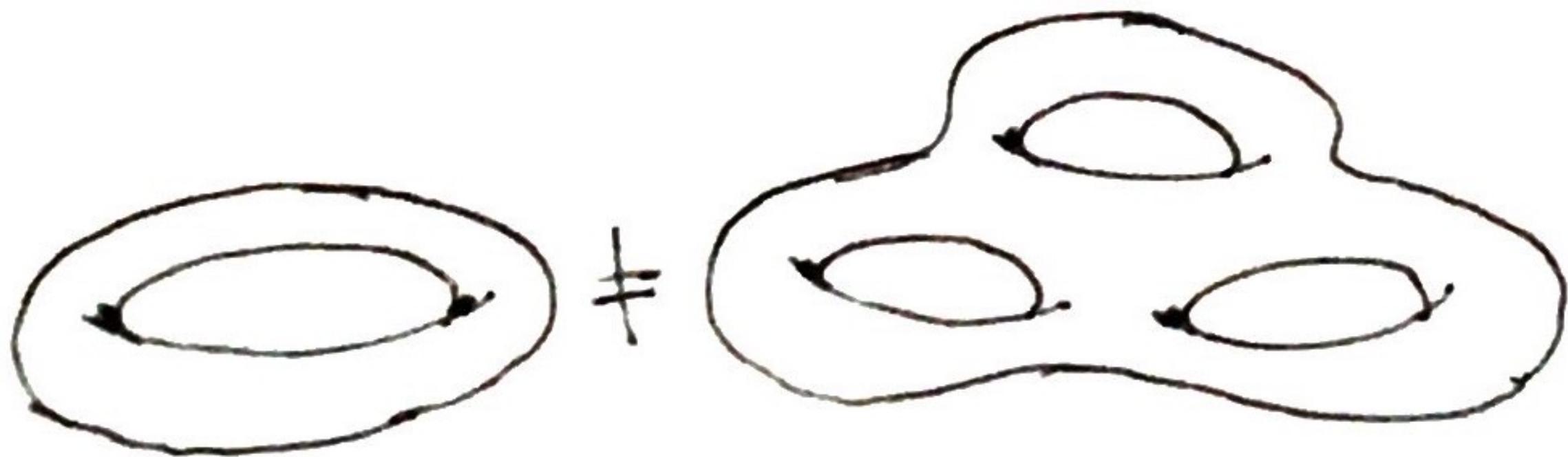


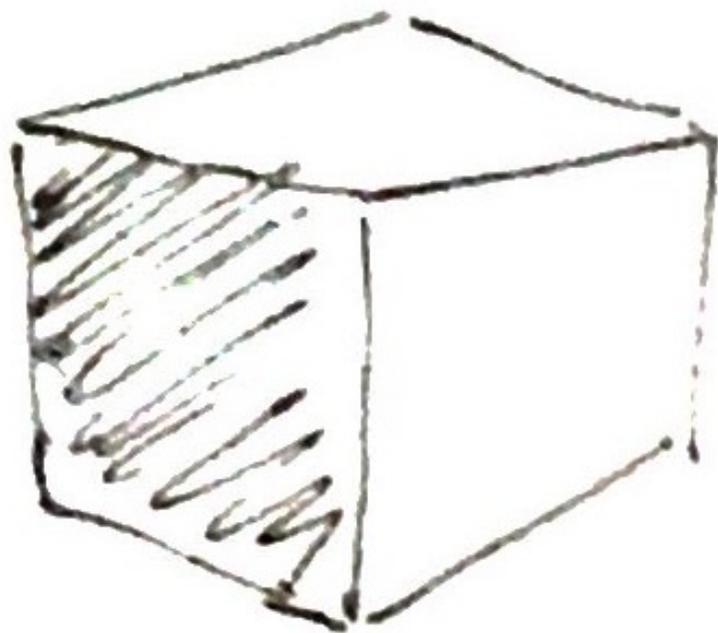
ne možeme  
proci  
TANJUR

Topologija promatra "svojstva"  
tijela koja su sačuvana prilikom  
nedesruktivnih "transformacija"

Topologija promatra "svojstva"  
tijela koja su sačuvana prilikom  
nedestruktivnih "transformacija"

Tanjur moramo probušiti da  
bi u njemu dobili rupu.





Ovalna deformacija (transformacija)

NEMA REZANJA

NEMA LIJEPLJENJA

Ovalva de formacija (transformacija)

NETA REZANTA

NETA LIJEPLJENJA

HOMEO MORFIZAM

(reverzibilna transformacija, ona i  
njen inverz su neprehidne)

Ovalna deformacija (transformacija)

NEMA REZANJA

NEMA LIJEPLJENJA

HOMEO MORFIZAM

(reverzibilna transformacija, ona i  
njen inverz su neprekidne)

HOMEO MORFNO znači TOPOLOŠKI ISTO

Procha je homeomorfna kugli

$\text{Archa}$  je homeomorfa  $\text{Ungli}$

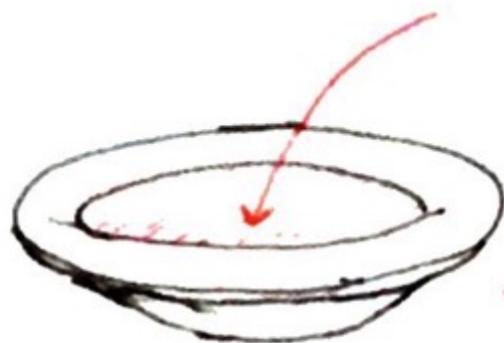
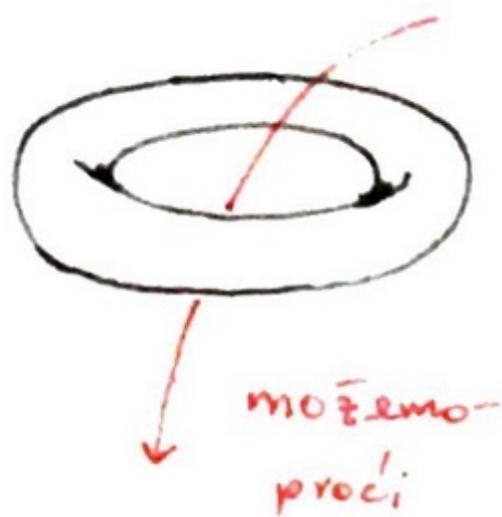
$\text{Kratna}$  je homeomorfa  $\text{Salici}$

koča je homeomorfa kugli

krafna je homeomorfa Sali

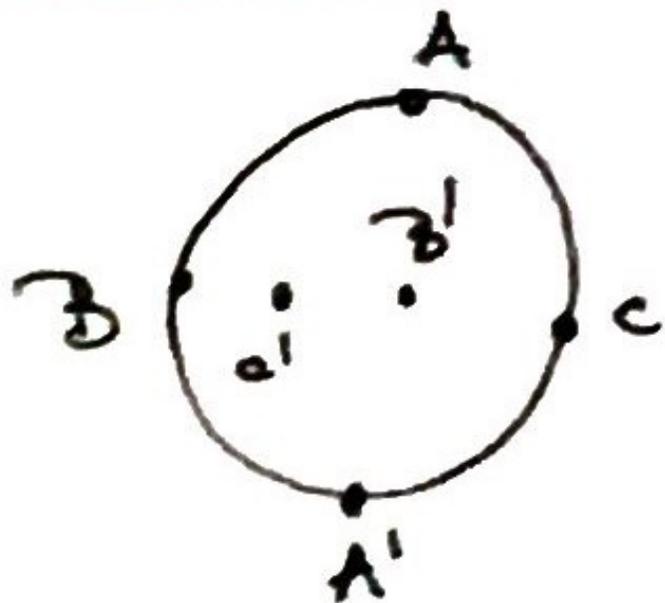
Salica nije homeomorfa torusu

koča je homeomorfna kugli  
krafna je homeomorfna šatici  
šatica nije homeomorfna tanjuru

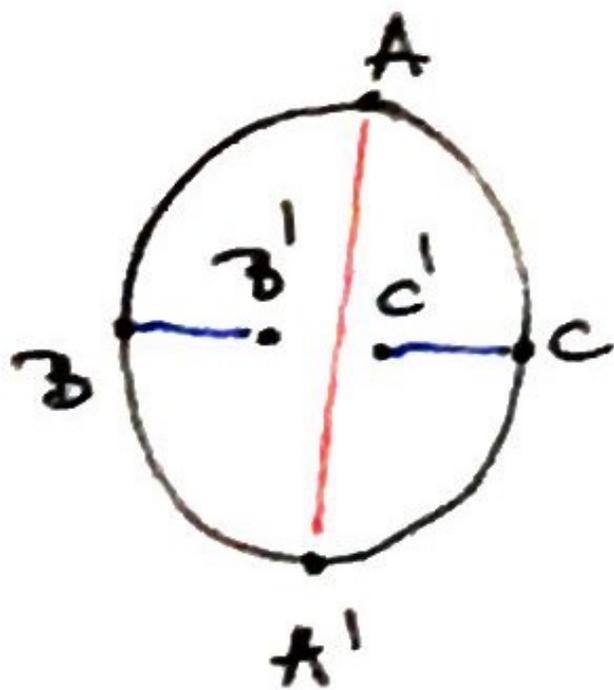
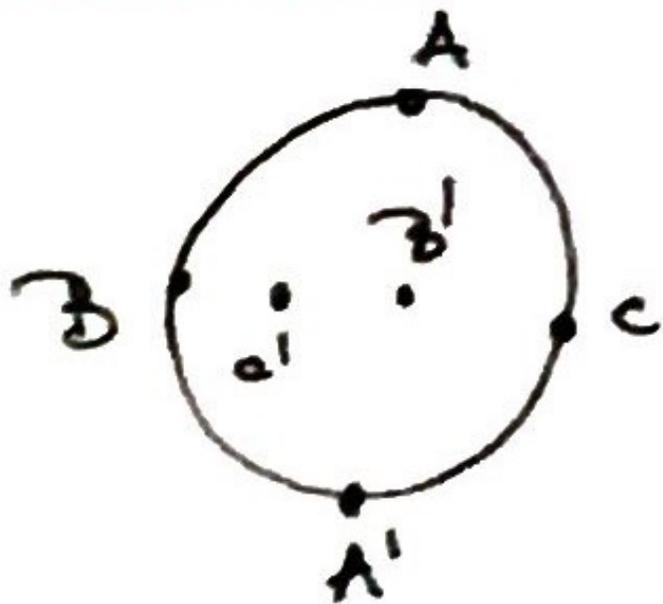


ne možemo  
proći

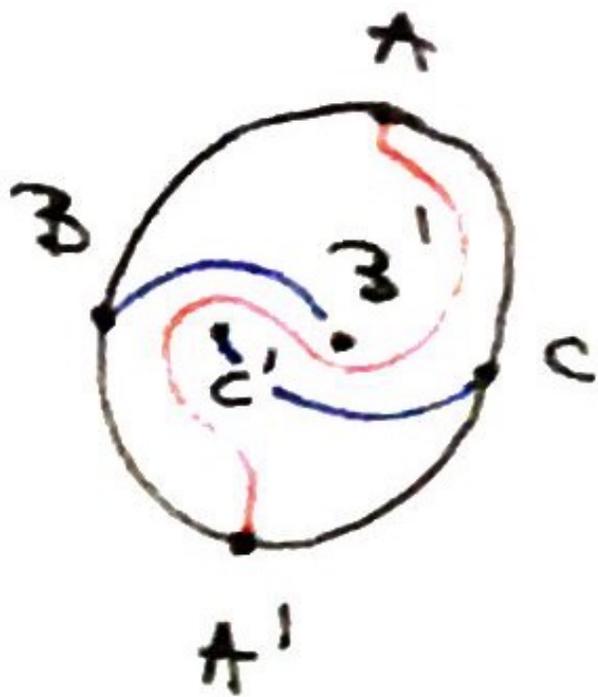
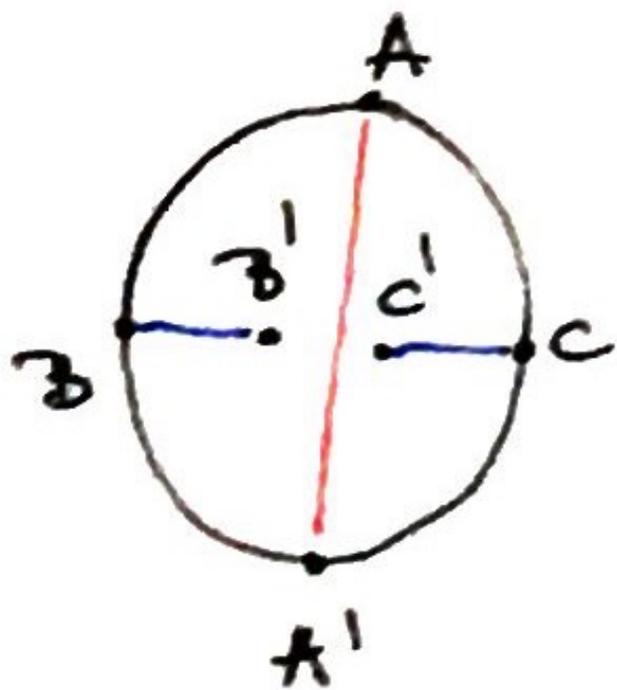
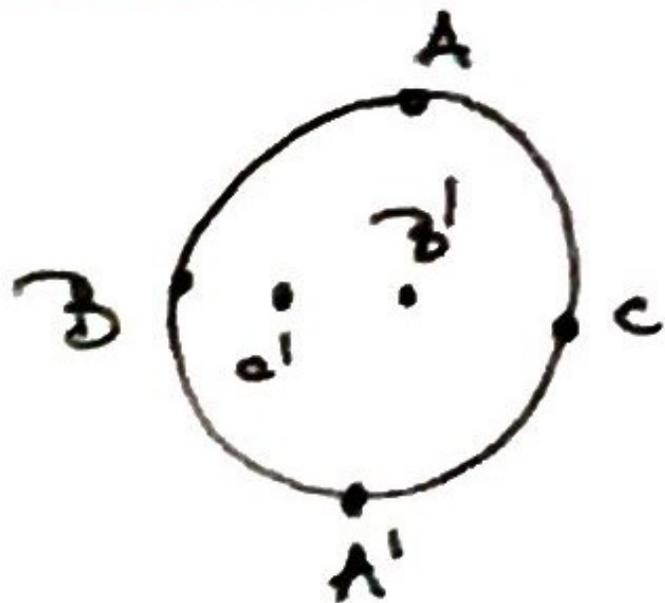
ЗАДАЧА



ЗАДАЧА



ЗАДАЧА



Kako znati koje oblici su homeomorfni?

Kako znati koje oblici su homeomorfni?

Topološke invarijante :-)

Kako znati koje oblici su homeomorfni?

Topološke invarijante :-)

EULEROVA KARAKTERISTIKA

$$\chi = \text{vrhovi} - \text{rubovi} + \text{stranice}$$

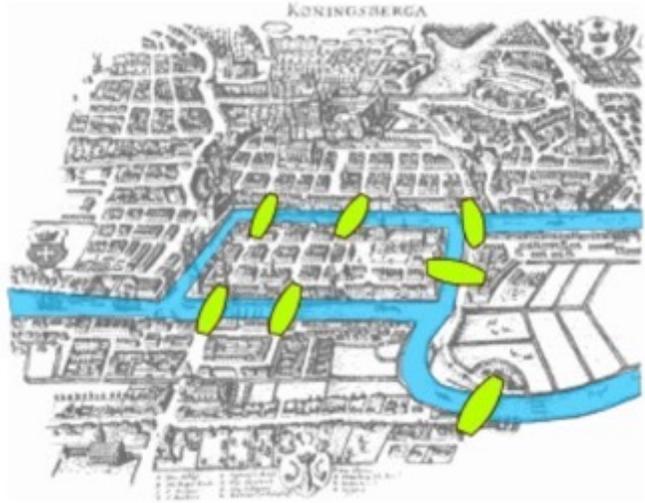


Švicarski matematičar. Jedan od najproduktivnijih znanstvenika u povijesti. Pokrenuo teoriju grafova s pričom o Königsbergškim mostovima.

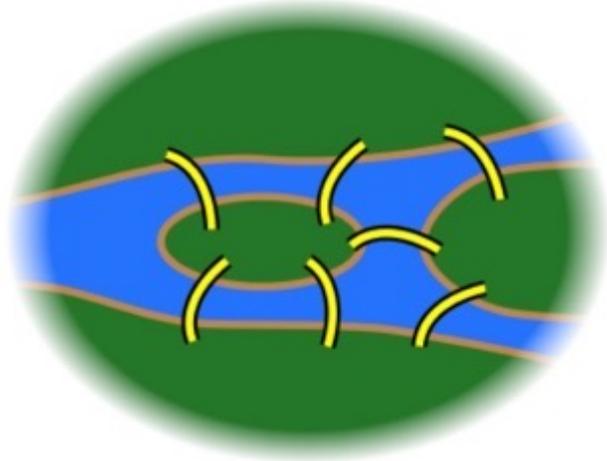
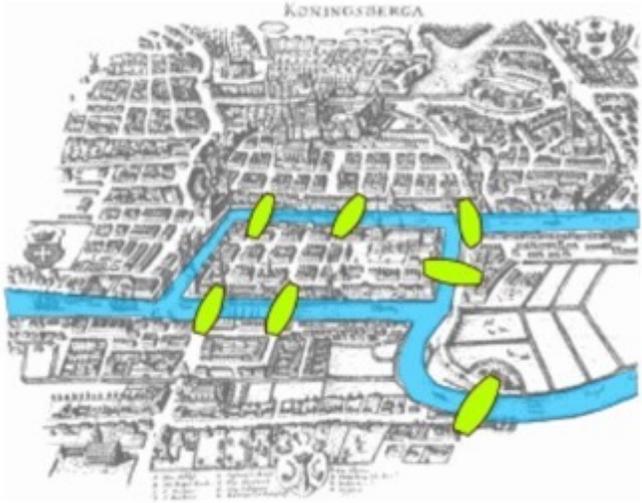


LEONHARD EULER  
15.4.1707. – 18.9.1783.

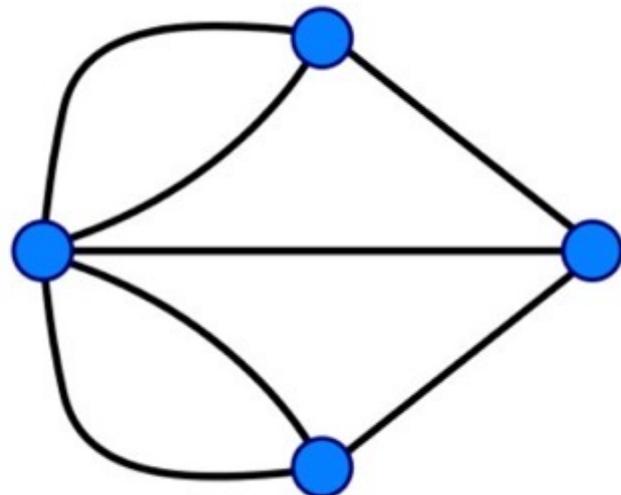
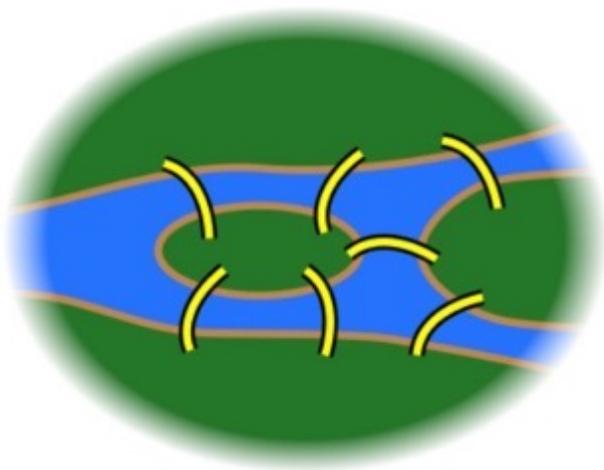
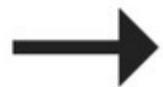
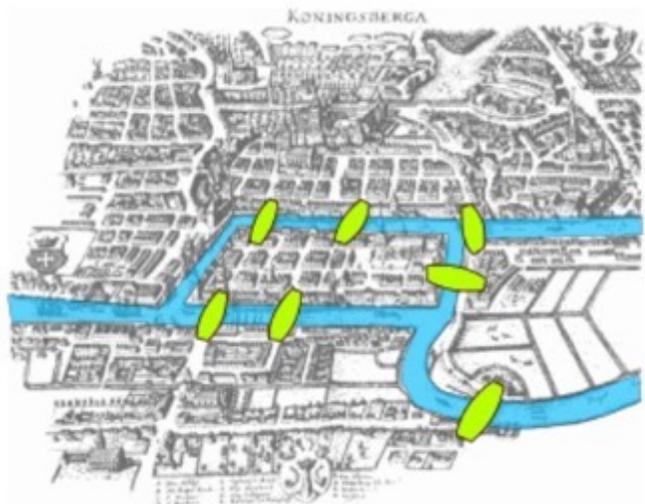
# SEVEN BRIDGES OF KÖNIGSBERG



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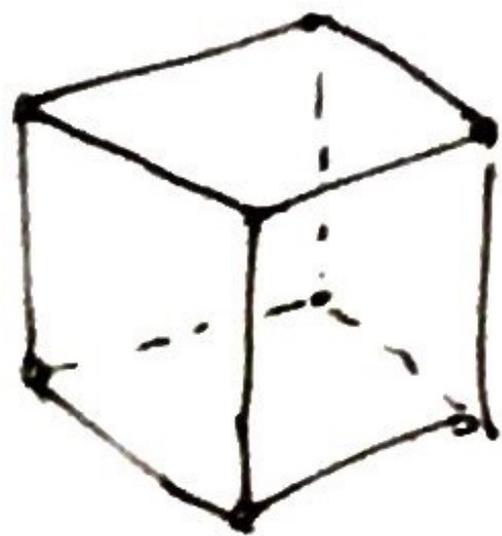


Pokrenno je razvoj  
dviye nove matematične  
grane, TOPOLOGIJU i  
TEORIJU GRAFOVA.



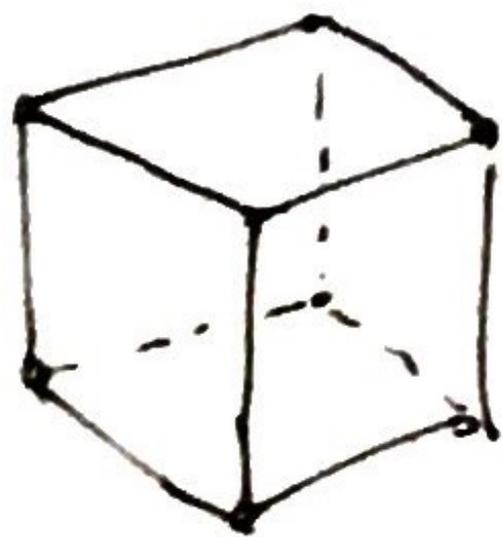
# EULEROVA KARAKTERISTIKA

$$\chi = \text{vrhovi} - \text{rubovi} + \text{stranice}$$



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$$\chi = \text{vrhovi} - \text{rubovi} + \text{stranice}$$



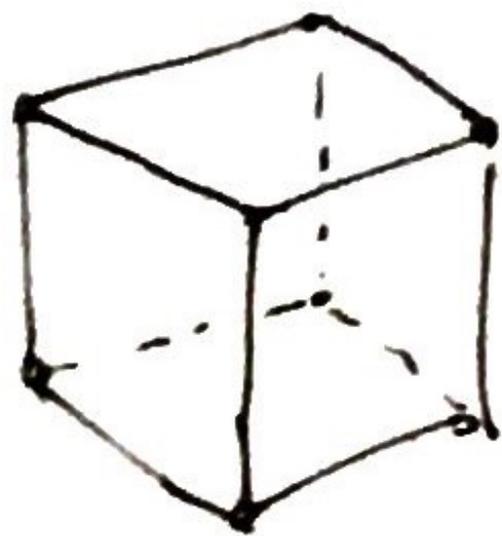
$$\# \text{ vrhova} = 8$$

$$\# \text{ rubova} = 12$$

$$\# \text{ stranice} = 6$$

# EULEROVA KARAKTERISTIKA

$$\chi = \text{vrhovi} - \text{rubovi} + \text{stranice}$$



$$\# \text{ vrhova} = 8$$

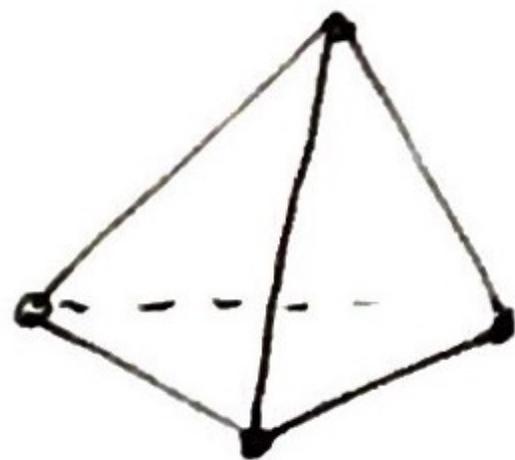
$$\# \text{ rubova} = 12$$

$$\# \text{ stranice} = 6$$

$$\chi = 8 - 12 + 6 = 2$$

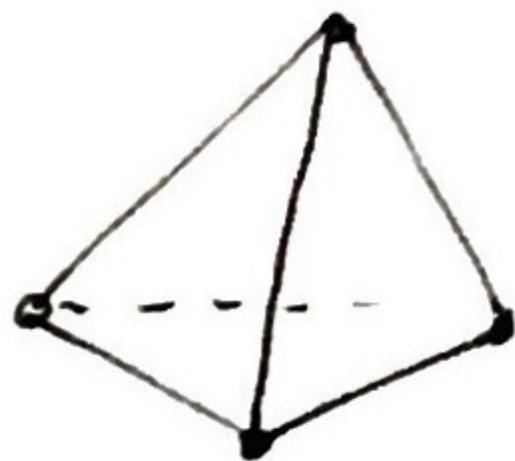
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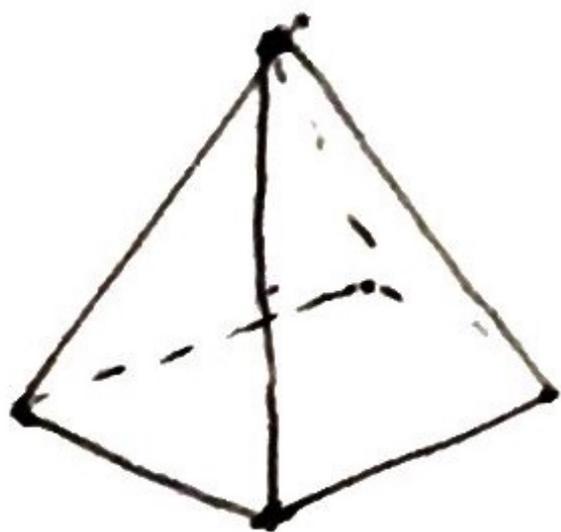


$$\begin{aligned} \# \text{ vrhova} &= 4 \\ \# \text{ rubova} &= 6 \\ \# \text{ stranice} &= 4 \end{aligned}$$

$$\chi = 4 - 6 + 4 = 2$$

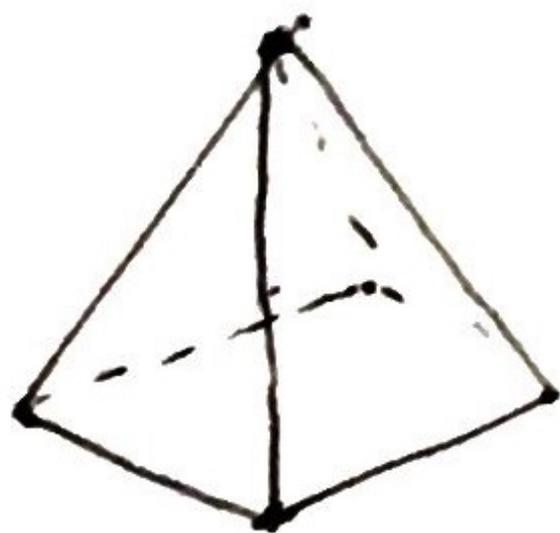
# EULEROVA KARAKTERISTIKA

$$\chi = \text{vrhovi} - \text{rubovi} + \text{stranice}$$



# EULEROVA KARAKTERISTIKA

$$\chi = \text{vrhovi} - \text{rubovi} + \text{stranice}$$



$$\# \text{ vrhova} = 5$$

$$\# \text{ rubova} = 8$$

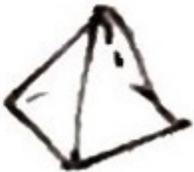
$$\# \text{ stranice} = 5$$

$$\chi = 5 - 8 + 5 = 2$$

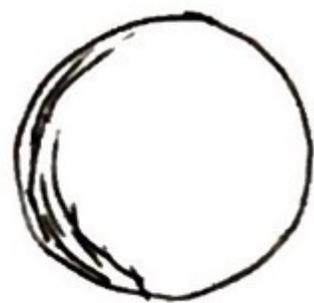
 ,  i  imaju istu Eulerovu  
karakteristiku  $\chi$  ( $\chi = 2$ ).

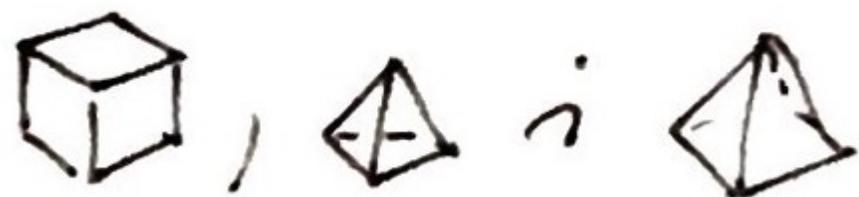
 ,  i  imaju istu Eulerovu  
karakteristiku  $\chi$  ( $\chi=2$ ).

Dvije plohe su **homeomorfne** ako i samo  
ako- imaju istu Eulerovu karakteristiku.

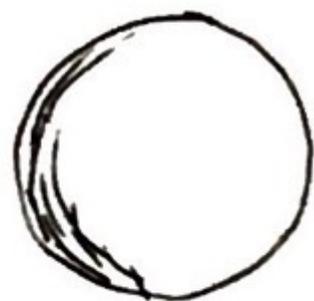
 ,  i  imaju istu Eulerovu  
karakteristiku  $\chi$  ( $\chi=2$ ).

Želimo da su koča i sfera topološki  
invariantne. Što su rubovi, vrhovi i  
stranice kod kugle?



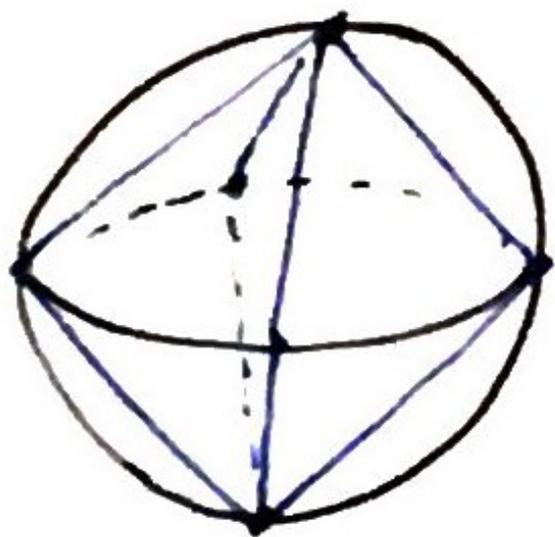
 imaju istu Eulerovu  
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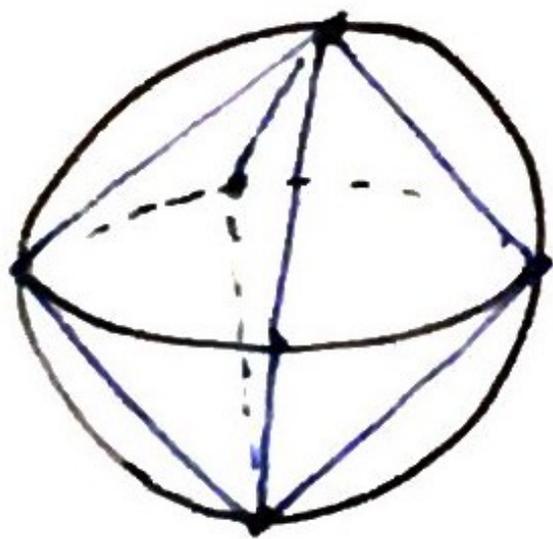
Želimo da su koča i sfera topološki  
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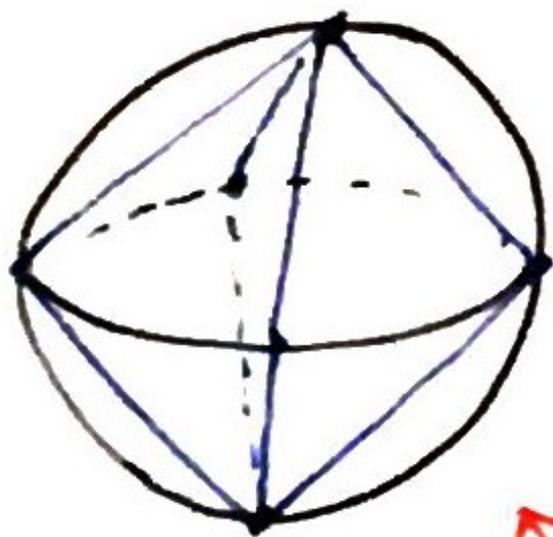
KAKO PRONADI EULEROVU  
KARAKTERISTIKU SFERE?

TRIANGULACIJA



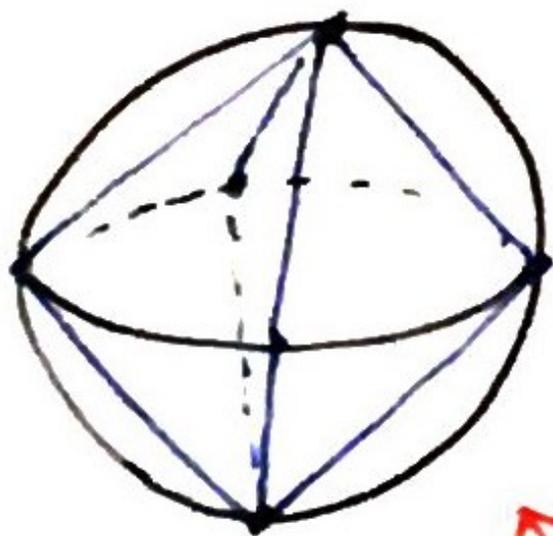


$$\begin{array}{l} \# \text{ vrhova} = 6 \\ \# \text{ rubova} = 12 \\ \# \text{ stranice} = 8 \end{array} \left. \vphantom{\begin{array}{l} \# \text{ vrhova} = 6 \\ \# \text{ rubova} = 12 \\ \# \text{ stranice} = 8 \end{array}} \right\} \chi = 6 - 12 + 8 = 2$$



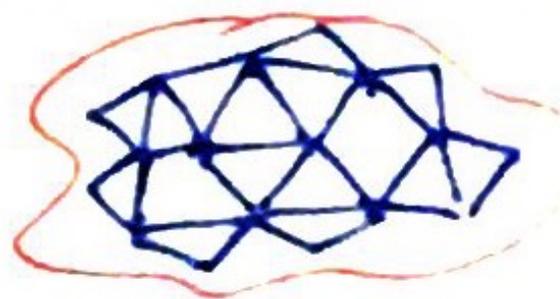
$$\begin{array}{l} \# \text{ vrhova} = 6 \\ \# \text{ rubova} = 12 \\ \# \text{ stranek} = 8 \end{array} \left\{ \begin{array}{l} \chi = 6 - 12 + 8 = 2 \end{array} \right.$$

↑ NITĚ VIŠĚ SFERA :-)



$$\begin{array}{l} \# \text{ vrhova} = 6 \\ \# \text{ rubova} = 12 \\ \# \text{ stranica} = 8 \end{array} \left\{ \chi = 6 - 12 + 8 = 2 \right.$$

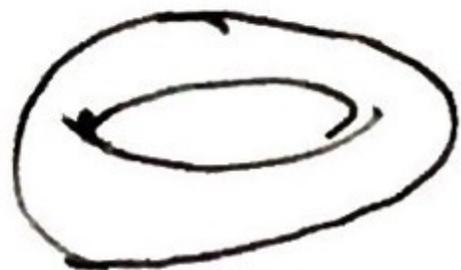
↑ NIJE VIŠE SFERA :-)



TEOREM: Svaka ploha homeomorfnu je:



$$\chi = 2$$



$$\chi = 0$$



$$\chi = -2$$



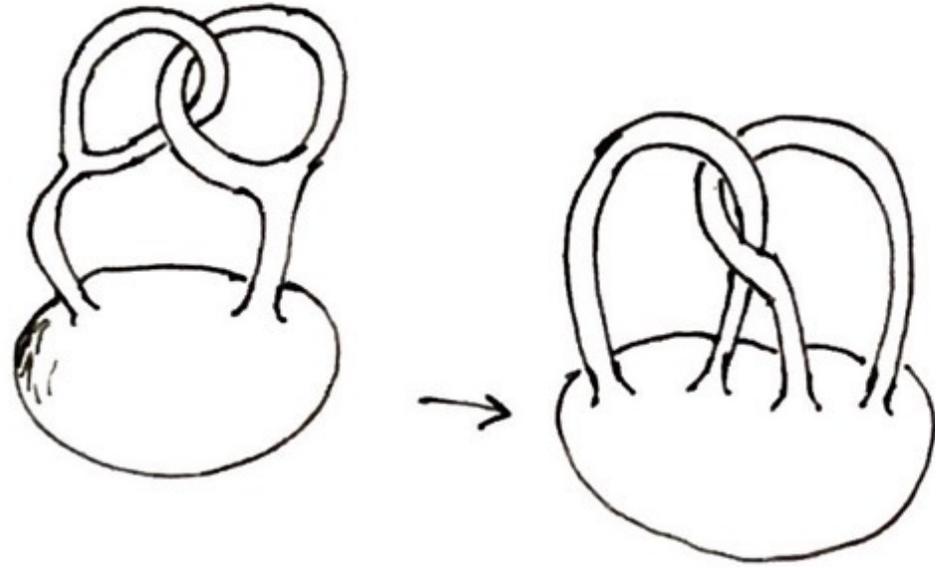
$$\chi = -4$$

...

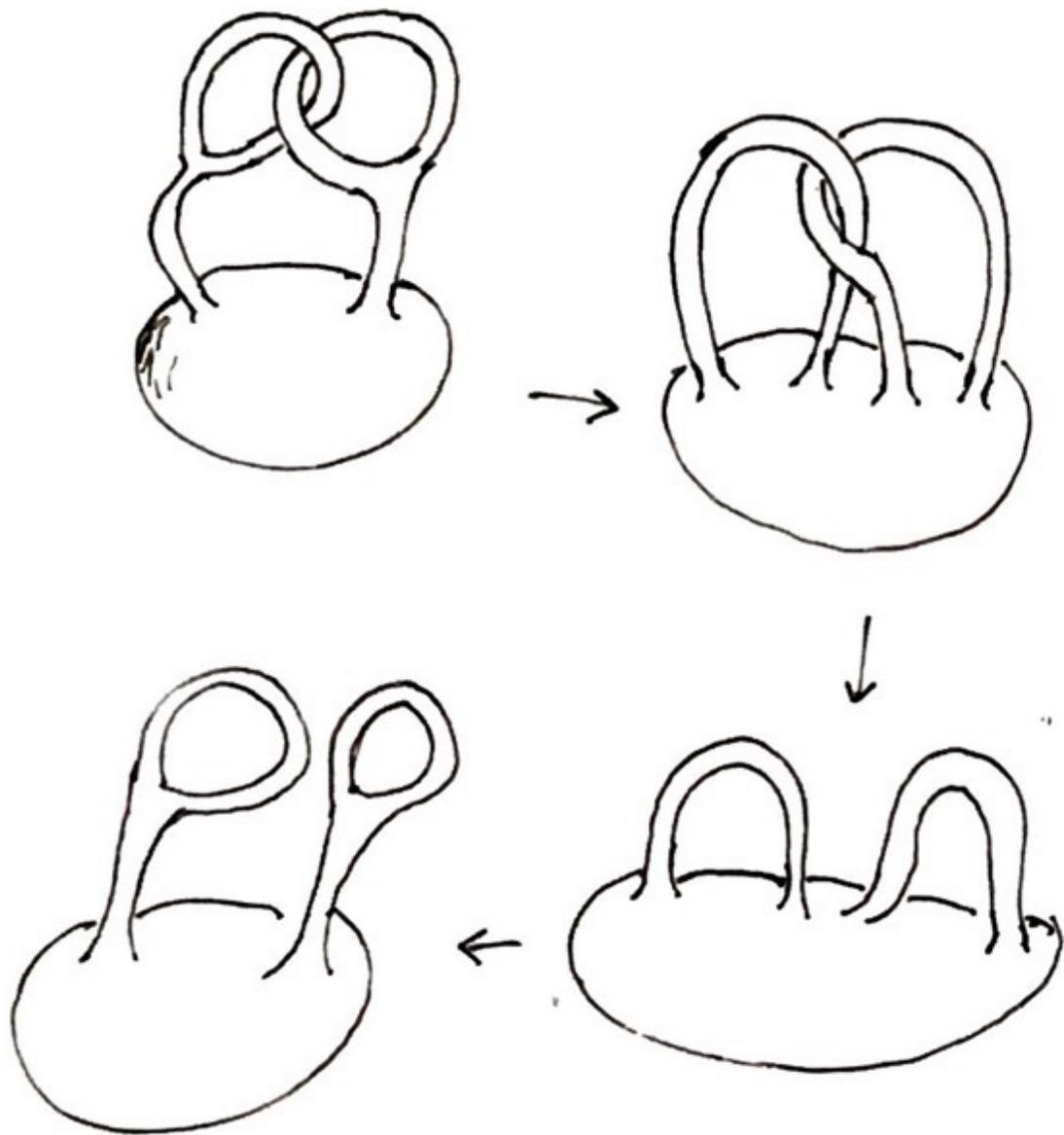
ZADATAK: Čemu je homeomorfno:



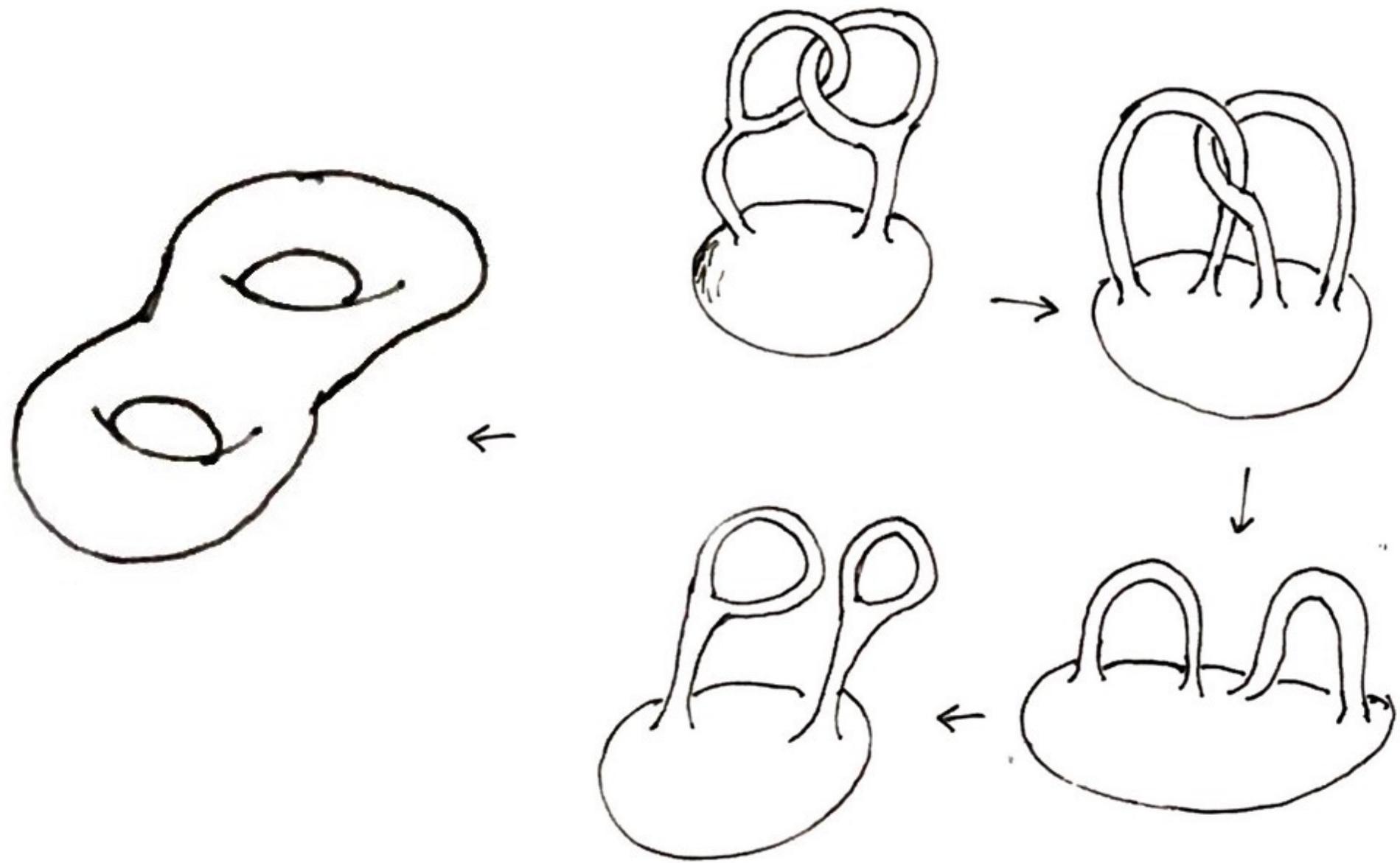
ZADATAK: Čemu je homeomorfno:



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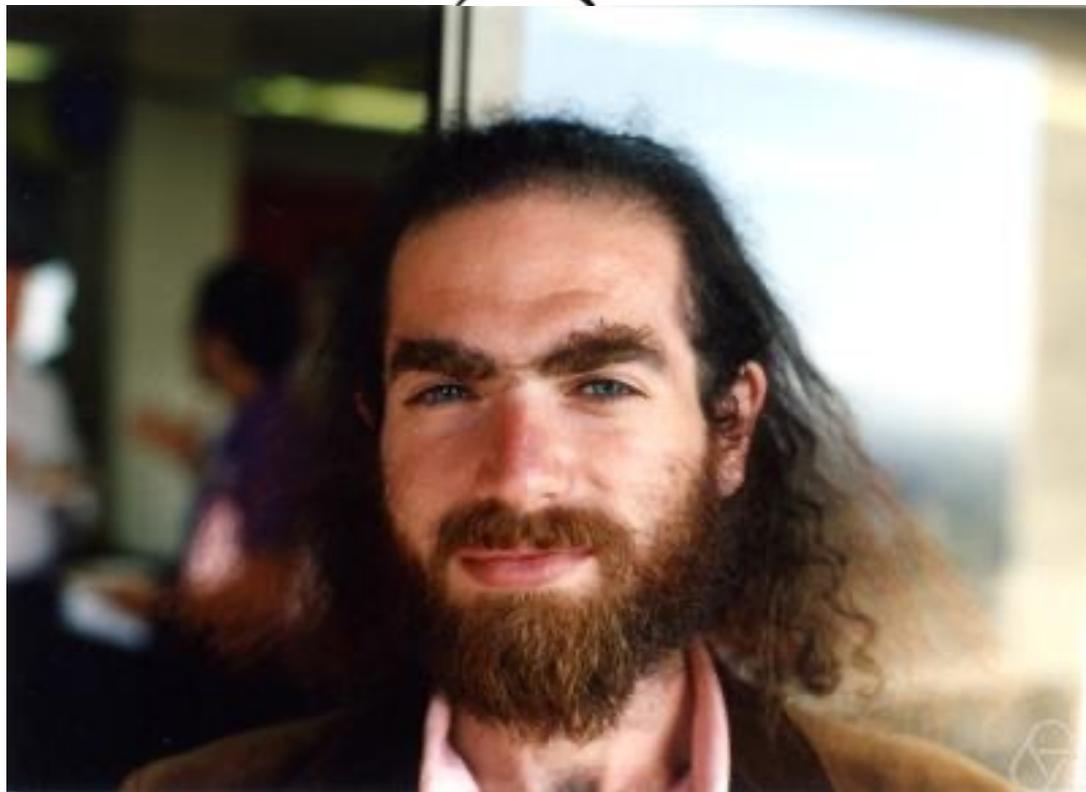


ZADATAK: Čemu je homeomorfno:



Kako znamo da je zemlja okrugla  
bez informacija iz Svemira

Magellan, Eratosten i ostali mjerili su  
na zemlji (ili plovili), ali to što su  
saznali konzistentno je i sa zemljom  
u obliku torusa.



GRIGORI YAKOVLEVICH PERELMAN

13.6.1966. —

2002. godine objavio na internetu  
znanstvene radove u kojima  
je dokazao Poincaréovu slutnju.  
Odbio Fieldsovu medalju i  
milijun dolara nagrade. Ne daje  
intervjue. Kaže da su znanstvenici  
pdivareni i da bradu ideje te da  
on ne želi sudjelovati u tome.



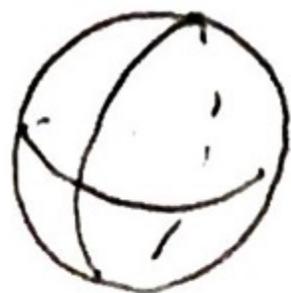


Francuski matematičar. Pokrenuo  
poje TOPOLOGIJE u današnjem  
smislu. Prvi je predvidio postojanje  
gravitacijskih valova, razumio  
relativnost i napisao  $E=mc^2$



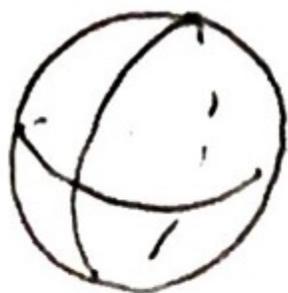
JULES HENRI POINCARÉ  
29. 4. 1854. - 17. 7. 1912.

# POINCARÉOVA SLUTNJA



2-sfera

# POINCARÉOVA SLUTNJA



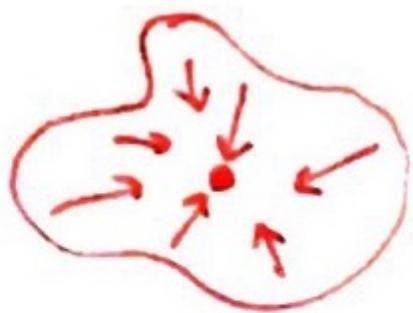
2-sfera

Svaka petlja može se  
stisnuti u točku

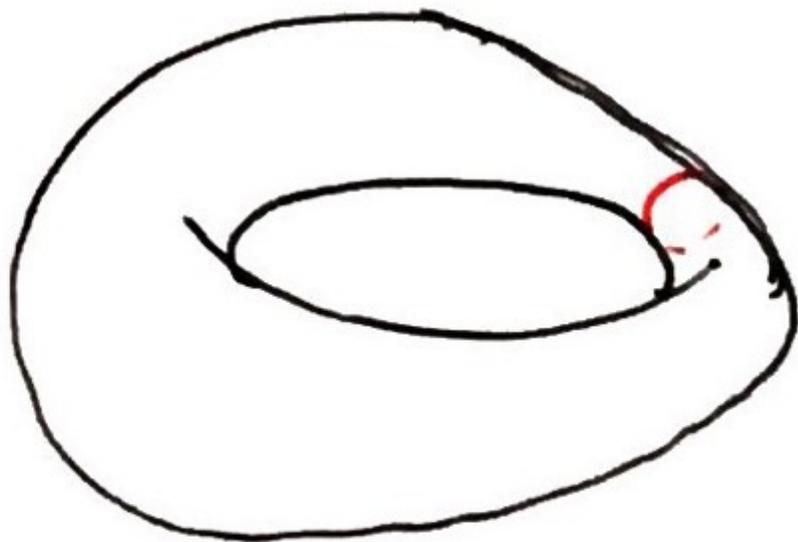
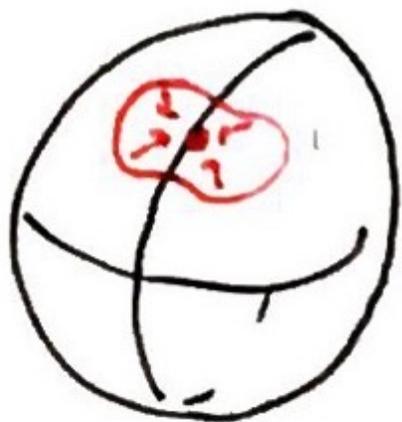


## POINCARÉOVA SLUTNĀ

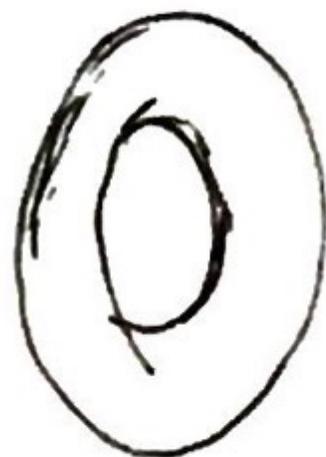
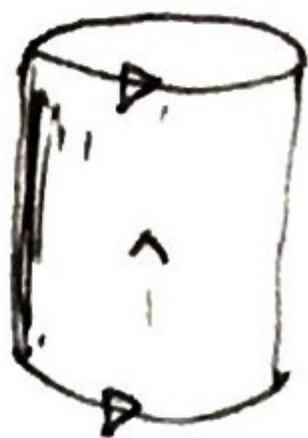
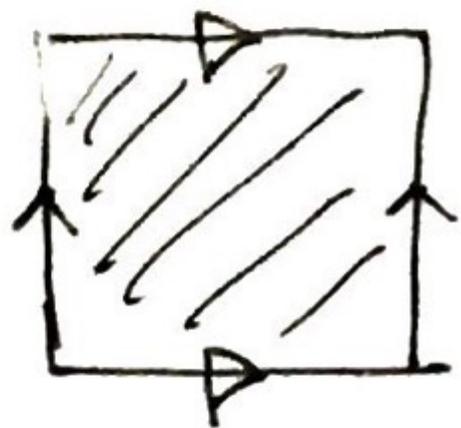
Every simply connected, closed  
3-manifold is homeomorphic  
to the 3-sphere.

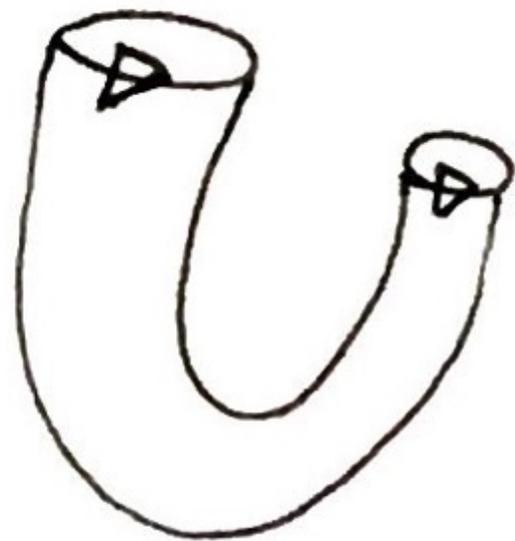
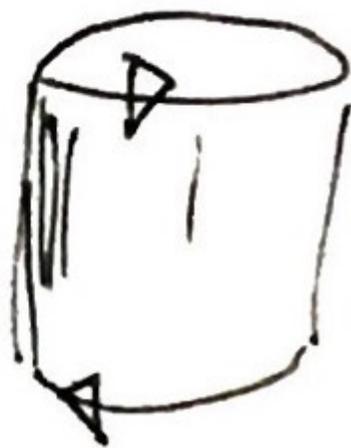
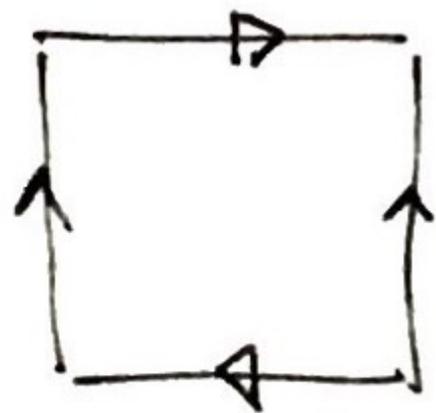


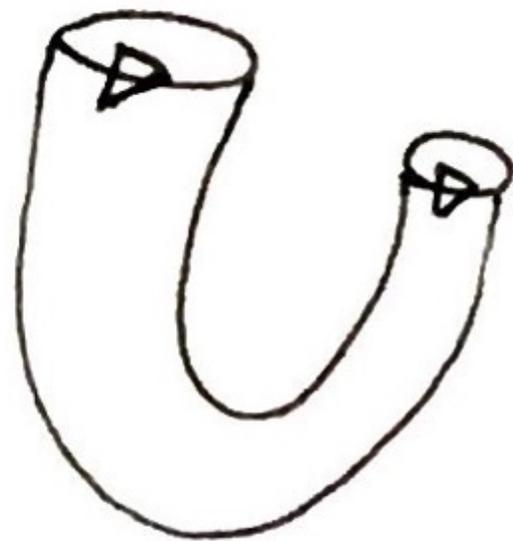
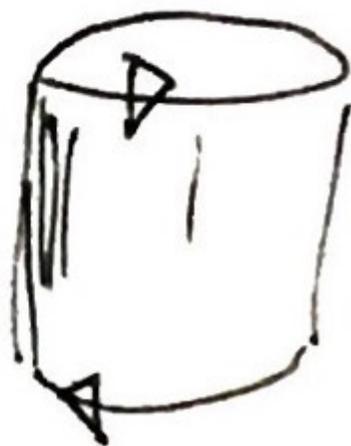
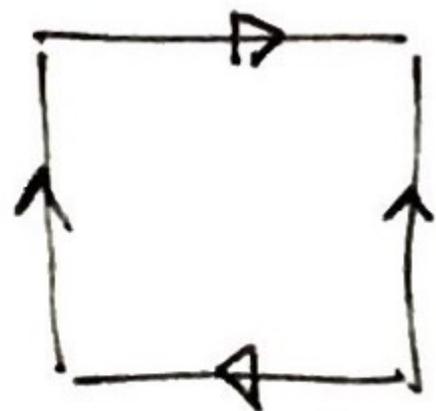
jednostruko povezano  
područje



ne može se  
stisnuti u  
točku

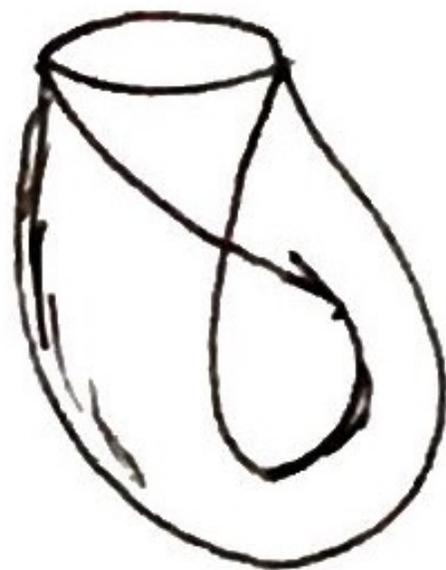






NITE MOGUĆE U 3D

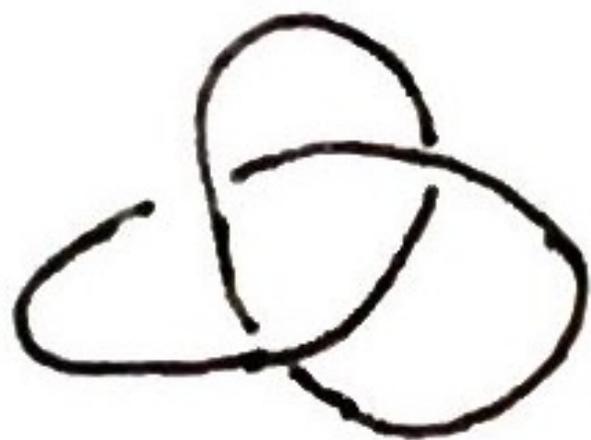
presijeca  
sama sebe

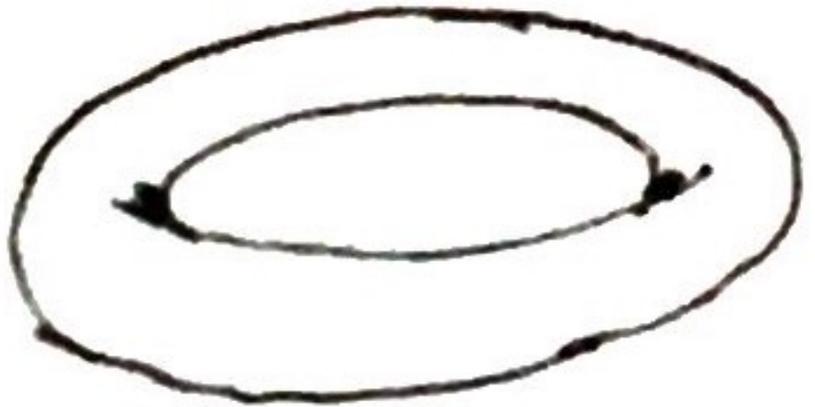
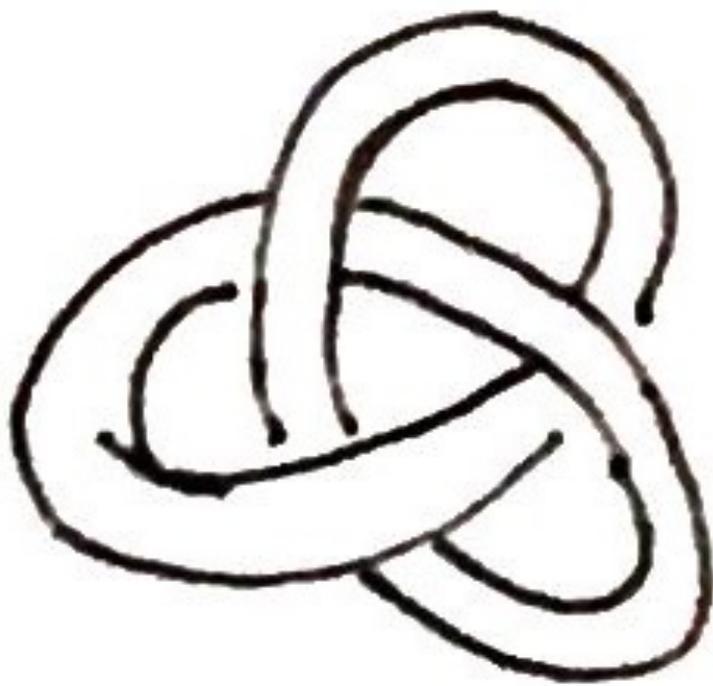


KLEINOVA  
BOGA



2.





**TEOREM:** Neka su  $K$  i  $L$  neki objekti, a  $M$  prostor u kojemu se nalaze.

$$\dim K + \dim L - \dim M = \text{overflow}$$

Ako je  $\text{overflow} < 0$  tada se  $K$  i  $L$  ne mogu sjediti u  $M$

Ako je  $\text{overflow} \geq 0$  tada se  $K$  i  $L$  sijeku

Ako je  $\text{overflow} < -1$  tada se mogu raspetljati



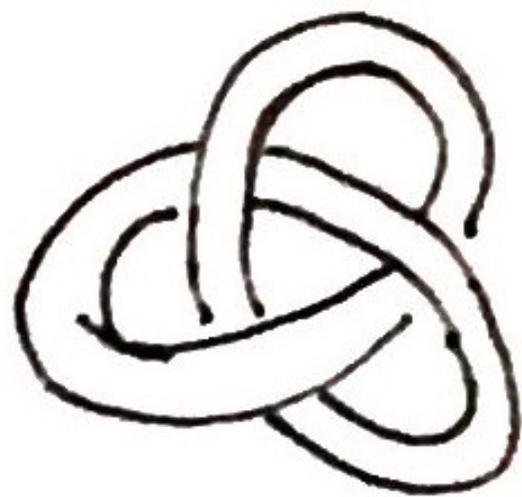
**TEOREM:** Svaki čvor uacimjen od krivulje (špage) moguće je razvezati u 4 prostorne dimenzije.

$$\dim K + \dim L - \dim M = \text{over flow}$$

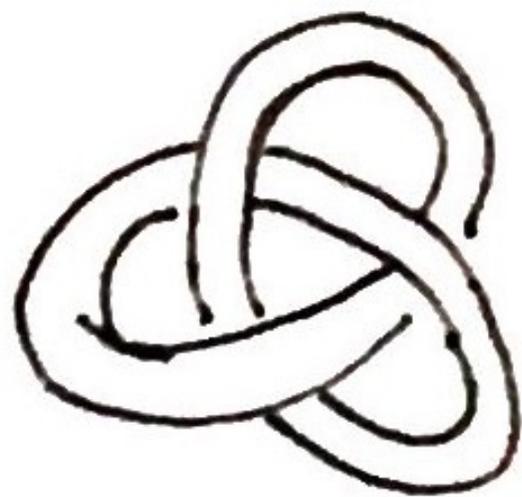
$$1 + 1 - \dim M < -1$$

$$- \dim M < -3$$

$$\dim M > 3 \quad \Rightarrow \quad \dim M = 4$$



Koliko dimenzija je  
potrebno za raspletjati  
čvor?

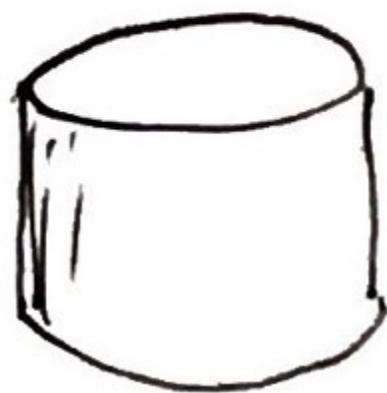
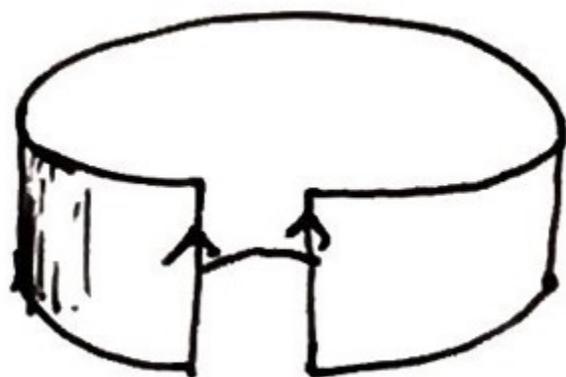
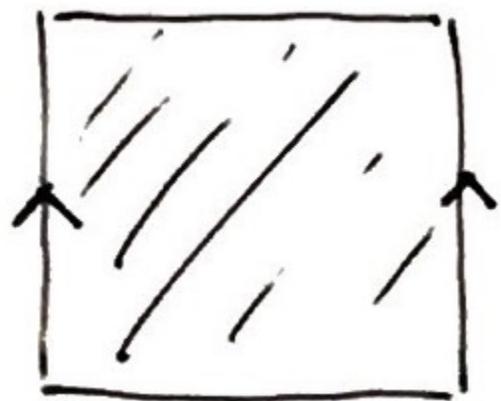


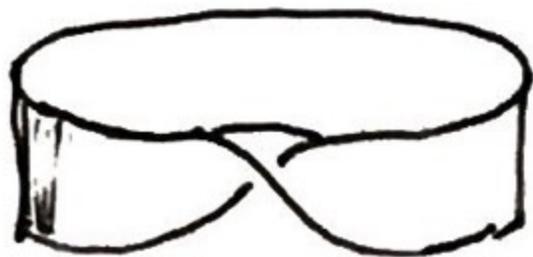
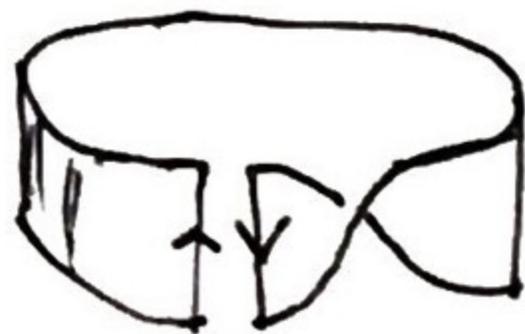
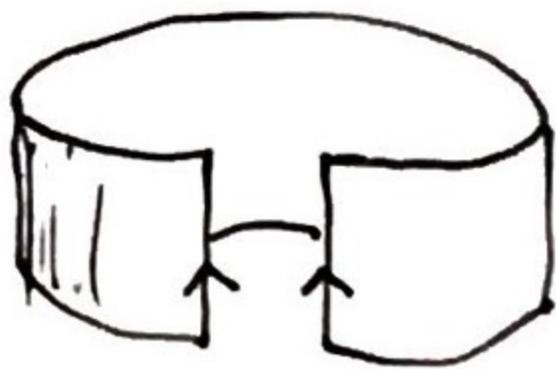
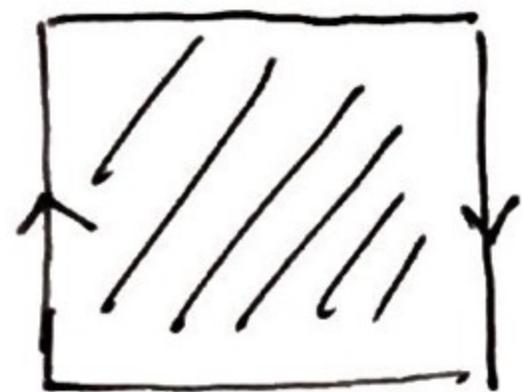
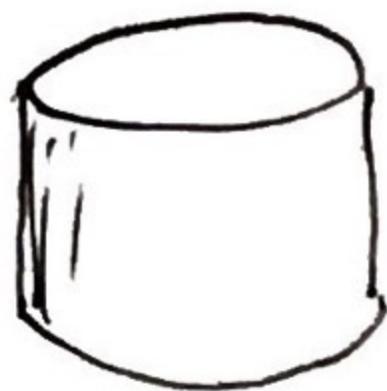
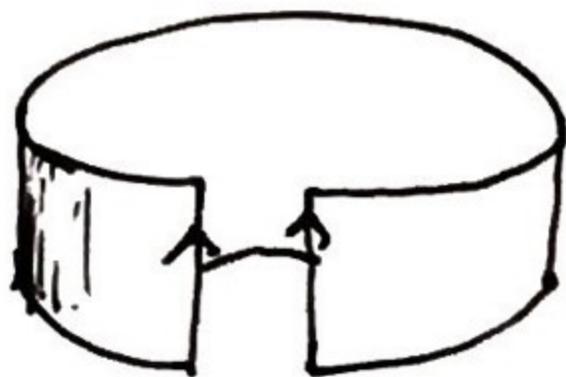
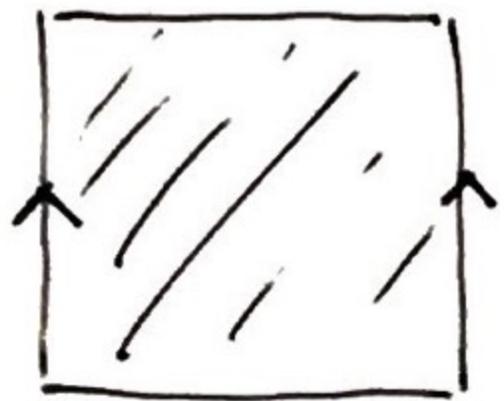
Koliko dimenzija je  
potrebno za raspletjati  
čvor?

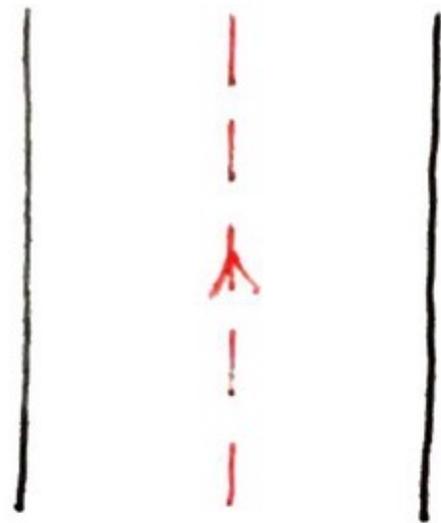
$$2+2 - \dim M < -1$$

$$- \dim M < -5$$

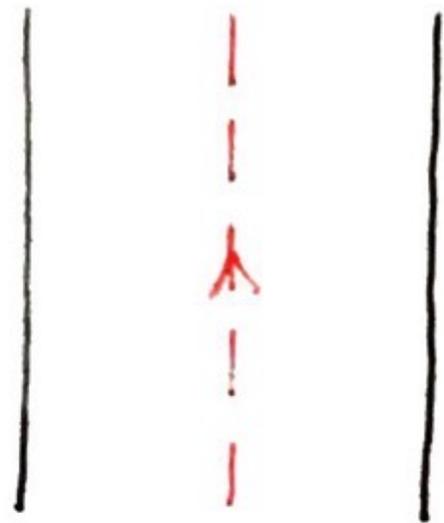
$$\dim M > 5 \Rightarrow \dim M = 6$$



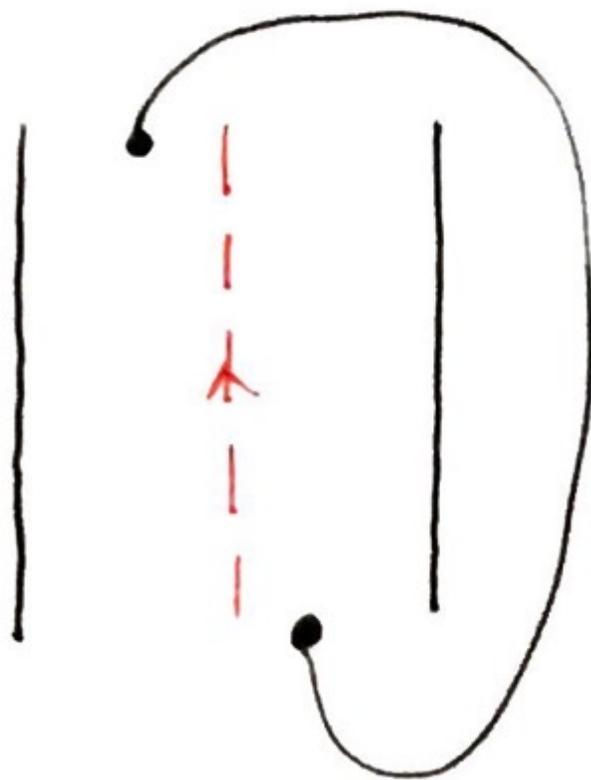


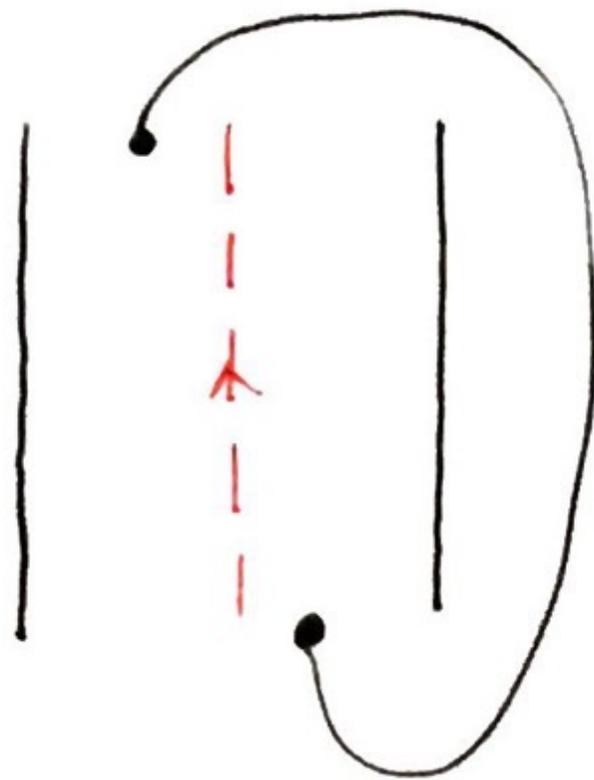
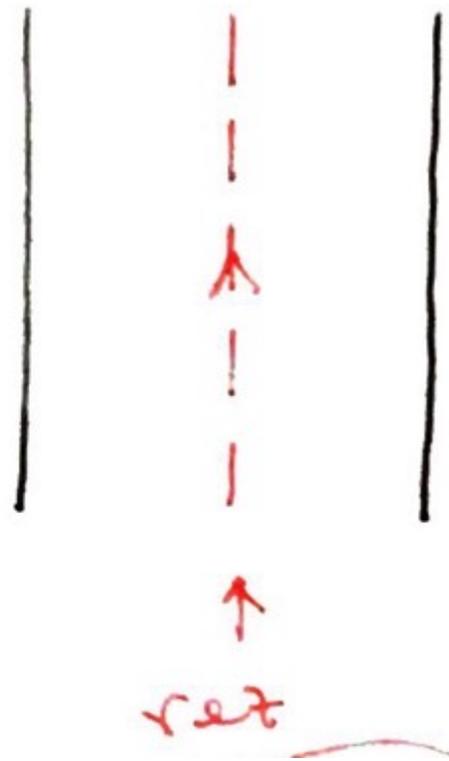


ret

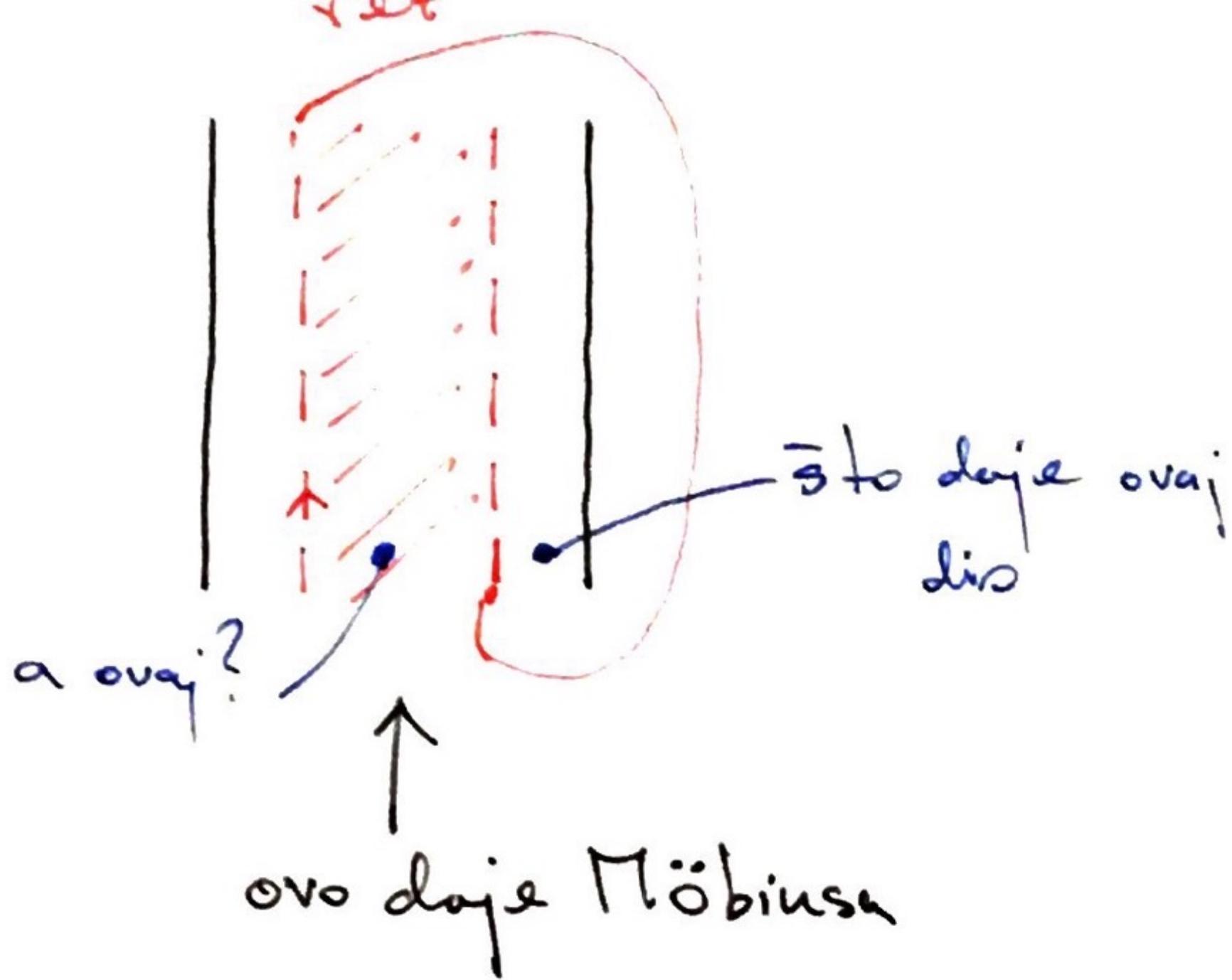


ret





ove dvije strane  
su spojene pa  
dobijemo duplo-  
dužu Möbiusovu



1. There is so much more to mathematics than numbers and formulas, for example, there is pictorial thinking music / score
2. Always draw pictures whenever you work on mathematics
3. There is so much more to pictures than photos of physical objects. In topology and geometry you should learn to see things that can not be seen physically (expanding our vision) and to work with such pictures reason on them

KRATJ