

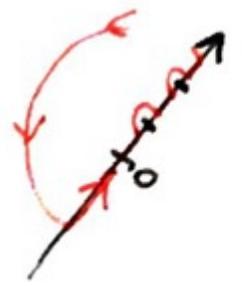
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10
- 11
- 12
- 13

- 1i.
- 2i.
- 3i.
- 4i.
- 5i.
- 6i.
- 7i.
- 8i.
- 9i.
- bi.

KOMPLEKSNI  
BROJEVI

by DARJO

$$\frac{1}{2\pi i} \oint_C f(z) dz = \sum \text{Res}$$

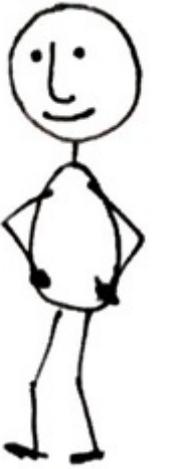


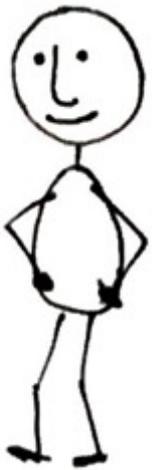
$\sqrt{-1} = i$ ,  $i^2 = -1$ , ma nije  
kema sense

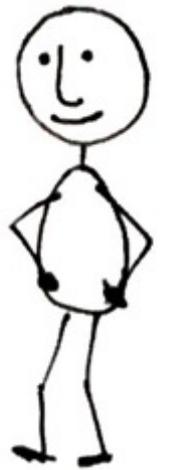
$$e^{\pi i} + 1 = 0$$

$$i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m} \Delta \psi - V\psi = 0$$

$$\delta(x - \frac{1}{2}) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} e^{in(x-\frac{1}{2})}$$

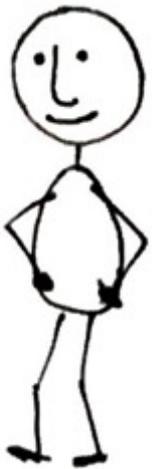








JA SAM PETAR.  
DOLAZIM S LITVE  
STRANE





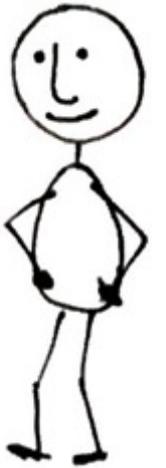
JA SAM PETAR.  
DOLAZIM S LIJEVE  
STRANE

A JA SAM IVAN  
I DOLAZIM S  
DESNE STRANE





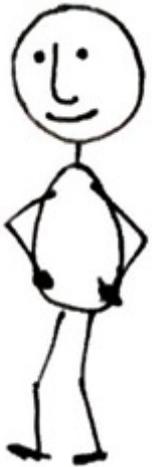
DANAS ČEMO  
RAZGOVARATI  
O BROJEVIMA





DANAS ĆEMO  
RAZGOVORATI  
O BROJEVIMA

ŠTO SU BROJEVI?  
KOLI BROJEVI POSTOJE?  
ZASTO IH KORISTIMO...  
SU NEKA OD PITANJA  
NA KOJACETMO DATI ODGOVORE



ZASTO TREBAMO

— BROJEVE?

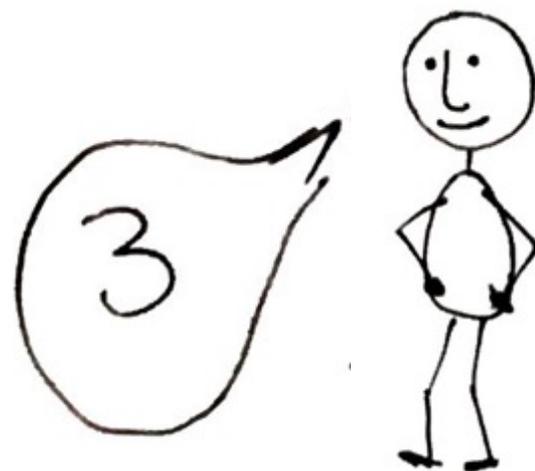


KOLIKO KRAVA  
PASE NA  
POLJU?



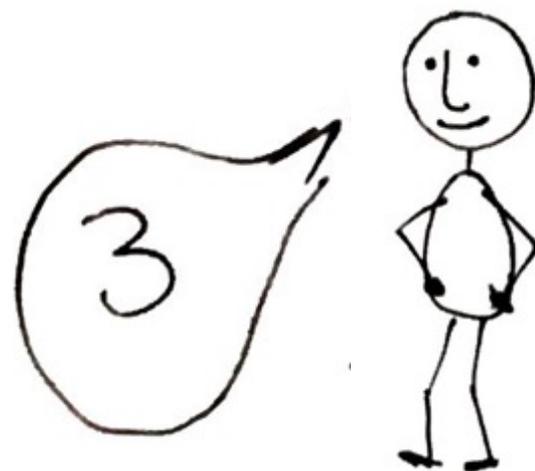


KOLIKO KRAVA  
PASE NA  
POLJU?





KOLIKO KRAVA  
PASE NA  
POLJU?



• • • • •  
1 2 3 4 5 6 ...

Brojevi su povezani s brojanjem

Brojevi su povezani s brojanjem

I II III

Brojevi su povezani s brojanjem

I II III

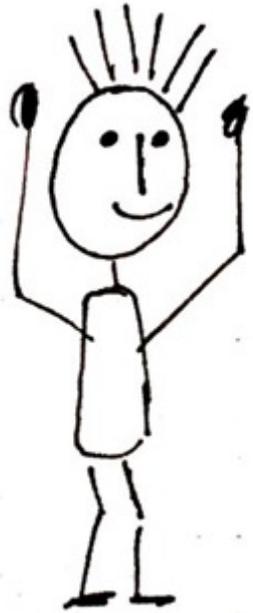
— = ≡

Brojevi su povezani s brojanjem

I II III

1 2 3

1 2 3



OVO SU SVI  
BROJEVI KOJI  
POSTOJE



• • • • •  
1 2 3 4 5 6 ...



OVO SU SVI  
BROJEVI KOJI  
POSTOJE

DA, DA, SVE  
VIŠE OD TEGA  
BILO BI BESKONечно



•   •   •   •   •   •   ...  
1   2   3   4   5   6

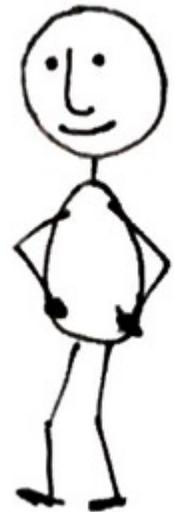
HM, HM, A ŽESU LI

TO ŽBILIA SVI

BRATEVI?



FRENDE, DAS  
MI MALO KRUTHA?  
GLADAN SAM

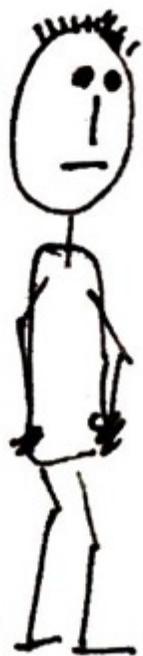




FRENDE, DAS  
MI MALO KRUKA?  
GLADAN SAM

IDE MI NA ŽIVCE,  
ALI NE SMIŽEM  
REČI DA MI NE DAM  
ZER ĆE ME NAZVATI  
PARLOM.

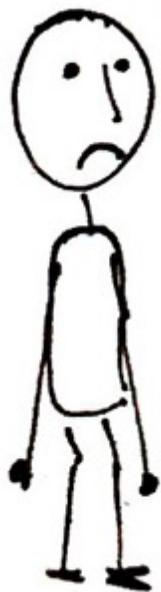




FRENDE, DAS  
MI MALO KRUIHA?  
GLADAN SAM

MOŽE FRENDE,  
EVO TI **NUKA**  
**KRUIHA**



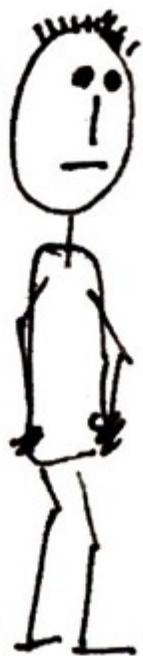


NUĻĀ ĒĒ BĀS  
GLUP BRŪL, POGOTĒVĒ  
KĀD SI GLĀDAN



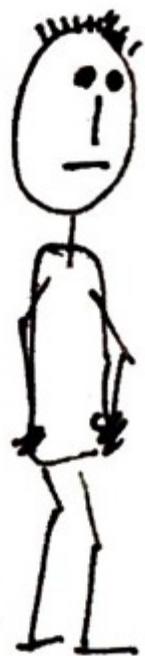
ċ 1 2 3 4 5 6 ...

DOBRO, A ŽESU LI  
SADA TO SVI  
BROTJEVI KOŽE  
TREBAJU?



FRENDE, DÁS  
MI KRUA?  
GLADAN SAM

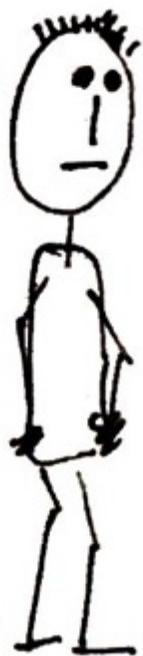




FRENDE, DÁS  
MI KRUPA?  
GLADAN SAM

HM, HM, CIZELI  
KRUPA BI BIO  
PREVIŠE, A AKO  
TU NE DAM RECI  
ČE DA SAM ZLOČESTI

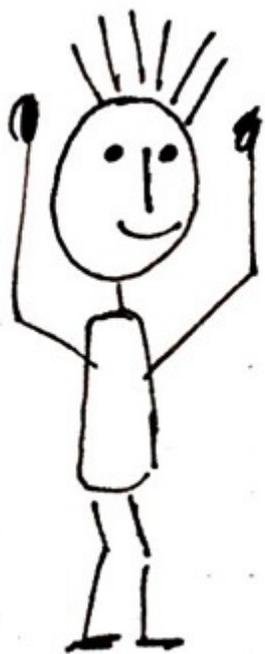




FRENDE, DAS  
MI KRUA?  
GILADAN SAM

PRIMATELU, EVO  
TOLA



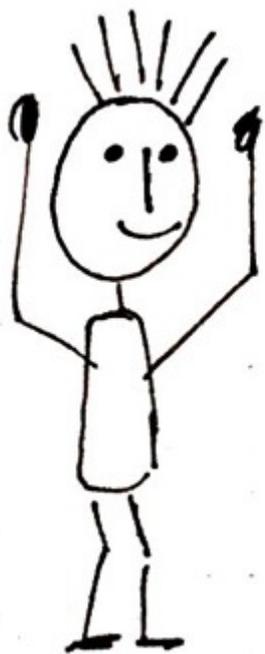


JUHUU, RAZLOMCI  
SU NAJBOLJA  
STVAR NA SVIJETU.  
RAZLOMCI SU BOLJI  
OD SUPERMANA.

RAZLOMKE VOLIM JA.  
TRALALALA LA

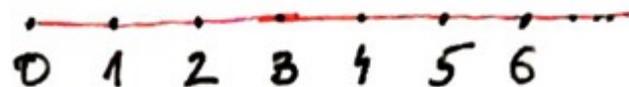
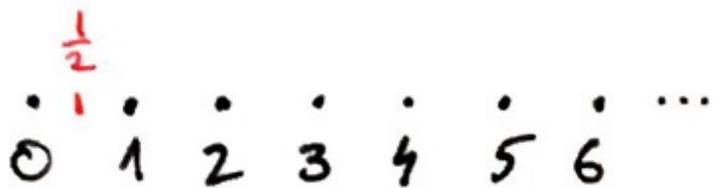


$\frac{1}{2}$   
• 0 • 1 • 2 • 3 • 4 • 5 • 6 • ...



JUHUU, RAZLOMCI  
SU NAJBOLJA  
STVAR NA SVIJETU.  
RAZLOMCI SU BOLJI  
OD SUPERMANA.

RAZLOMKE VOLIM JA.  
TRALALALALA



ДОБРО, А ТЕСУ ЛИ  
САДА ТО СВИ  
БРОТЕВИ КОГЕ  
ТРЕБАЛО?



AKO IMAS 3 TABUKE,  
A 7A TI DAM DUITE  
KO4KO TABUKA IMAS?





AKO IMAS 3 TABUKE,  
A 7A TI DAM DUITE  
KOLIKO TABUKA IMAS?

5





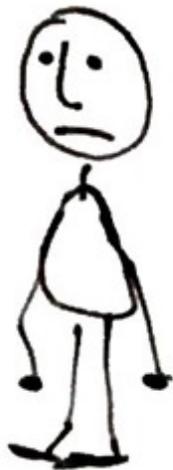
AKO IMAS 3 TABUKE,  
#7A TI UZMEMDVNE  
KOLIKO TABUKA IMAS?





AKO IMAS 3 TABUKE,  
#7A TI UZMEM DVNE  
KOLIKO TABUKA IMAS?

SAMO JEDNU, A  
ZA PITU TREBAM  
DVNE... SMRC





ALCO IMĀS 3 ŽABUKĒ,  
A TA TĪ UZŅEM 4  
KŌ LIKŌ ŽABUKĀ IMĀS





AKO IMAS 3 TABUKE,  
A ZA TI UZNETI 4  
KO LIKO TABUKA IMAS

ETO NECE ICI.  
KAKO CES MI UZETI  
4 AKO IMAM SAMO 3?





TAKO DA ITAS NEGATIVAN  
BRO? JABUKA. IPAT  
CES MINUS JEDNU  
JABUKU

ETO NEĆE IĆI.  
KAKO CES MI UZETI  
4 AKO ITAM SAMO 3?

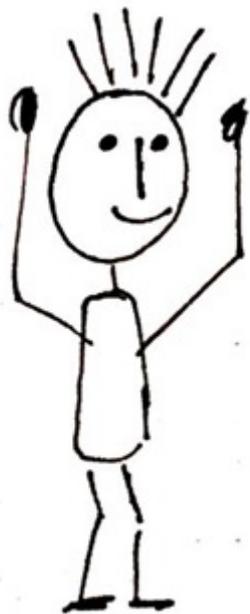




TAKO DA NIŠ NĚGATIVN  
BRŮ? JABUKA. NIŠ  
ČES MINUS JEDNU  
JABUKU

ŠTO JE TO NĚGATIVN  
BRŮ? KAKO JABUKA  
MŮŽE BĚTI NĚGATIVNA?





ΜΟΞΕ ΜΟΞΕ. ΤΟ ΓΕ  
ΑΝΤΙΓΑΒΟΥΚΑ. ΣΤΟ  
ΝΙΣΙ ΕΥΟ ΖΑ  
ΑΝΤΙΝΑΤΕΡΟΥ

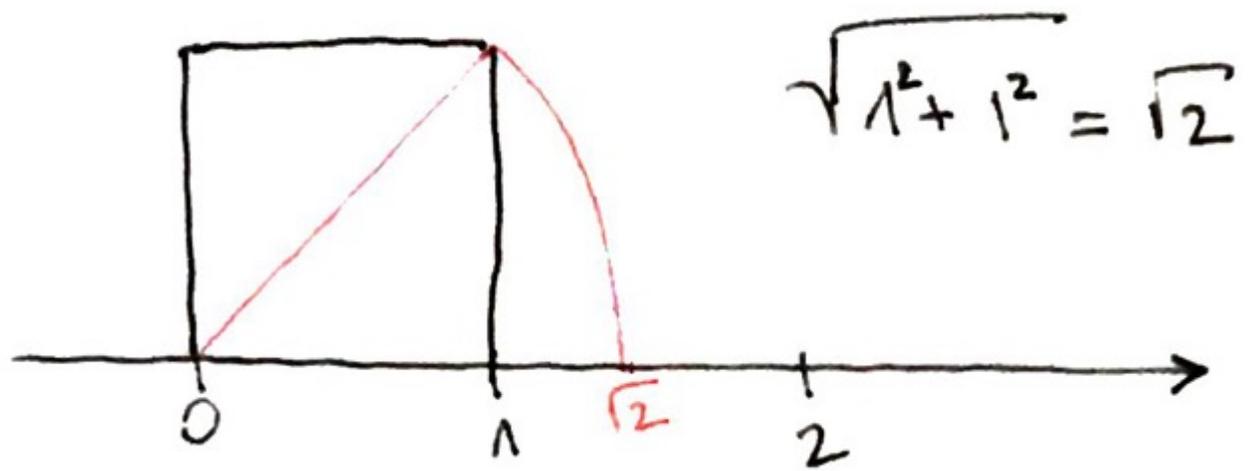
ΣΤΟ ΓΕ ΤΟ ΝΕΓΑΤΙΒΑΝ  
ΒΡΟΥ? ΚΑΚΟ ΓΑΒΟΥΚΑ  
ΜΟΞΕ ΒΙΤΙ ΝΕΓΑΤΙΒΝΑ?





NEGATIVNI BROJEVI  
SU ODLIČNI ZA  
BILJEŽENJE DUGOVA.  
ALO MI JE DUŽAN 300 \$  
PISEM - 300 \$

DOBRO, A ŽESU LI  
SADA TO SVI  
BROTJEVI KOŽE  
TREBAJU?



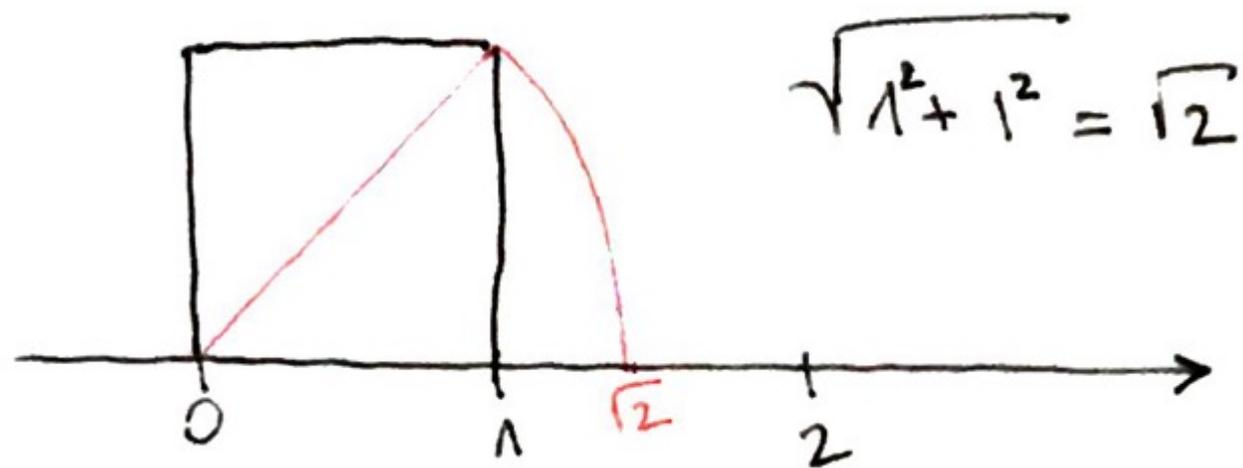
$$\sqrt{1^2 + 1^2} = \sqrt{2}$$



Pitagorejci su propovijedali da se svi brojevi mogu napisati kao razlomci. Hipasusu se pripisuje otkriće iracionalnih brojeva zbog čega su ga utopili u moru.



HIPPASUS  
530 - 450 BC



Babilonci su dali aproksimaciju 1 24 51 10  
u brojevnom sustavu s bazom 60

$$1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} = 1.41421296$$

SADA MOŽEMO BEZ PROBLEMA

- ZBRANATI
- ODUZIMATI
- MNŌŽITI
- DILEZITI
- VADITI KORĪEN
- KVADRIRATI
- ⋮

JEDNADŽBA

$$x - 5 = 0$$

$$x + 2 = 0$$

$$2x - 1 = 0$$

$$x^2 - 5 = 0$$

$$x^2 + 1 = 0$$

REZENTE

SKUP

JEDNADŽBA

$$x - 5 = 0$$

$$x + 2 = 0$$

$$2x - 1 = 0$$

$$x^2 - 5 = 0$$

$$x^2 + 1 = 0$$

REZULTAT

$$x = 5$$

SKUP

$\mathbb{N}$

JEDNADŽBA

$$x - 5 = 0$$

$$x + 2 = 0$$

$$2x - 1 = 0$$

$$x^2 - 5 = 0$$

$$x^2 + 1 = 0$$

REŠENIE

$$x = 5$$

$$x = -2$$

SKUP

$\mathbb{N}$

$\mathbb{Z}$

JEDNADŽBA

$$x - 5 = 0$$

$$x + 2 = 0$$

$$2x - 1 = 0$$

$$x^2 - 5 = 0$$

$$x^2 + 1 = 0$$

REŠENÍ

$$x = 5$$

$$x = -2$$

$$x = \frac{1}{2}$$

SKUP

$\mathbb{N}$

$\mathbb{Z}$

$\mathbb{Q}$

JEDNADŽBA

$$x - 5 = 0$$

$$x + 2 = 0$$

$$2x - 1 = 0$$

$$x^2 - 5 = 0$$

$$x^2 + 1 = 0$$

REŠENÍ

$$x = 5$$

$$x = -2$$

$$x = \frac{1}{2}$$

$$x = \pm\sqrt{5}$$

SKUP

$\mathbb{N}$

$\mathbb{Z}$

$\mathbb{Q}$

$\mathbb{R}$

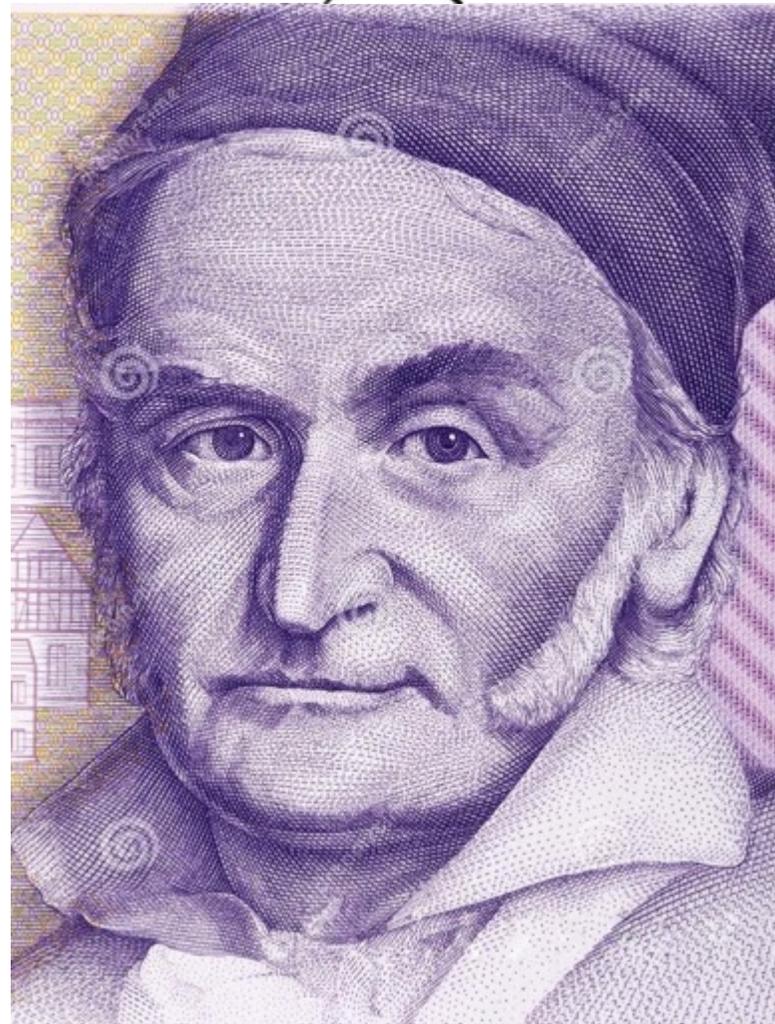
KAKO RIZESITI  $x^2 + 1 = 0$  ?

KAKO RIZEŠITI  $x^2 + 1 = 0$  ?

ŠTO JE  $\sqrt{-1}$  ?

TREBA LI NAM  $\sqrt{-1}$  ?

Njemački matematičar i fizičar  
s velikim doprinosom u algebri,  
astronomiji, geometriji, magnetizmu  
i ne-euklidskoj geometriji...



JOHANN CARL FRIEDRICH GAUSS

30.4.1777. – 23.2.1855.

C. F. GAUSS : OSNOVNI TEOREM ALGEBRE

$$a_n X^n + a_{n-1} X^{n-1} + \dots + a_2 X^2 + a_1 X + a_0 = 0$$

IMA  $n$  NULTOCAKA



JOHANN CARL FRIEDRICH GAUSS

30.4.1777. - 23.2.1855.

C. F. GAUSS : OSNOVNI TEOREM ALGEBRE

$$a_n X^n + a_{n-1} X^{n-1} + \dots + a_2 X^2 + a_1 X + a_0 = 0$$

IMA  $n$  NULTOČAKA

$$2x + 4 = 0 \quad 2x = -4 \quad x = -2$$



JOHANN CARL FRIEDRICH GAUSS

30.4.1777. – 23.2.1855.

# C. F. GAUSS : OSNOVNI TEOREM ALGEBRE

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0 = 0$$

IMA  $n$  NULTOCĀKA

$$2x + 4 = 0 \quad 2x = -4 \quad x = -2$$

$$x^2 - 9 = 0 \quad x^2 = 9 \quad x_1 = 3, x_2 = -3$$



JOHANN CARL FRIEDRICH GAUSS

30.4.1777. – 23.2.1855.

C. F. GAUSS : OSNOVNI TEOREM ALGEBRE

$$a_n X^n + a_{n-1} X^{n-1} + \dots + a_2 X^2 + a_1 X + a_0 = 0$$

IMA  $n$  NULTOČAKA

$$X^2 + 1 = 0$$



JOHANN CARL FRIEDRICH GAUSS

30.4.1777. – 23.2.1855.

C. F. GAUSS : OSNOVNI TEOREM ALGEBRE

$$a_n X^n + a_{n-1} X^{n-1} + \dots + a_2 X^2 + a_1 X + a_0 = 0$$

IMA  $n$  NULTOČAKA

---

$$X^2 + 1 = 0 \Rightarrow X^2 = -1$$



JOHANN CARL FRIEDRICH GAUSS

30.4.1777. – 23.2.1855.

## C. F. GAUSS : OSNOVNI TEOREM ALGEBRE

$$a_n X^n + a_{n-1} X^{n-1} + \dots + a_2 X^2 + a_1 X + a_0 = 0$$

IMA  $n$  NULTOČAKA

$$X^2 + 1 = 0 \Rightarrow X^2 = -1$$

$$X_1 = \sqrt{-1}, X_2 = -\sqrt{-1}$$



JOHANN CARL FRIEDRICH GAUSS

30.4.1777. – 23.2.1855.

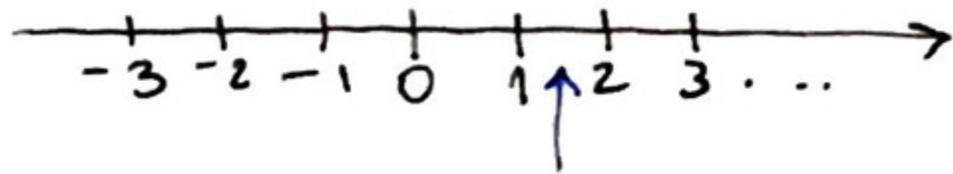
2 BOG LAKSEG PISANJA  $\sqrt{-1} = i$  (imaginary)

2 BOG LAKSEG PISANTA  $\sqrt{-1} = i$  (imaginary)

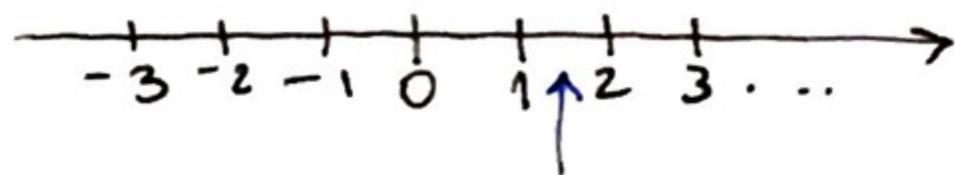
3 - REALAN BRO?

$2i$  - IMAGINARAN  
BRO?

$3 + 2i$  ← COMPLEKSAN BRO?  
↑            ↑  
REALAN        IMAGINARAN



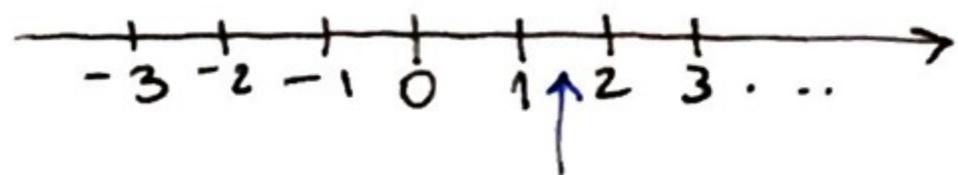
SVI BROJEVI KOJE SMO  
SPOMENULI ISKUNJAVAJU  
BROJEVNI PRAK. .  
NEMA VIŠE RUPA.



SVI BROJEVI KOJE SMO  
SPOMENULI ISKUNZAVAJU  
BROJEVNI PRAK. .  
NEMA VIŠE RUPA.

GDJE STAVITI  $\sqrt{-1}$ ?

U JEDNOJ DIMENZIJI  
NEMA VIŠE PROSTORA.

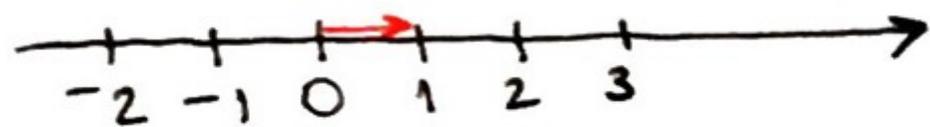


SVI BROJEVI KOJE SMO  
SPOMENULI ISKUNZAVAJU  
BROJEVNI PRAK. .  
NEMA VIŠE RUPA.

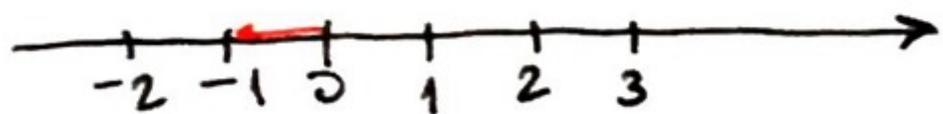
GDJE STAVITI  $\overline{1}$ ?

U JEDNOJ DIMENZIJI  
NEMA VIŠE PROSTORA.

TREBAMO  $70\overline{8}$   
PROSTORA,  $70\overline{9}$   
SLOBODE :->

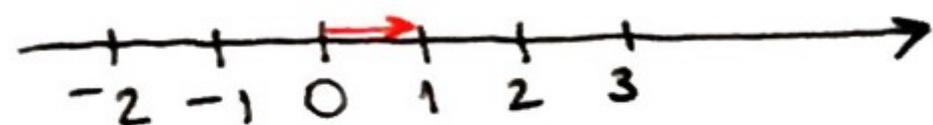


$$1 \cdot 2 = 2$$

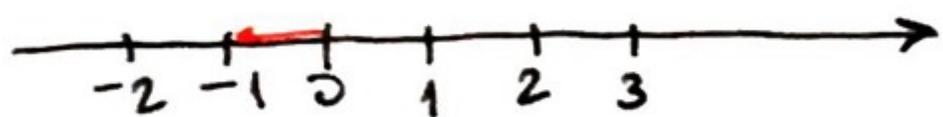


$$(-1) \cdot 2 = -2$$

$$(-1) \cdot (-2) = 2$$



$$1 \cdot 2 = 2$$



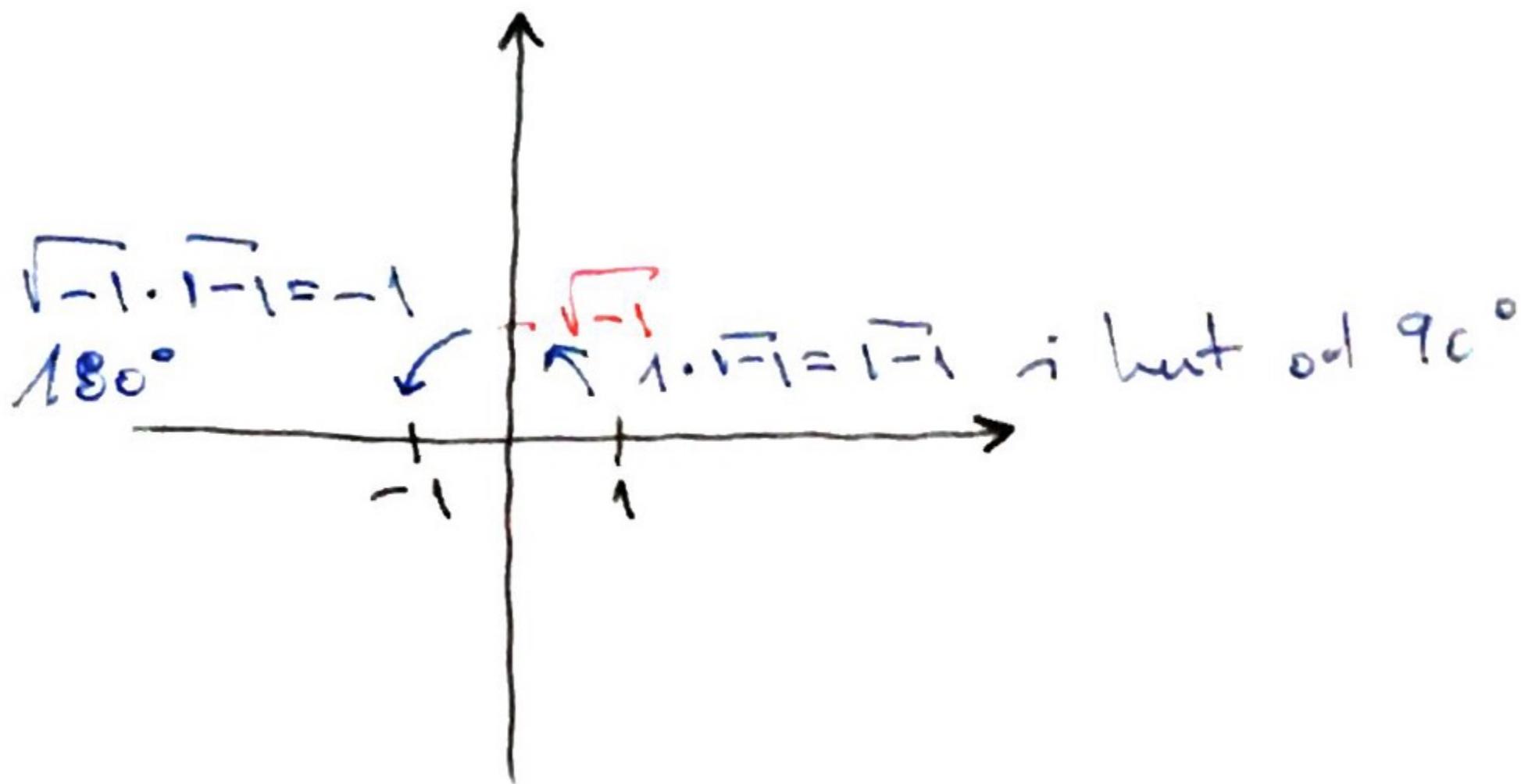
$$(-1) \cdot 2 = -2$$

$$(-1) \cdot (-2) = 2$$

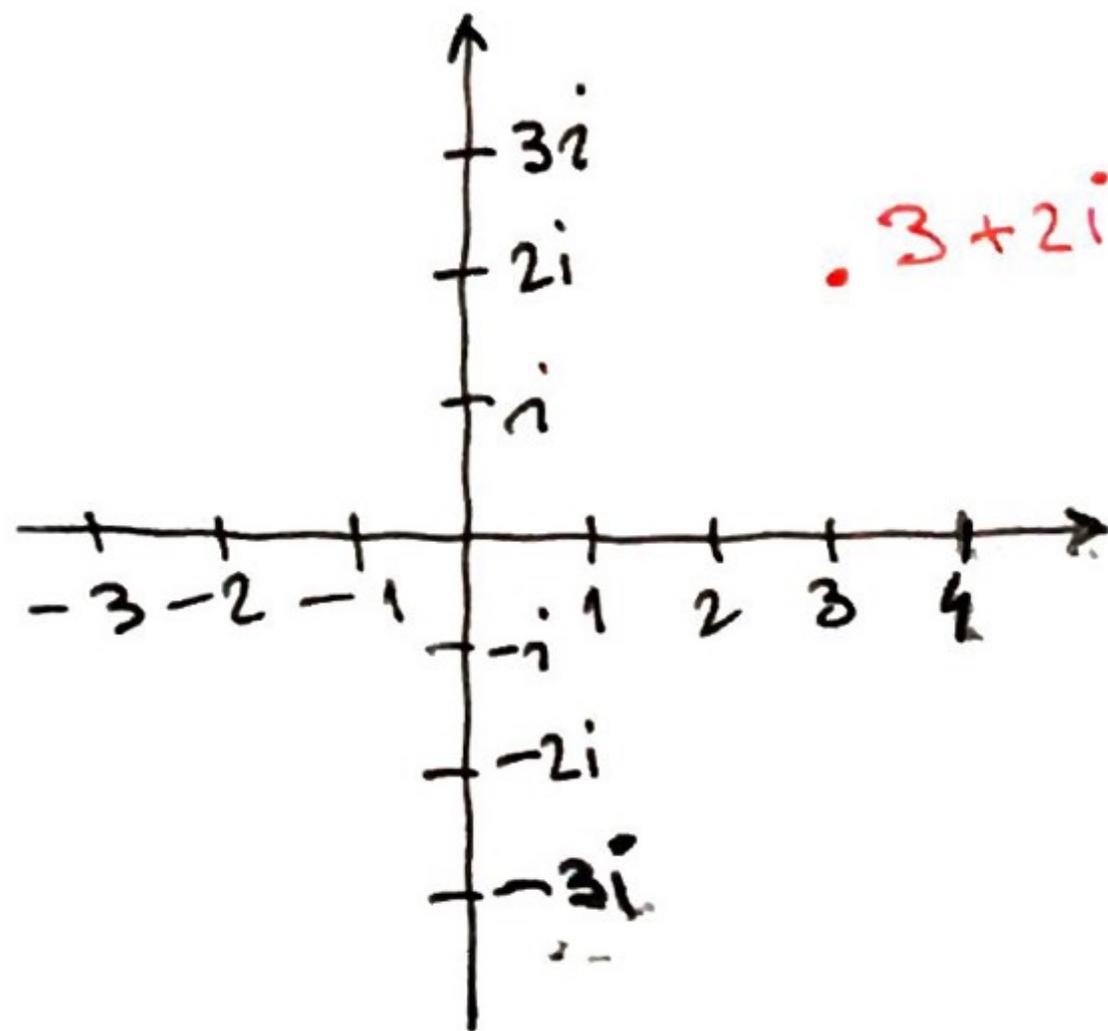
VEKTOR JE PROMIENIO KUT

$-1$  je pod kutem od  $180^\circ$ , ali  
 $(-1)^2$  je pod kutem od  $0^\circ$

$(\sqrt{-1})^2 = -1$  je pod kutem od  $180^\circ$

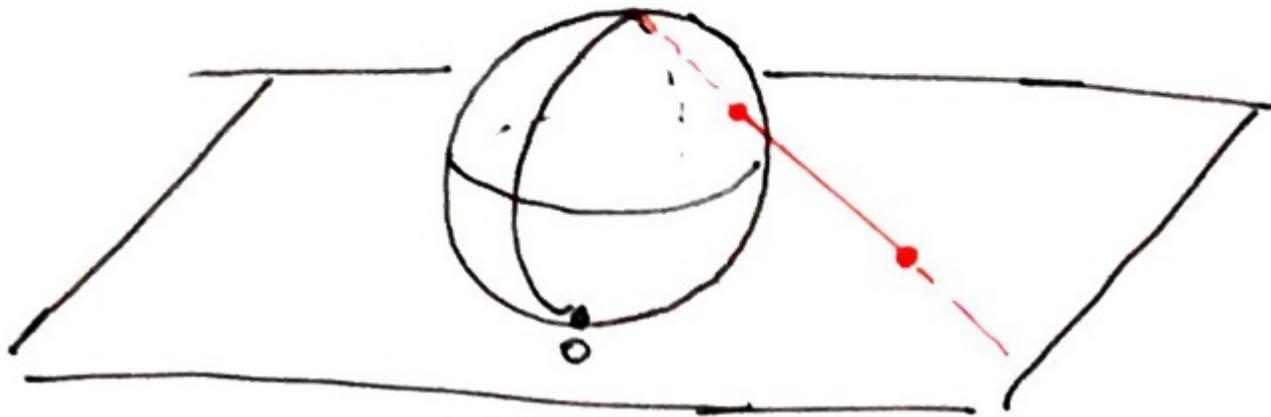


# KOMPLEKSNA RAVNINA $\mathbb{C}$

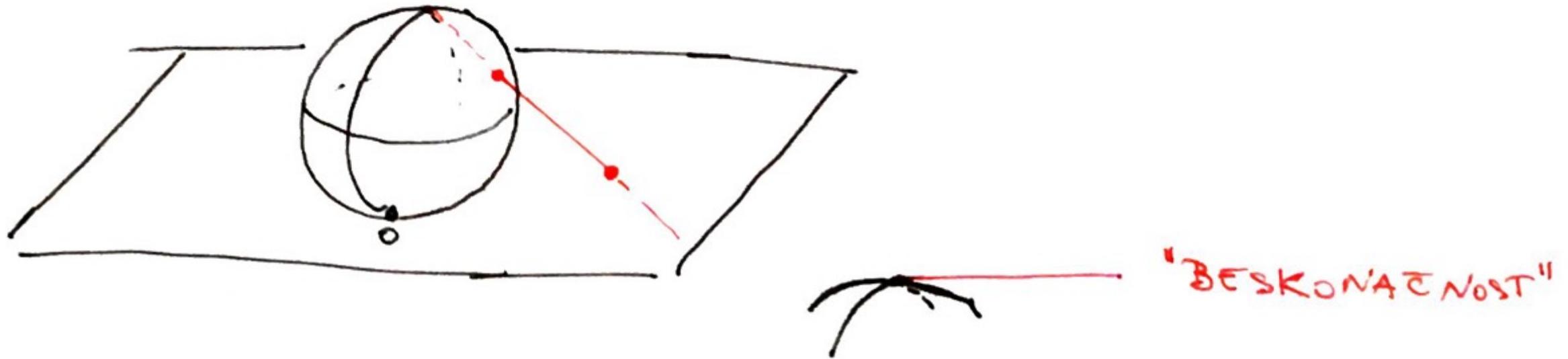


KOJEJE JE OBLIKA KOMPLEKSNA  
RAVNINA?

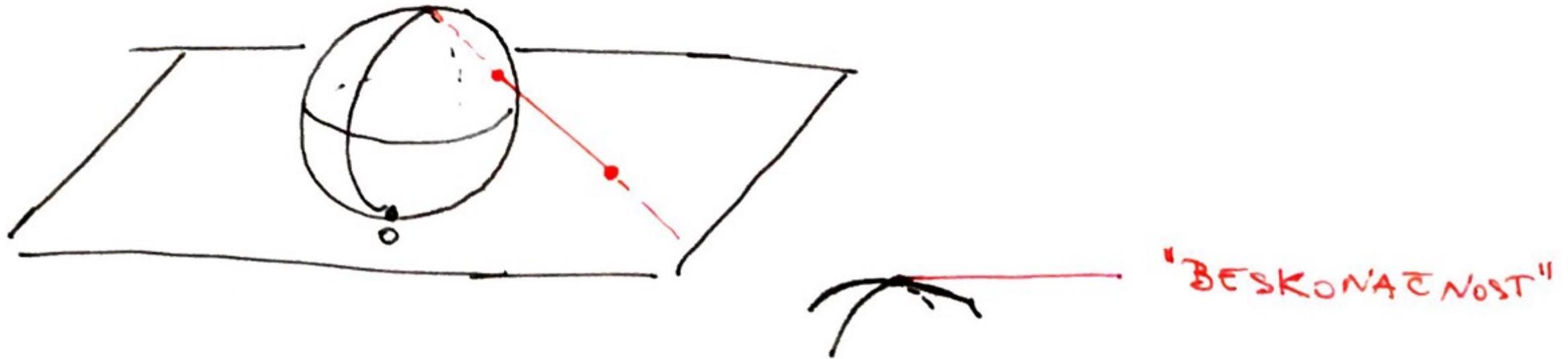
KOJE JE OBLIKA KOMPLEKSNA  
RAVNINA?



KOJE JE OBLIKA KOMPLEKSNA  
RAVNINA?

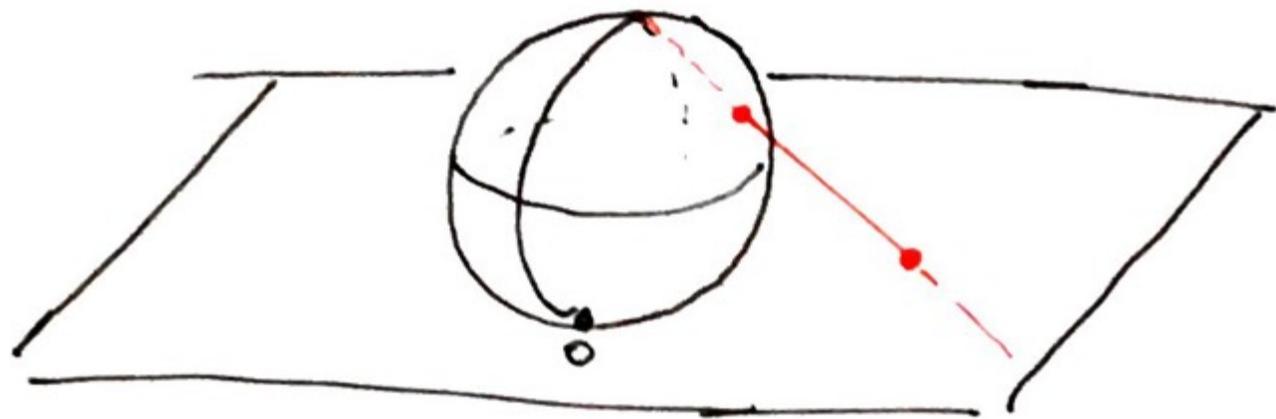
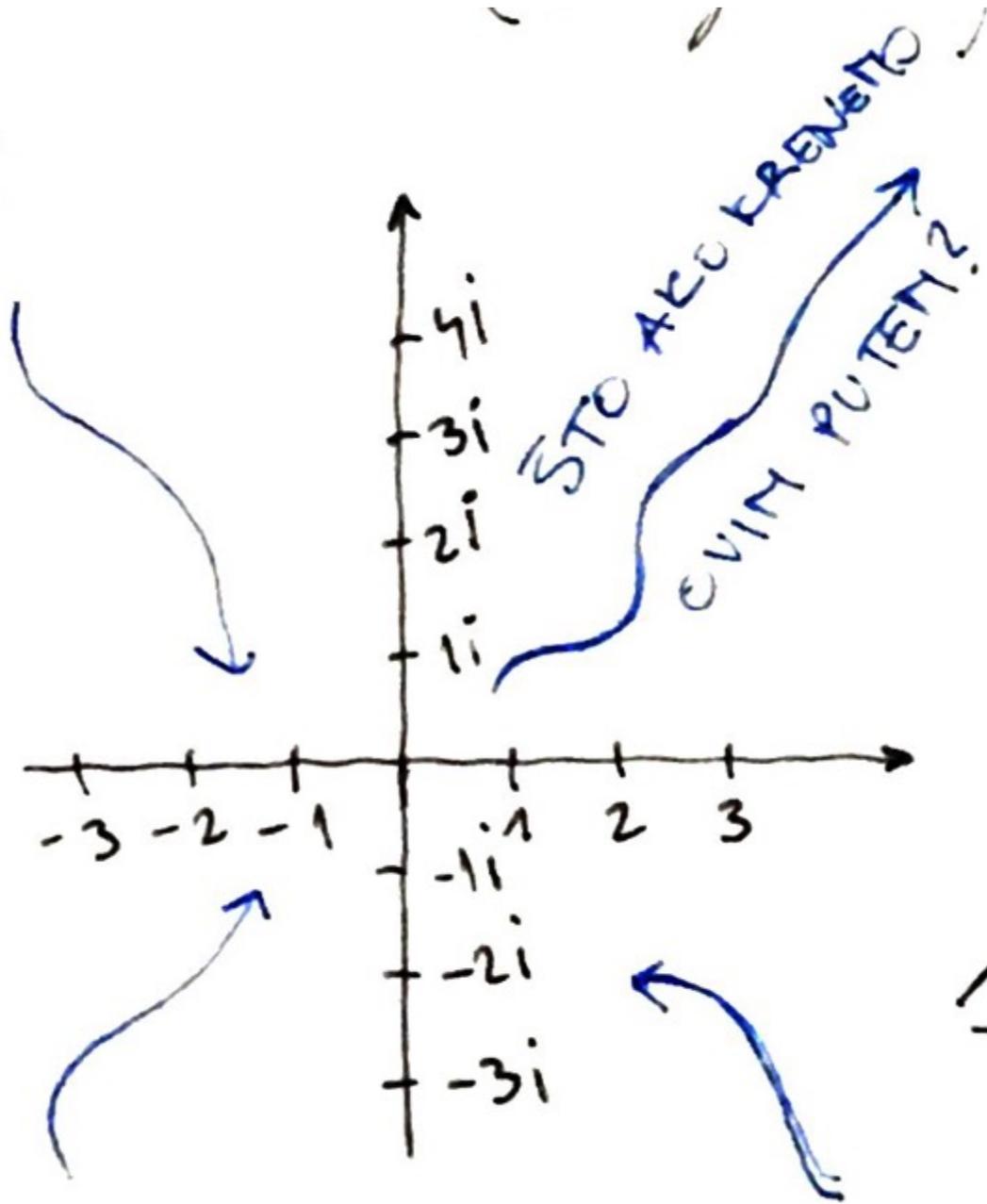


KOJEJE JE OBLIKA KOMPLEKSNA  
RAVNINA?



KOMPLEKSNA RAVNINA JE LOPTA :-)

A



$$1 + 1 + 1 + 1 + \dots + 1 + 1 + \dots =$$

$$1 + 1 + 1 + 1 + \dots + 1 + 1 + \dots = +\infty$$

$$1 + 1 + 1 + 1 + \dots + 1 + 1 + \dots = +\infty$$

$$1 + 2 + 3 + 4 + 5 + \dots + n + (n+1) + (n+2) + \dots =$$

$$1 + 1 + 1 + 1 + \dots + 1 + 1 + \dots = +\infty$$

$$1 + 2 + 3 + 4 + 5 + \dots + n + (n+1) + (n+2) + \dots = -\frac{1}{12}$$

$$1 + 1 + 1 + 1 + \dots + 1 + 1 + \dots = +\infty$$

$$1 + 2 + 3 + 4 + 5 + \dots + n + (n+1) + (n+2) + \dots = -\frac{1}{12}$$

$$1 + 2 + 4 + 8 + 16 + \dots + 2^n + 2^{n+1} + \dots =$$

$$1 + 1 + 1 + 1 + \dots + 1 + 1 + \dots = +\infty$$

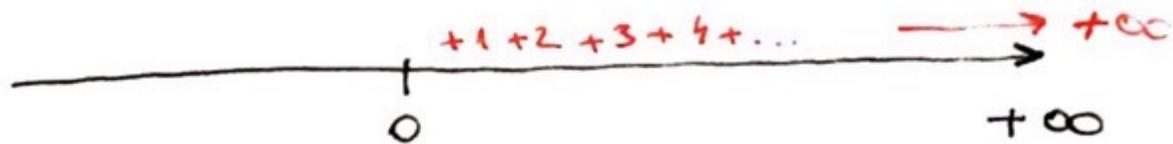
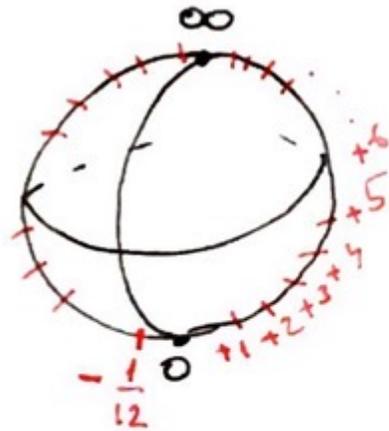
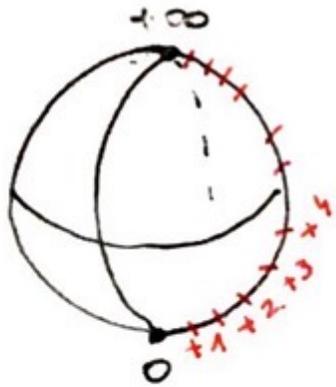
$$1 + 2 + 3 + 4 + 5 + \dots + n + (n+1) + (n+2) + \dots = -\frac{1}{12}$$

$$1 + 2 + 4 + 8 + 16 + \dots + 2^n + 2^{n+1} + \dots = -1$$

$$1 + 1 + 1 + 1 + \dots + 1 + 1 + \dots = +\infty$$

$$1 + 2 + 3 + 4 + 5 + \dots + n + (n+1) + (n+2) + \dots = -\frac{1}{12}$$

$$1 + 2 + 4 + 8 + 16 + \dots + 2^n + 2^{n+1} + \dots = -1$$



KAKO MNOŽITI KOMPLEKSNE BROJEVE?

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$$\begin{aligned}(2+i) \cdot (4+2i) &= 2 \cdot 4 + 2 \cdot 2i + i \cdot 4 + i \cdot 2i \\ &= 8 + 4i + 4i + 2i^2 = 8 + 8i - 2 \\ &= 6 + 8i\end{aligned}$$

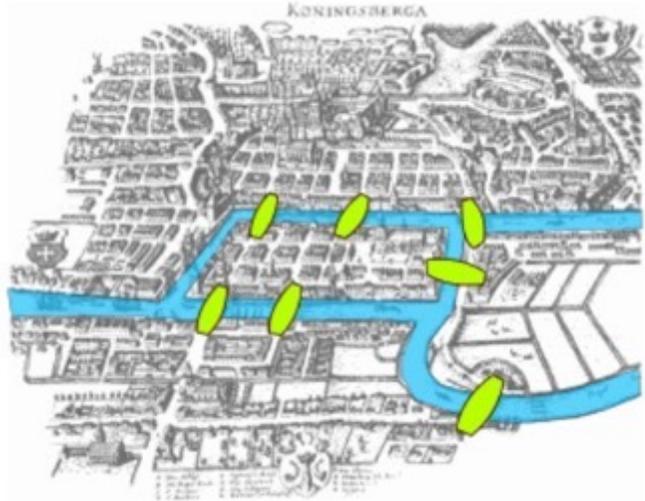


Švicarski matematičar. Jedan od najproduktivnijih znanstvenika u povijesti. Pokrenuo teoriju grafova s pričom o Königsbergškim mostovima.

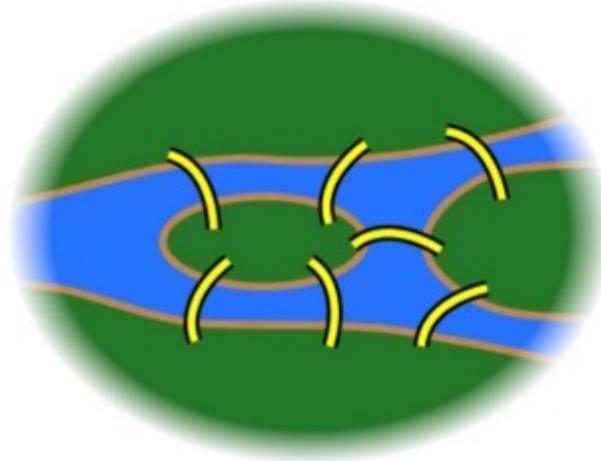
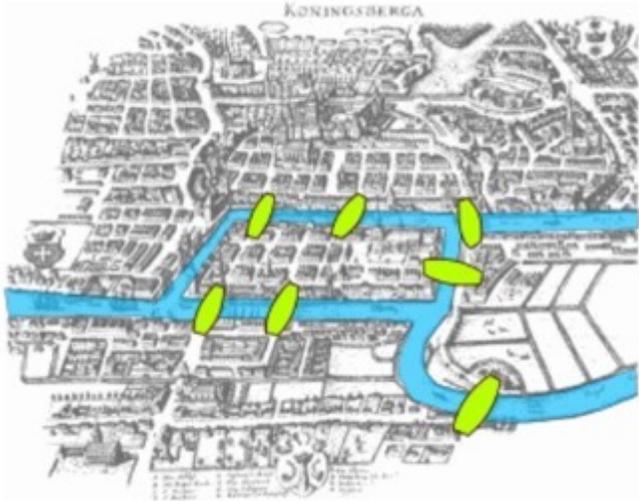


LEONHARD EULER  
15.4.1707. – 18.9.1783.

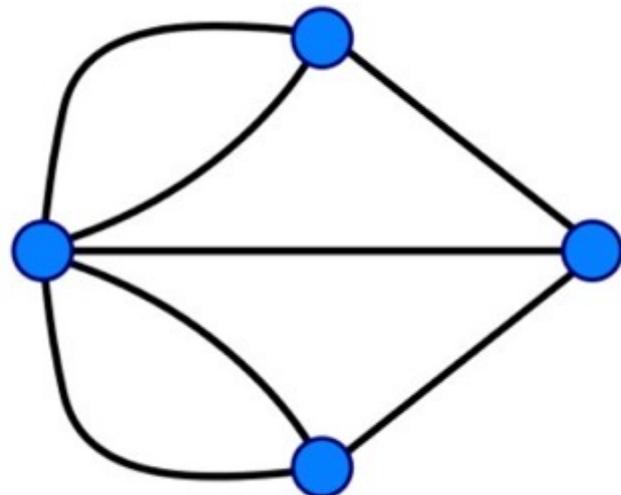
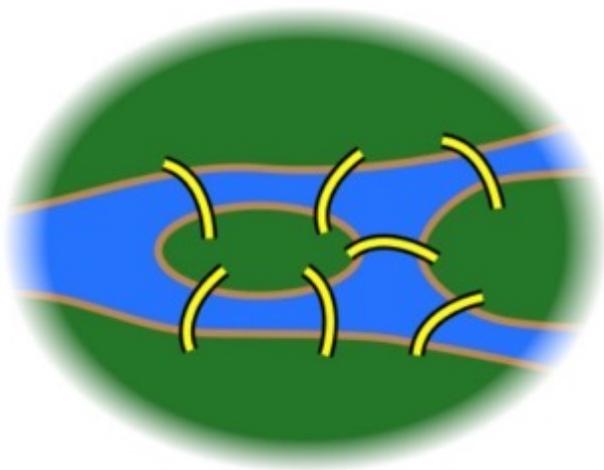
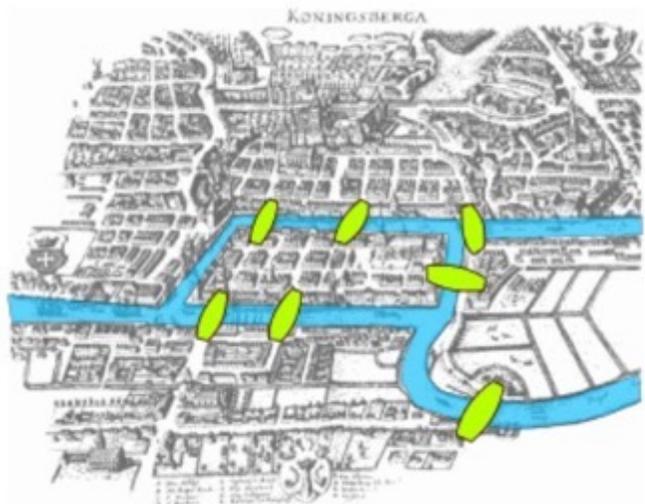
# SEVEN BRIDGES OF KÖNIGSBERG



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# SEVEN BRIDGES OF KÖNIGSBERG



Pokrenno je razvoj  
dviye nove matematične  
grane, TOPOLOGIJU i  
TEORIJU GRAFOVA.



# EULEROVA FORMULA

$$e^{ix} = \cos(x) + i \sin(x)$$



LEONHARD EULER  
15.4.1707. - 18.9.1783.

# EULEROVA FORMULA

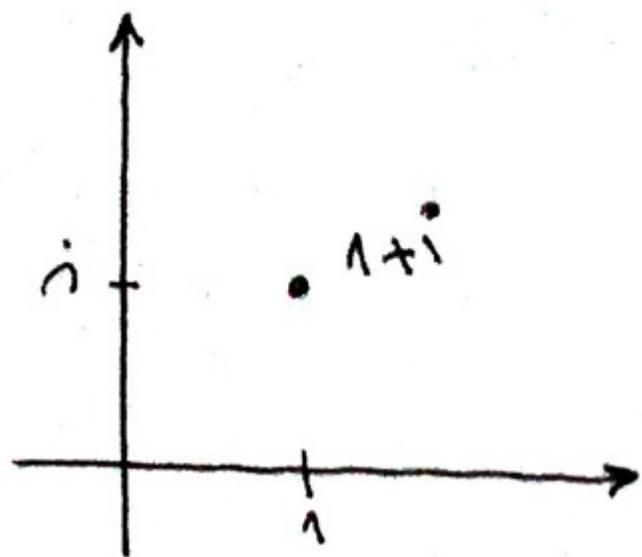
$$e^{ix} = \cos(x) + i \sin(x)$$

$$e^{i\pi} + 1 = 0$$

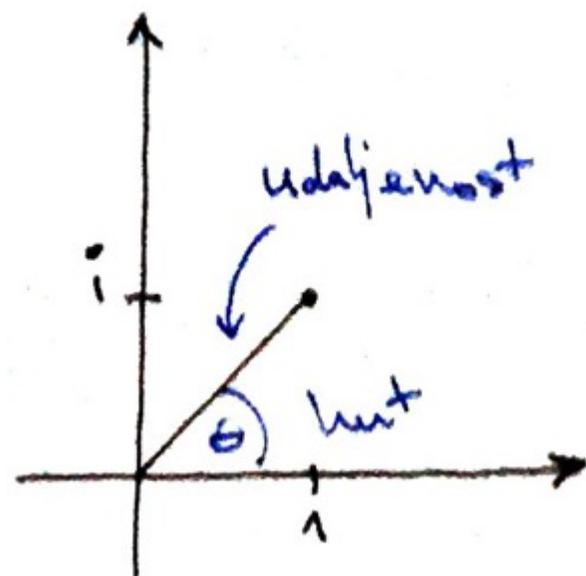
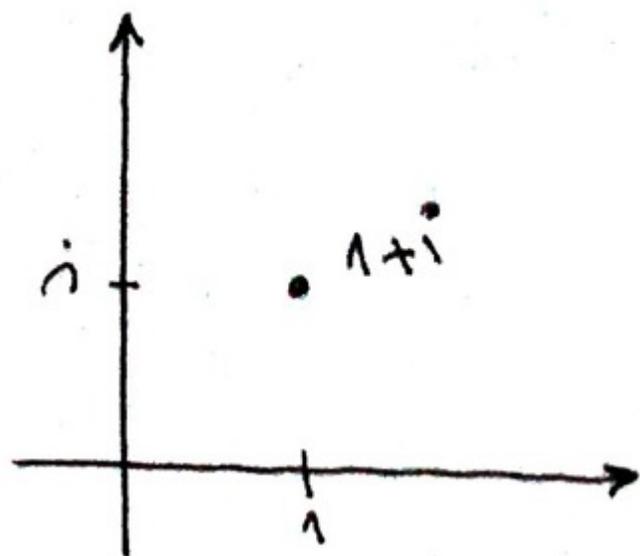


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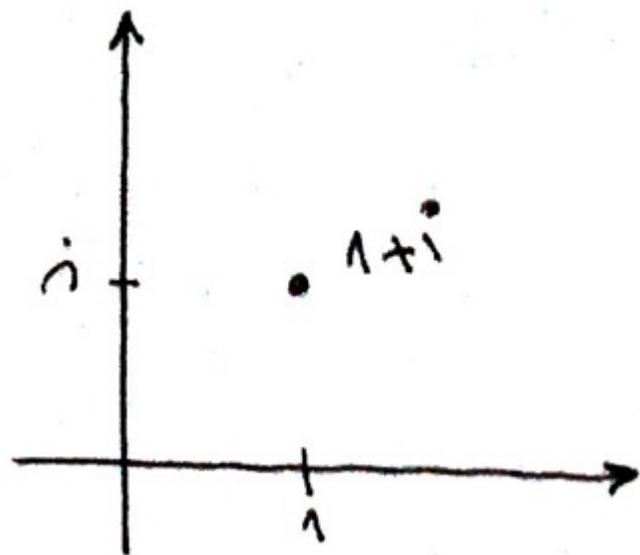
KAKO MNOŽITI KOMPLEKSNE BROJEVE?



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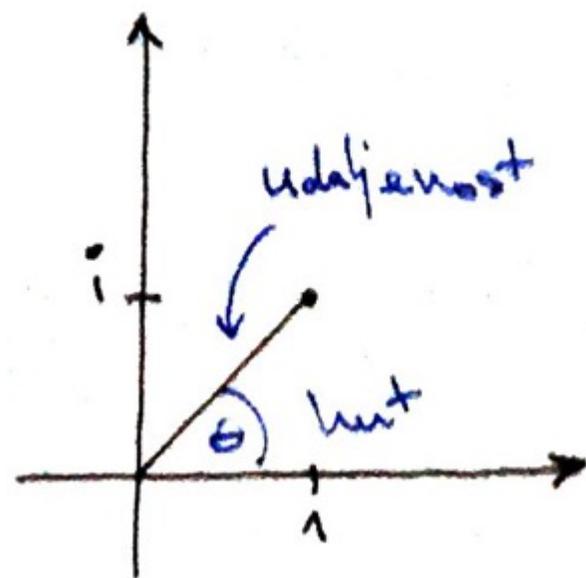


# KAKO MNOŽITI KOMPLEKSNE BROJEVE?



$$\text{UDALJENOST} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\text{KUT} = 45^\circ$$



$$1+i = \sqrt{2} \angle 45^\circ$$

$$1+i = \sqrt{2} e^{i \frac{\pi}{4}}$$

KAKO MNOŽITI KOMPLEKSNE BROJEVE?

$$(1+i) \cdot i = i + i \cdot i = i + i^2 = -1 + i$$

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$$(1+i) \cdot i = \sqrt{2} e^{i \frac{\pi}{4}} \cdot 1 e^{i \frac{\pi}{2}}$$

$$= \sqrt{2} \cdot 1 \cdot e^{i \left( \frac{\pi}{4} + \frac{\pi}{2} \right)}$$

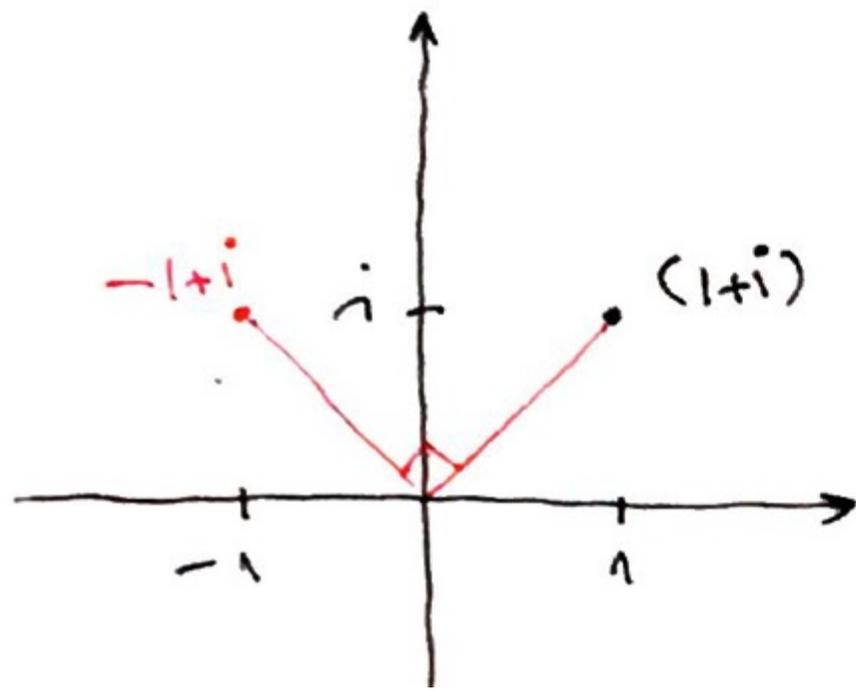
$$= \sqrt{2} e^{i \frac{3\pi}{4}}$$

$$= \sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

$$= \sqrt{2} \cdot \left( -\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) = -1 + i$$

ΚΑΚΟ ΜΝΟΪΤΗ ΚΟΜΠΛΕΚΣΝΕ ΒΡΩΤΕΥΕ?

$$(1+i) \cdot i = i + i \cdot i = i + i^2 = -1 + i$$



ΠΝΟΪΖΕΝΤΕ ΣΙ  
ΠΟΤΙΡΑΛΟ ΤΕ ΒΡΩΤ  
1+i ΖΑ 90°

$$x^3 = 1$$

$$x^3 = 1$$

$$x \cdot x \cdot x = 1$$

$$X^3 = 1$$

$$X \cdot X \cdot X = 1$$

$$1L^* \cdot 1L^* \cdot 1L^* = 1L^0 \text{ ;i} 1L^{\cancel{360}}$$

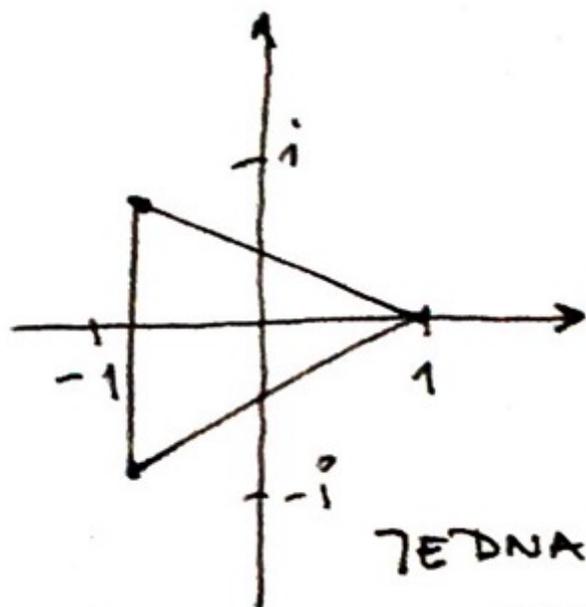
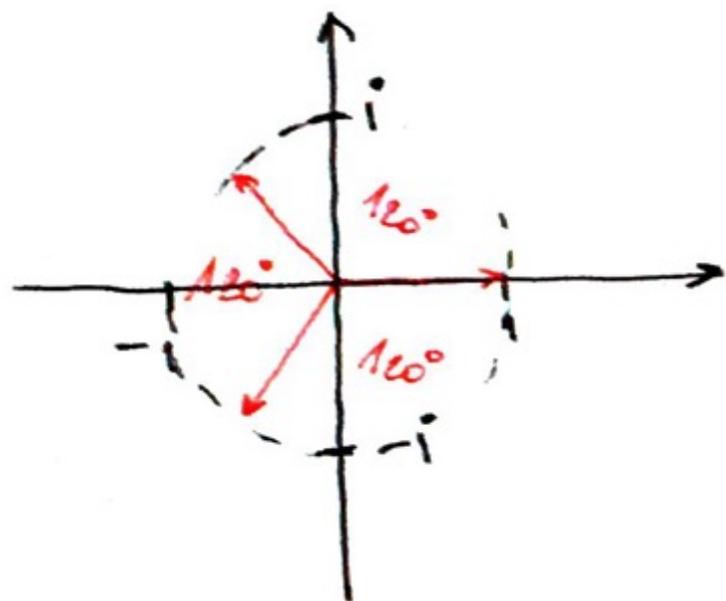
$$L^* + L^* + L^* = 360 \Rightarrow L^* = 120^\circ$$

$$X^3 = 1$$

$$X \cdot X \cdot X = 1$$

$$1 \angle^* \cdot 1 \angle^* \cdot 1 \angle^* = 1 \angle 0 \text{ ili } 1 \angle 360^\circ$$

$$\angle^* + \angle^* + \angle^* = 360 \Rightarrow \angle^* = 120^\circ$$



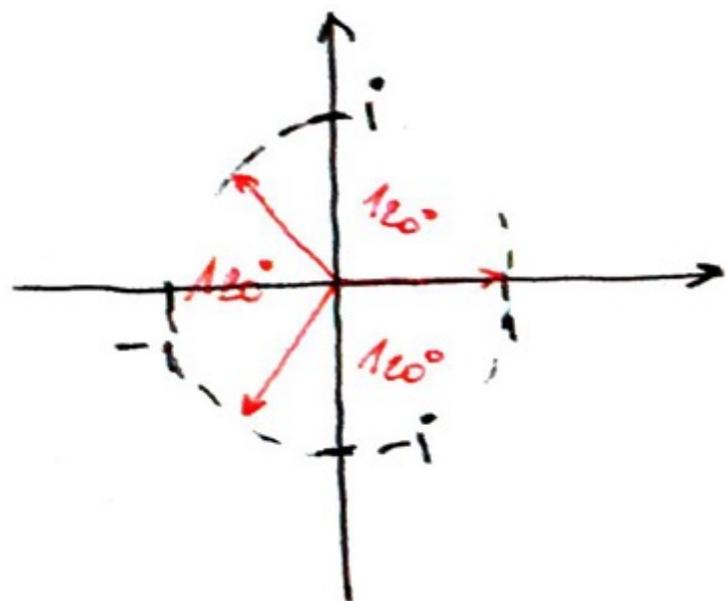
TRÓJNĄK  
JEDNA KO STRONIC  
TROKUT

$$X^3 = 1$$

$$X \cdot X \cdot X = 1$$

$$1 \angle^* \cdot 1 \angle^* \cdot 1 \angle^* = 1 \angle 0 \text{ ili } 1 \angle 360^\circ$$

$$\angle^* + \angle^* + \angle^* = 360 \Rightarrow \angle^* = 120^\circ$$



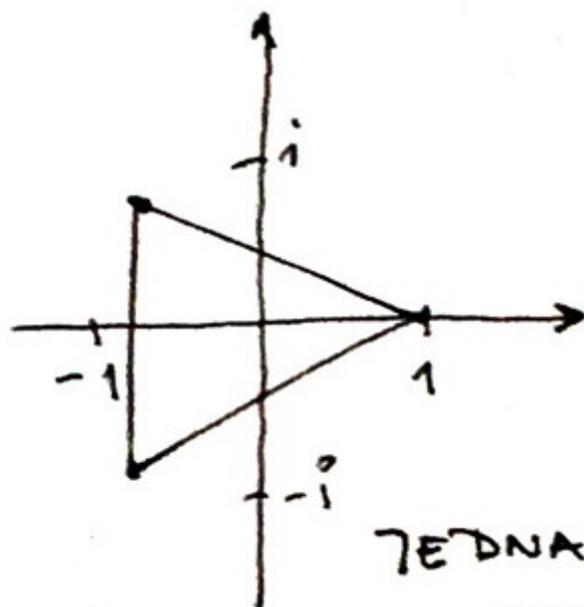
$$X^7 = 1$$

DATE JEDNACOSTRANICAN

SEDMEROKUT

$$X^n = 1$$

DATE PRAVILNI n-TEROKUT



JEDNACOSTRANICAN

TROKUT

BRZA FOURIEROVA TRANSFORMACIJA (FFT)

Složenost proračuna pada s  $O(N^2)$  na  $O(N \log N)$

# BRZA FOURIEROVA TRANSFORMACIJA (FFT)

Složenost proračuna pada s  $O(N^2)$  na  $O(N \log N)$

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i \frac{2\pi kn}{N}}$$

↑  
kompleksni korijen  
iz jedinice

## BRZA FOURIEROVA TRANSFORMACIJA (FFT)

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$$X_k = \sum_{n=0}^{N-1} x_n e^{-i \frac{2\pi kn}{N}}$$

↑  
kompleksni korijen  
iz jedinice

TOP 10  
ALGORITAMA

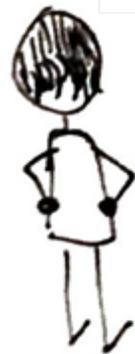
# BRZA FOURIEROVA TRANSFORMACIJA (FFT)

Složenost proračuna pada s  $O(N^2)$  na  $O(N \log N)$

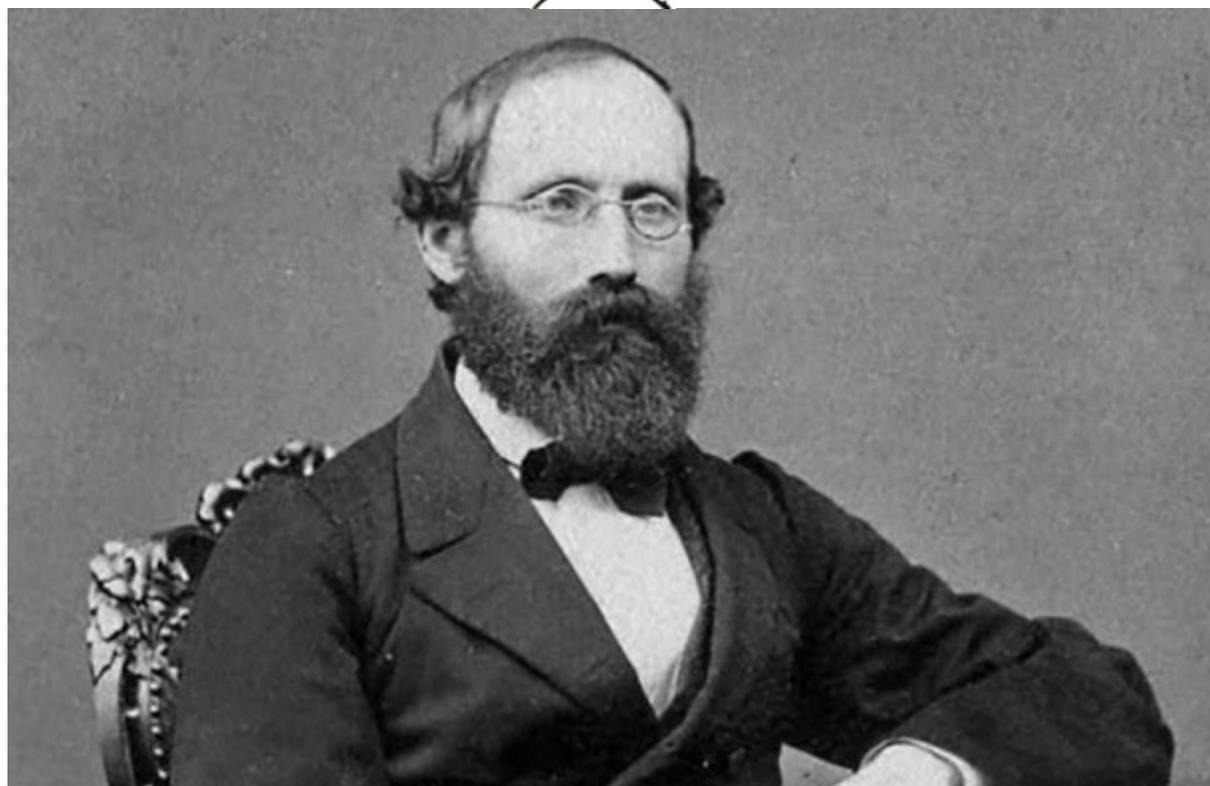
$$X_k = \sum_{n=0}^{N-1} x_n e^{-i \frac{2\pi kn}{N}}$$

kompleksni korijen  
iz jedinice

TOP 10  
ALGORITAMA



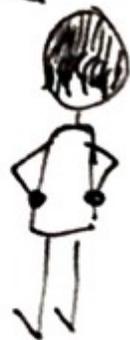
An algorithm is a sequence of finite computational steps that transforms an input into an output.



BERNHARD RIEMANN

17.9.1826. – 20.7.1866.

Njemački matematičar s ogromnim doprinosom u analizi, teoriji brojeva, kompleksnoj analizi i diferencijalnoj geometriji. Postavio temelje opće teorije relativnosti. Najpoznatiji je po RIEMANNOVOJ HIPOTEZI



# RIEMANNOVA ZETA FUNKCIA

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z} = \frac{1}{1^z} + \frac{1}{2^z} + \frac{1}{3^z} + \dots$$

# RIEMANNOVA ZETA FUNKCIJA

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \dots$$



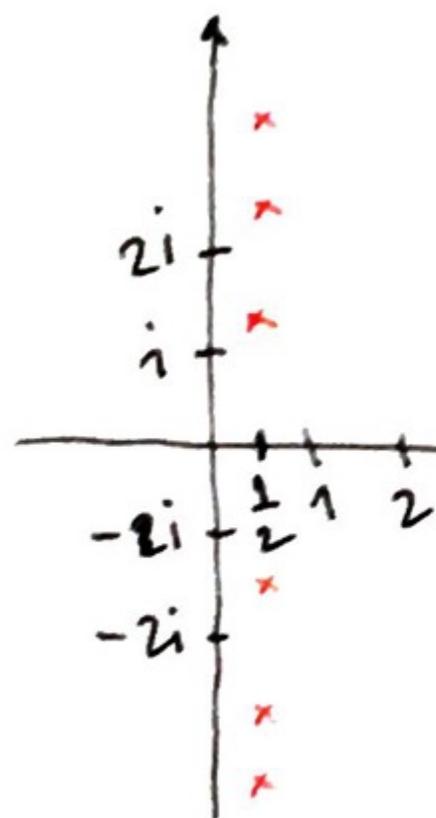
## RIEMANNOVA ZETA FUNKCIJA

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z} = \frac{1}{1^z} + \frac{1}{2^z} + \frac{1}{3^z} + \dots$$

## PILEMITSKI PROBLEM

### RIEMANNOVA HIPOTEZA

Sve netrivialne nultocke Riemannove zeta funkcije imaju realni dio jednak  $\frac{1}{2}$ .



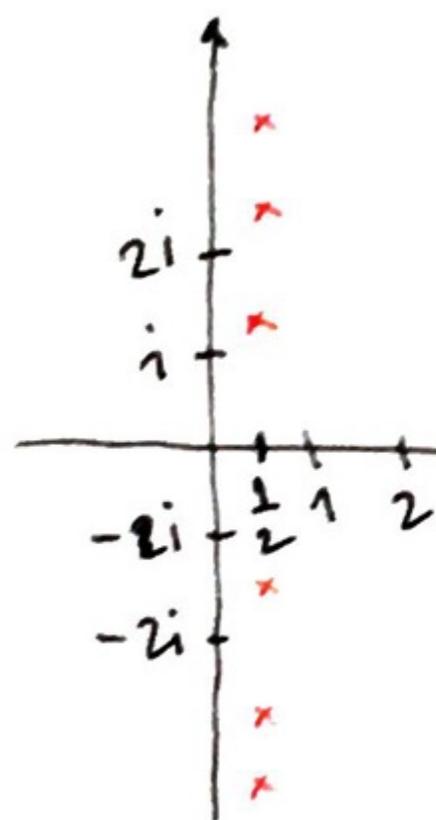
DANAS ZNAMO PRVIH

10 000 000 000 000

NULTOČAKA I SVE

IMAJU REALNI

DIO JEDNAK  $\frac{1}{2}$



DANAS ZNAMO PRVIH

10 000 000 000 000

NULTOČAKA I SVE

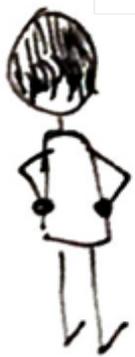
IMAJU REALNI

DIO JEDNAK  $\frac{1}{2}$

ŠTO JE 10 000 000 000 000

NAPRAMA BESKONAČNOSTI?

David Hilbert je jedan od najvećih matematičara u povijesti. Radio na razvoju varijacijskog računa, algebre, algebarske teorije brojeva, geometrije, teorije operatora, fizike i temelja matematike. Radio s Einsteinom na općoj relativnosti.



DAVID HILBERT

23.1.1862. - 14.2.1943

HILBERT: 23 NAJBITNIJA PROBLEMA  
KOJA TREBA RIJEŠITI U 20. STOLJEĆU



DAVID HILBERT  
23.1.1862. - 14.2.1943

# HILBERT: 23 NAJBITNIJA PROBLEMA KOJA TREBA RIJEŠITI U 20. STOLJEĆU

Problem	Brief explanation	Status	Year Solved
1st	The <i>continuum hypothesis</i> (that is, there is no set whose cardinality is strictly between that of the integers and that of the real numbers)	Proven to be impossible to prove or disprove within Zermelo–Fraenkel set theory with or without the Axiom of Choice (provided Zermelo–Fraenkel set theory is consistent, i.e., it does not contain a contradiction). There is no consensus on whether this is a solution to the problem.	1940, 1963
2nd	Prove that the axioms of arithmetic are consistent.	There is no consensus on whether results of Gödel and Gentzen give a solution to the problem as stated by Hilbert. Gödel's second incompleteness theorem, proved in 1931, shows that no proof of its consistency can be carried out within arithmetic itself. Gentzen proved in 1936 that the consistency of arithmetic follows from the well-foundedness of the ordinal $\epsilon_0$ .	1931, 1936
3rd	Given any two polyhedra of equal volume, is it always possible to cut the first into finitely many polyhedral pieces that can be reassembled to yield the second?	Resolved. Result: No, proved using Dehn invariants.	1900
4th	Construct all metrics where lines are geodesics.	Too vague to be stated resolved or not. <sup>[1]</sup>	–
5th	Are continuous groups automatically differential groups?	Resolved by Andrew Gleason, depending on how the original statement is interpreted. If, however, it is understood as an equivalent of the Hilbert–Smith conjecture, it is still unsolved.	1953?
6th	Mathematical treatment of the axioms of physics (a) axiomatic treatment of probability with limit theorems for foundation of statistical physics (b) the rigorous theory of limiting processes "which lead from the atomistic view to the laws of motion of continua"	Partially resolved depending on how the original statement is interpreted. <sup>[1]</sup> Items (a) and (b) were two specific problems given by Hilbert in a later explanation. <sup>[1]</sup> Kolmogorov's axiomatics (1933) is now accepted as standard. There is some success on the way from the "atomistic view to the laws of motion of continua." <sup>[1][2]</sup>	1933–2002?
7th	Is $a^b$ transcendental, for algebraic $a \neq 0, 1$ and irrational algebraic $b$ ?	Resolved. Result: Yes, illustrated by Gelfond's theorem or the Gelfond–Schneider theorem.	1934
8th	The Riemann hypothesis ("the real part of any non-trivial zero of the Riemann zeta function is $1/2$ ") and other prime number problems, among them Goldbach's conjecture and the twin prime conjecture	Unresolved.	–
9th	Find the most general law of the reciprocity theorem in any algebraic number field.	Partially resolved. <sup>[1]</sup>	–
10th	Find an algorithm to determine whether a given polynomial Diophantine equation with integer coefficients has an integer solution.	Resolved. Result: Impossible, Matijasevič's theorem implies that there is no such algorithm.	1970
11th	Solving quadratic forms with algebraic numerical coefficients.	Partially resolved. <sup>[1][2]</sup>	–
12th	Extend the Kronecker–Weber theorem on Abelian extensions of the rational numbers to any base number field.	Unresolved.	–
13th	Solve 7-th degree equation using algebraic (variant: continuous) functions of two parameters.	The problem was partially solved by Vladimir Arnold based on work by Andrei Kolmogorov. <sup>[1]</sup>	1957
14th	Is the ring of invariants of an algebraic group acting on a polynomial ring always finitely generated?	Resolved. Result: No, a counterexample was constructed by Masayoshi Nagata.	1959
15th	Rigorous foundation of Schubert's enumerative calculus.	Partially resolved.	–
16th	Describe relative positions of ovals originating from a real algebraic curve and as limit cycles of a polynomial vector field on the plane.	Unresolved, even for algebraic curves of degree 8.	–
17th	Express a nonnegative rational function as quotient of sums of squares.	Resolved. Result: Yes, due to Emil Artin. Moreover, an upper limit was established for the number of square terms necessary.	1927
18th	(a) Is there a polyhedron that admits only an anisohedral tiling in three dimensions? (b) What is the densest sphere packing?	(a) Resolved. Result: Yes (by Karl Reinhardt). (b) Widely believed to be resolved, by computer-assisted proof (by Thomas Callister Hales). Result: Highest density achieved by close packings, each with density approximately 74%, such as face-centered cubic close packing and hexagonal close packing. <sup>[1]</sup>	(a) 1928 (b) 1998
19th	Are the solutions of regular problems in the calculus of variations always necessarily analytic?	Resolved. Result: Yes, proven by Ennio de Giorgi and, independently and using different methods, by John Forbes Nash.	1957
20th	Do all variational problems with certain boundary conditions have solutions?	Resolved. A significant topic of research throughout the 20th century, culminating in solutions for the non-linear case.	?
21st	Proof of the existence of linear differential equations having a prescribed monodromic group	Partially resolved. Result: Yes/No/Open depending on more exact formulations of the problem.	?
22nd	Uniformization of analytic relations by means of automorphic functions	Unresolved.	?
23rd	Further development of the calculus of variations	Too vague to be stated resolved or not.	–



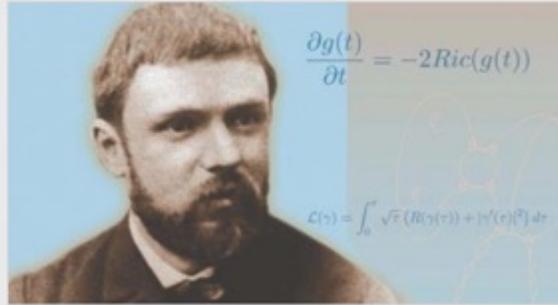
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23.1.1862. – 14.2.1943



## Call for Proposals

CMI invites proposals under the Enhancement and Partnership Program for fiscal year 2020-21 (1 October 2020-30 September 2021) and later. The principal aim of the program is to enhance activities that are already planned and financially viable. The next deadline for submissions is 1 September 2019.



2000. 8 MILENITSKIH PROBLEMA  
- nagrada MILIJUN DOLARA

### Millennium Problems

#### Yang-Mills and Mass Gap



Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

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Congratulations to former Clay Research Fellows Manjul Bhargava

(Princeton) and Akshay Venkatesh (IAS) and former Clay Senior Scholar Christopher Hacon (Utah) on their election as Fellows of the...

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[More news](#)

### Events

#### Quantum Field Theory and Manifold Invariants

Sunday, June 30, 2019

#### Thermodynamic Formalism: Applications to Probability, Geometry and Fractals

Monday, July 1, 2019

#### Aspects of Geometric Group Theory

Monday, July 8, 2019

#### PROMYS Europe

Sunday, July 14, 2019

#### 2019 Clay Research Conference and Workshops

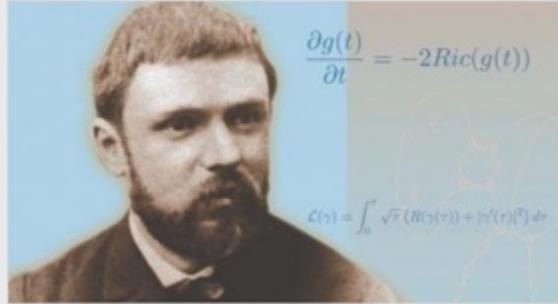
Sunday, September 29, 2019

[More events](#)



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P versus NP

Hodge Conjecture

Riemann Hypothesis

Yang-Mills existence and mass gap

Navier-Stokes existence and smoothness

Birch and Swinnerton-Dyer Conjecture

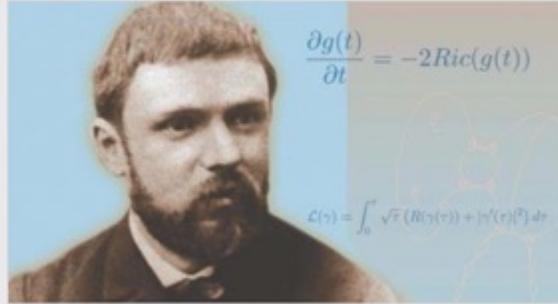
Poincare Conjecture





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Aspects of Geometric Group Theory  
Monday, July 8, 2019

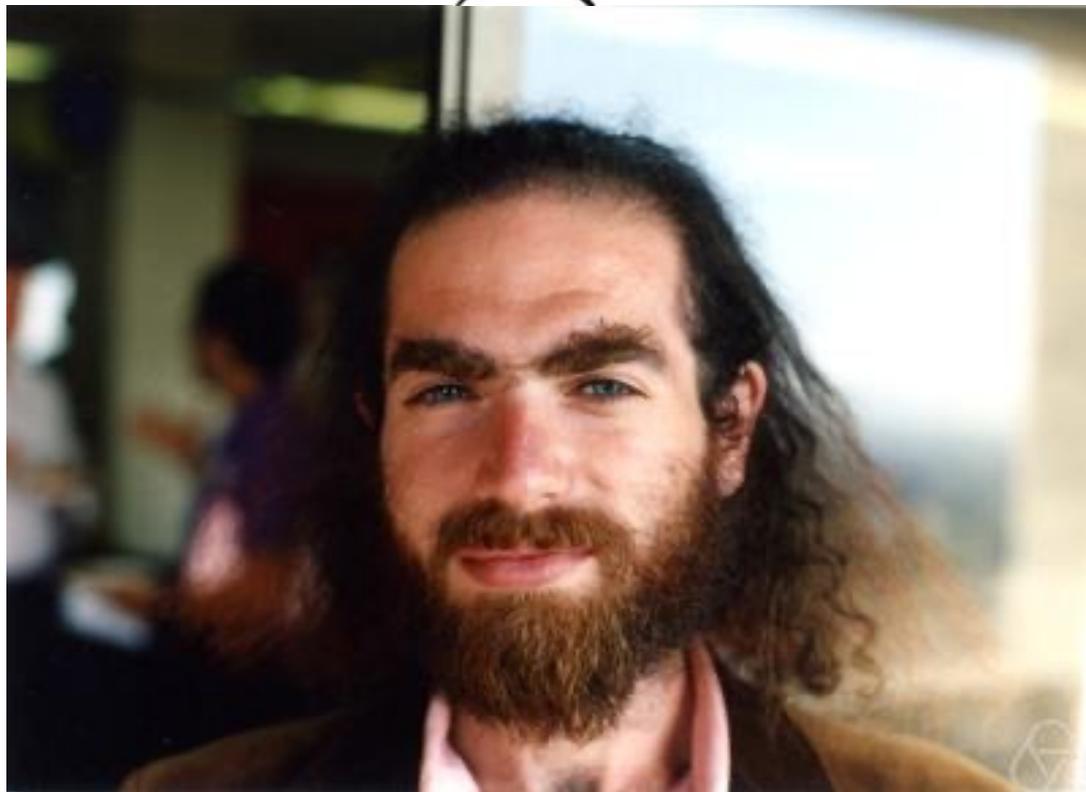
PROMYS Europe  
Sunday, July 14, 2019

2019 Clay Research Conference and Workshops  
Sunday, September 29, 2019

[More events](#)

IMA 1 LAKŠIH NAČINA  
ZA ZARADITI MILIJUN  
DOLARA



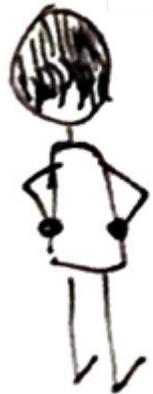


GRIGORI YAKOVLEVICH PERELMAN

13.6.1966. —

2002. godine objavio na internetu  
znanstvene radove u kojima  
je dokazao Poincaréovu slutnju.  
Odbio Fieldsovu medalju i  
milijun dolara nagrade. Ne daje  
intervjue. Kaže da su znanstvenici  
pdivareni i da bradu ideje te da  
on ne želi sudjelovati u tome.



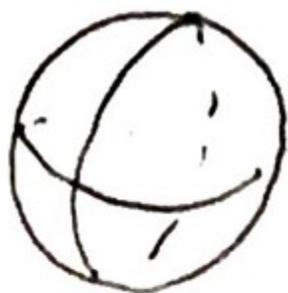


Francuski matematičar. Pokrenuo  
poje TOPOLOGIJE u današnjem  
smislu. Prvi je predvidio postojanje  
gravitacijskih valova, razumio  
relativnost i napisao  $E=mc^2$



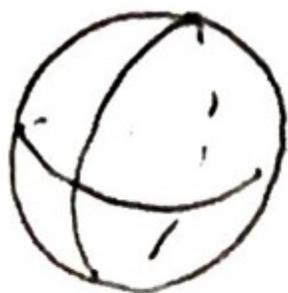
JULES HENRI POINCARÉ  
29. 4. 1854. - 17. 7. 1912.

# POINCARÉOVA SLUTNJA



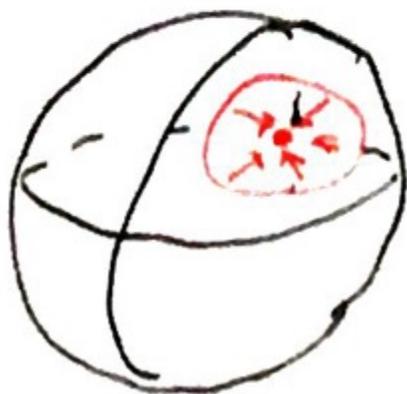
2-sfera

# POINCARÉOVA SLUTNJA



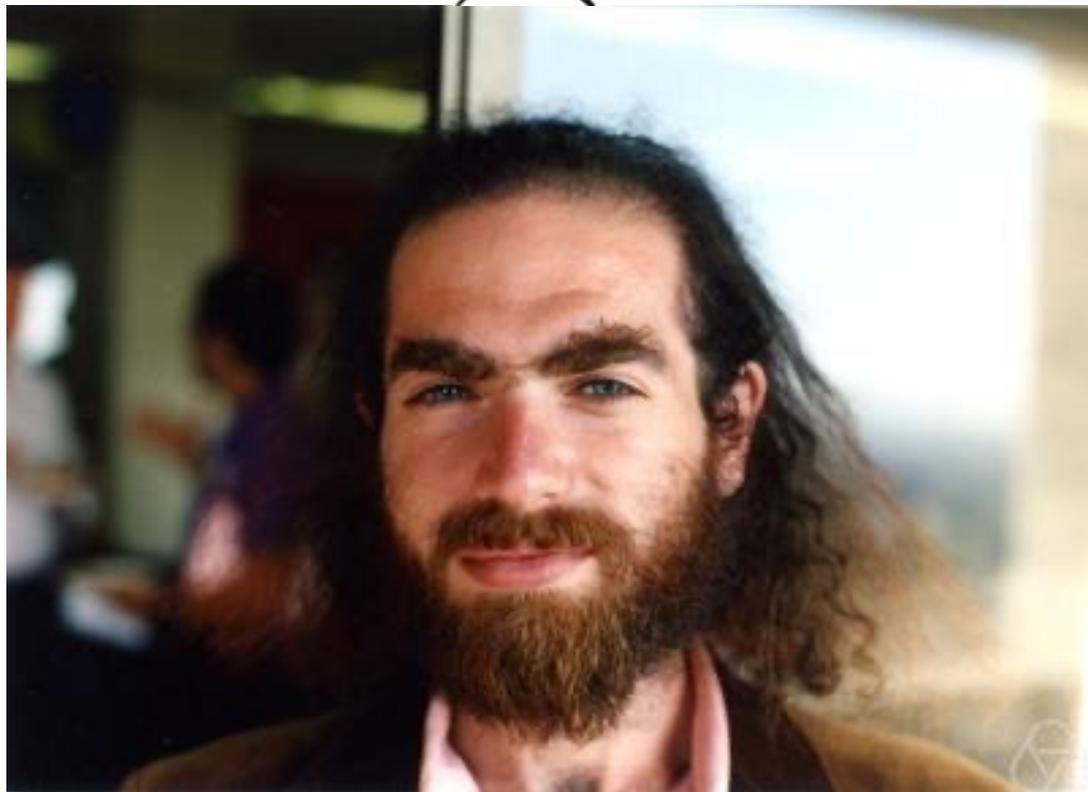
2-sfera

Svaka petlja može se  
stisnuti u točku



## POINCARÉOVA SLUTNĀ

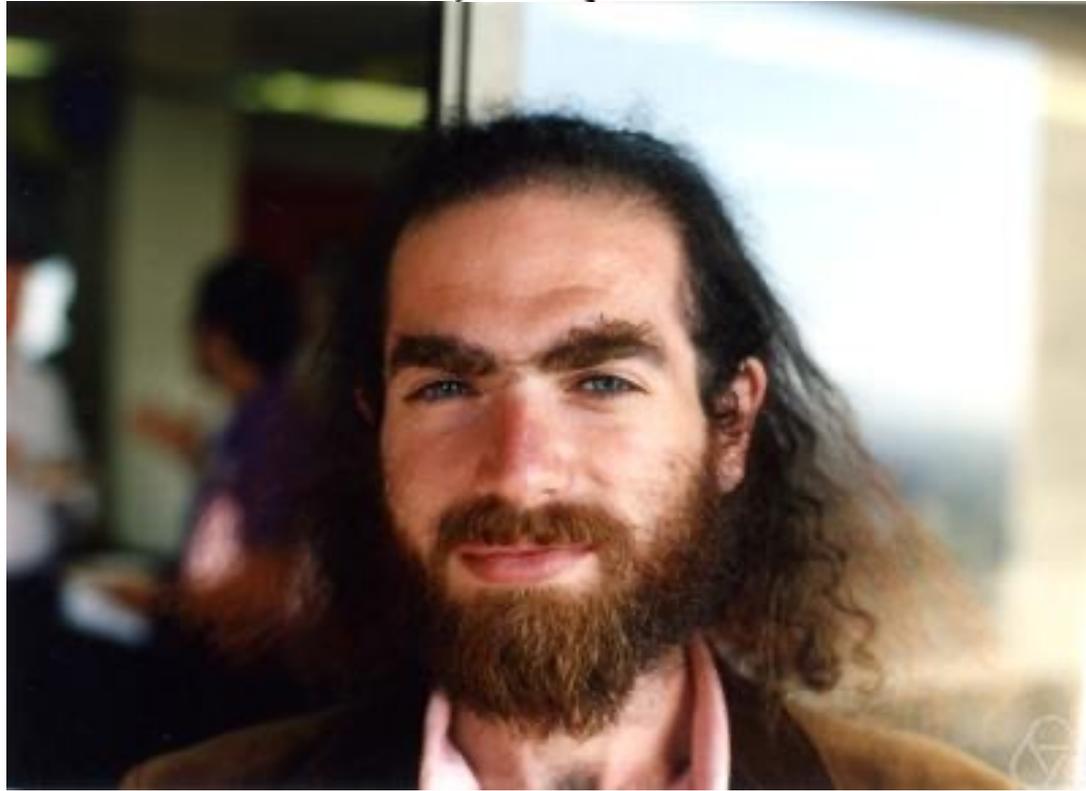
Every simply connected, closed  
3-manifold is homeomorphic  
to the 3-sphere.



GRIGORI YAKOVLEVICH PERELMAN  
13.6.1966. —

Odbio je milijun dolara i  
nakon što ga je zvao  
PUTIN osobno.





GRIGORI YAKOVLEVICH PERELMAN

13.6.1966. —

Emptiness is everywhere  
and it can be calculated,  
which gives us a great  
opportunity. I know how  
to control the universe.  
So tell me, why should  
I run for a million?

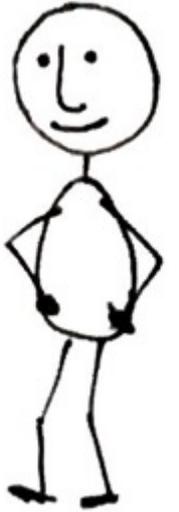


Emptiness is everywhere and it can be calculated, which gives us a great opportunity. I know how to control the universe. So tell me, why should I run for a million?

Grigori Perelman



OK. To su sada validni  
svi brojevi koje  
trebamo.





OK. To su sada valjda  
svi brojevi koje  
trebamo.

Čekaj, čekaj, a HIPERKOMPLEKSNI  
BROJEVI, KVATERNIONI...



