Informational gain guided model predictive motion planning for autonomous underwater vehicles

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Abstract— **During sea floor exploration missions, it is often of great importance to get the sonar scans of some interesting areas as detailed as possible, while some less informational areas can even be left out. In contrast to the standard coverage solutions, i.e. the lawnmower pattern, which only geometrically solve the problem, here a novel solution for adaptive sonar scanning is proposed. The solution is based on the estimated informational gain of the scanned sea floor, which leads the path planner to give maximum information paths for the vehicle to follow. Model predictive controller has the task to generate feasible and optimized system state and control trajectories to follow the reference path, or to incorporate the path planner and/or informational gain in its optimal control problem's criterion.**

Index Terms— **model predictive control, path planning, path following, motion planning, autonomous underwater vehicles, sonar scanning**

I. Introduction

It is known that 71% of planet Earth's surface is covered with water. Saltwater (seas and oceans) takes approximately 97.5% of this amount, and less than 2.5% of seas and oceans have been explored. This fact alone shows how important the exploration of the marine environments is, and that it has a great potential for research in many fields of science. An important part of the marine exploration is, among others, exploration of the sea bottom. This includes the exploration of the biosphere (e.g. sea weed Posidonia, animal species such as shells, coral reefs etc.), exploration of underwater archaeological sites (e.g. sunken historical ships, sunken ancient settlements and cities etc.), marine safety: searching for naval mines and neutralizing them, patrolling along the maritime borders, and many other applications.

Many of the above mentioned missions include sonar based sea bottom scanning. The lawnmower pattern is one of the most commonly used solutions for the 2D coverage problem of the area to be scanned. In marine robotics, it is typically used as a reference path for vehicles used for sonar scanning, namely remotely operated vehicles (ROVs), autonomous underwater vehicles (AUVs), autonomous surface vehicles (ASVs), etc. For a marine vehicle to be able to execute these missions, it has to have a path planning and motion control module in its control architecture.

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Our long-term research goal is to develop an adaptive sea floor sonar scanning algorithm, in such a way that the marine vehicle does not necessarily need to traverse the whole length of all the lawnmower's legs. During the mission, this algorithm would steer the vehicle to scan some interesting areas in more detail, while skipping areas which are less interesting, as shown in Fig. 1. The marine vehicle starts the mission (from the lower left waypoint "start", see Fig. 1) of sonar scanning by following the waypoints of the set lawnmower pattern (shown as red circles). The vehicle in this setting is equipped with a forward looking sonar (shown as white lined triangles). The vehicle scans the sea bottom and estimates its informational gain (blue areas denote low, yellow medium, and red areas denote high informational gain areas). As it spots some medium and high informational gain areas in its sonar range, it diverges from the low informational gain area, i.e. the current lawnmower leg, and moves along the maximum informational gain path until it reaches the next waypoint. The path of the vehicle is shown as a series of green circles. Moving from the third waypoint (upper right red circle) to the fourth waypoint (lower right red circle), the vehicle scans the already detected interesting area in more detail, and in the end converges to the end waypoint.

The definition of which areas are interesting and which are not, i.e. some measure of information gain, depends on the mission itself, but also on the scientific

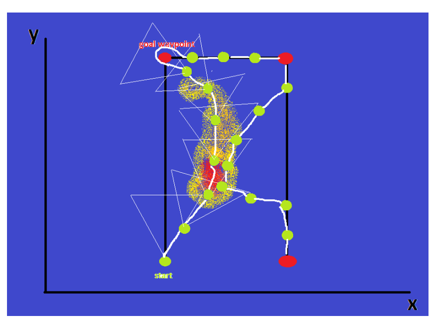

Fig. 1. An example of the AUV behavior in the presence of the interesting parts of the sea floor out of the lawnmower lines. Red circles denote the lawnmower's waypoints, green circles denote the vehicles position in discrete time intervals, while white lined triangles denote the forward-looking sonar range. Blue, yellow and red areas denote low, medium, and high information gain areas of the sea floor, respectively.

field for which the marine vehicle is being utilized. In the field of marine biology, coral reefs can be detected by measuring the sea floor roughness. In other words, coral reefs cause a greater elevation variation in the sea floor scans, than for example barren sand fields. In fields such as marine archaeology and safety, informational gain can be an output of some shape/pattern recognition algorithm, where some structures specified by the mission itself bring more information than other structures.

With this in mind, we plan to use model predictive control (MPC) as a control methodology. MPC has the advantage that it can include a function of the above mentioned informational gain into its cost function, and give such controls which maximize the informational gain. Another advantage of MPC is that, during control optimization, it explicitly takes into account the constraints on system states and controls, thus generating dynamically feasible solutions.

An overview of the algorithms needed to accomplish the set goal is given in this paper. Namely, mathematical models of the AUV and path following are given in Section II. Classical algorithms for path planning in general, with a special reference to the ones used in marine robotics, as well as the classical guidance and control methods for line following, are presented in Section III. Section IV gives a short introduction to the MPC and also an overview of the MPC schemes used for navigation of ground robots on rough terrains, as well as MPC based motion planning frameworks in marine robotics. The proposed conceptual solution of the above formulated research problem is described in Section V.

II. Mathematical modeling

A. Underwater marine vehicle model

In order to discuss navigation, guidance, and control of the (underactuated) underwater marine vehicle, it is necessary to acquire the mathematical model of the vehicle. The notation and the model given in this paper follow the notation defined in [1]. The AUV LUPIS, which will be used for research, is an underactuated 3-DOF marine vehicle. It is controllable w.r.t. the linear speeds of surge and heave, and also w.r.t. the rotational yaw rate. Both the kinematic and the dynamic models of the vehicle, given below, are expressed in the body-fixed coordinate frame.

Lets denote the velocity vector with $\boldsymbol{\nu} = [u\,w\,r]^T$ where u, w, and r are surge, heave and yaw speed, respectively. Also, lets denote the vector of position/Euler angles with $\boldsymbol{\eta} = [x\,y\,z\,\psi]^T$, where x, y, z, and ψ denote the x, y, and z coordinates of the vehicle's position, respectively, while ψ denotes the yaw angle or heading of the vehicle. Since the vehicle has to maintain a constant depth during the sonar scanning missions, the generalized kinematic model of the vehicle, given as $\boldsymbol{\eta} = \boldsymbol{J}_\Theta(\boldsymbol{\eta})\boldsymbol{\nu}$, can be rewritten as:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ r \end{bmatrix} \tag{1}$$

The dynamic model of the vehicle is given in a general matrix form:

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} = -\boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} - \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{\tau} + \boldsymbol{g}(\boldsymbol{\eta}), \tag{2}$$

where $\boldsymbol{M}$ is the matrix of system inertia, $\boldsymbol{C}(\boldsymbol{\nu})$ is the Coriolis matrix, $\boldsymbol{D}(\boldsymbol{\nu})$ is the damping matrix, $\boldsymbol{\tau} = [X\,Z\,N]^T$ is the vector of forces and moments which the actuators apply to the vehicle, where X, Z, and N are surge and heave forces, and yaw moment, respectively. Vector of generalized gravitational and buoyancy forces is denoted by $\boldsymbol{g}(\boldsymbol{\eta})$ [1].

B. Path following models

There are three possible motion control scenarios, and they are described as follows. The first one is setpoint regulation, i.e. point-stabilization, in which only the (constant) position and orientation (attitude) references are set for the vehicle to achieve. The second scenario is trajectory tracking, in which the reference values for system states are parametrized by time. The third scenario is path following/tracking, in which the desired reference values are not parametrized by time [1]. In this scenario, the geometric properties of the path to be followed are used in order to achieve system's convergence to the desired path. The path following models are given in this Subsection.

1) Generalized path following: Lets consider the vehicle as a point mass particle in a 2D plane. Its position is denoted as $\mathbf{p} = [x,\, y]^T$, and its speed is denoted as $\mathbf{v} = [v_x,\, v_y]^T$. The modulus of the speed vector is denoted by $U = ||\mathbf{v}||_2$, and it's orientation by $\psi = \arctan(v_y/v_x)$. The desired path, defined as a curve in a plane, is denoted by $\mathbf{p}_d(\theta) = [x_d(\theta),\, y_d(\theta)]^T$, where θ is a scalar parameter of the curve's shape.

The main objective of the path following problem is that the vehicle converges to, and stays on the desired path. Let us assume that the value θ^* (see Fig. 2), which minimizes the Euclidean distance between $\mathbf{p}$ and $\mathbf{p_d}$, is known. The local reference frame at $\mathbf{p_d}$ should now be rotated for an angle:

$$\psi_d(\theta^*) = \arctan\left(\frac{y_d'(\theta)}{x_d'(\theta)}\right) \tag{3}$$

relative to the inertial I frame and thus transformed into the PP frame. Notation $x_d'(\theta) = \frac{dx_d}{d\theta}(\theta)$ has been used. This means that the x-axes of the local reference frame at $\mathbf{p_d}$ is aligned with the tangential vector to the path at $\mathbf{p_d}(\theta^*)$, see Fig. 2.

The error vector between $\mathbf{p}$ and $\mathbf{p_d}(\theta^*)$ expressed in the rotated PP frame is given by:

$$\epsilon = \mathbf{R}_p^T(\psi_d)\left(\mathbf{p} - \mathbf{p}_d(\theta^*)\right), \tag{4}$$

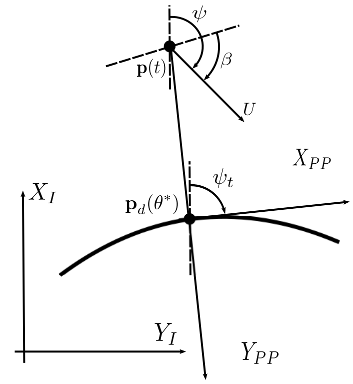

Fig. 2. Geometrical representation of the general path following problem [2].

where

$$\mathbf{R}_p^T(\psi_d) = \begin{bmatrix} \cos\psi_d & -\sin\psi_d \\ \sin\psi_d & \cos\psi_d \end{bmatrix} \tag{5}$$

is the rotation matrix from the inertial I frame to the PP frame. By definition $\epsilon = [0, d]^T$, where d is called cross-track error, and it represents the lateral distance of the vehicle to the path-tangent at $\mathbf{p_d}(\theta^*)$, which is illustrated in Fig. 2. The overall task of the path following is to achieve $d \to 0$ as $t \to \infty$ [2].

The basic model of the path following in this case is given by:

$$\begin{aligned} \dot{d} &= U\sin(\psi - \psi_d) \\ \dot{\psi} &= r. \end{aligned} \tag{6}$$

2) Line-of-sight (LOS) path following model: Generally speaking, the control based on the model given by (6) forces the vehicle to approach the line as soon as possible, saturating the actuators, and causing the vehicle to overshoot over the set path. It is better to use common sense reasoning and approach the line by following the so-called line-of-sight point with coordinates (x_{LOS}, y_{LOS}) at a distance of n vehicle lengths L_{pp} ahead of the vehicle. This way, the vehicle approaches the path much smoother, and usually without the above mentioned overshoot. It is important to mention that LOS approach assumes that the path which is followed is defined as a line, or as a set of linearly interpolated waypoints $\{\dots, W_{k-1} = (x_{k-1}, y_{k-1}), W_k = (x_k, y_k), \dots\}$ in a horizontal plane.

Now the reference heading (yaw angle) ψ_d, the so-called LOS angle ψ_{LOS}, is calculated as [3]:

$$\psi_d = \psi_{LOS} = \arctan\left(\frac{y_{LOS} - y(t)}{x_{LOS} - x(t)}\right) \tag{7}$$

The interested reader is referred to [3] and the references therein for other definitions of the LOS angle.

It is possible to substitute the heading error $\beta = (\psi - \psi_d)$ in (6) with a virtual input for stabilizing d, as:

$$\beta(e) = \beta_{LOS} = -\arctan\left(\frac{d}{\Delta}\right), \tag{8}$$

where $\Delta > 0$ is a guidance parameter shaping the convergence of the vehicle to the path. It is a so-called lookahead distance [2].

Other definitions of this so-called LOS angle could be [4]:

$$\beta_{LOS}(d) = -\frac{d}{\sqrt{d^2 + \Delta^2}} \tag{9}$$

Using linear-like expression for ψ_{LOS} and β_{LOS} instead of $\arctan$-like functions has been elaborated in [5].

In the case of general path following, heading error dynamics can be modeled as [6]:

$$\begin{aligned} \dot{\beta}_{LOS} &= \dot{\psi} - \dot{\psi}_d \\ &= \frac{\kappa}{1 - d\kappa}(u\sin\beta - v\cos\beta) + r, \end{aligned} \tag{10}$$

where κ denotes the curvature of the path to be followed. The path curvature is calculated in the intersection point of the line passing through the vehicle position point (which is orthogonal to the set path), and the path itself.

All of the above described approaches do not take into account the possible underactuated properties of the marine vehicles following a path under the influence of the disturbance, i.e. sea current. An extension of the LOS approach, called Integral LOS (ILOS) is presented in [7], solves this problem.

3) Line following model: Assuming that the marine vehicle is moving in a horizontal plane (at a constant depth), pitch, roll, heave, and sway motions can be neglected. With sonar scanning in mind, it is preferable for the marine vehicle to maintain a constant surge speed with respect to water $U_r = const. > 0$, in order to get sonar measurements in equidistant points along the followed lawnmower lines. The task of the control algorithm is to steer the vehicle to follow the current straight line of the lawnmower pattern, thus reducing the distance to the line, denoted as d, to zero.

Taking into account all the above mentioned model simplifications due to vehicle's movement in the horizontal plane, vehicle's position and orientation $[x\,y\,\psi]$ in the earth-fixed frame $\langle e \rangle$ are expressed as

$$\dot{x} = U_r\cos\psi + \nu_x \tag{11}$$
$$\dot{y} = U_r\sin\psi + \nu_y \tag{12}$$
$$\dot{\psi} = r \tag{13}$$

where ν_x, ν_y are x and y components of the current speed, respectively, and r is yaw rate [8].

In case that the sea current, which is considered to be a disturbance to the system, is modeled, we obtain the following set of equations for the kinematic model of the line following in the horizontal plane

$$\dot{d} = U_r \sin\beta + \nu \simeq U_r\beta + \nu \tag{14}$$
$$\dot{\beta} = r \tag{15}$$

where the symbol $\simeq$ denotes linear approximation for small values of β, and ν is the intensity of the sea current speed vector projection onto a vector orthogonal to the followed straight line [8].

III. Classical navigation, guidance, and control methods in marine robotics

A. Path planning algorithms used in marine robotics

For a vehicle to reach some predefined goal position, starting from some arbitrary position, while at the same time avoiding the obstacles, it has to use some kind of navigation to successfully execute this task. The most often used path planning algorithms which solve this navigation problem, are: A* for known grid-based maps of the environment, D*-like algorithms for unknown or partially known grid based maps of the environment, Probabilistic Road Map (PRM) algorithms, RRT-like algorithms, and the Potential Field algorithms.

Most of the above mentioned algorithms have been used for mobile ground robots in the past decades, but it is interesting to note how is it possible to transfer their basic concepts into the domain of autonomous marine vehicles. A comprehensive overview of the path planning algorithms used in marine robotics is given in [9].

There exist several concepts when approaching the path planning problem in marine robotics. Some of the approaches use optimization techniques to optimize the path of the vehicle while avoiding the obstacles, namely: constrained optimization and semi-infinite constrained optimization [10], sequential quadratic programming (SQP) with constructive solid geometry (CSG) representation of the obstacles [11], mixed integer linear programming for adaptive sampling of the ocean measurements [12].

Another method is to use Virtual Force Fields (VFF) [13]. One of the algorithms which is being used really often in the robotics field is RRT, and its RRT* extension. It is a relatively simple and fast sample-based algorithm, which generates a connectivity tree of nodes from the vehicle's current location to the goal location [14].

Meta heuristic algorithms have also been used to solve the time-energy-optimal path planning problems for marine vehicles, such as genetic and evolutionary algorithms in [15], [16], and also ant colony algorithm in [17].

Grid based path planning algorithms have also been used, i.e. A^* algorithm for minimum energy consumption path in strong currents environments in [18], Fast Marching* (FM and FM*) algorithms for maximum turning rate constrained path generation with a multiresolution method to speed up the path planning process in [19], and Sliding Wavefront Expansion (SWE) Dijkstra-like algorithm for generating minimum duration paths for the AUVs in strong currents [20].

B. Path following methods used in marine robotics

The output of the above mentioned high-level path planning algorithms is the geometrical curve, or a series of waypoints in the workspace of the vehicle. In order for the vehicle to traverse this set path, it needs a mid-level tracking or so-called guidance architecture which will ensure that the set path is followed. Control solutions, both linear and nonlinear, have been devised for fully actuated, as well as underactuated marine vehicles. Several approaches have been proposed. For example nonlinear control techniques in [21], backstepping and Lyapunov-based approaches in [6], [22]–[26]. Also, line-of-sight approaches have been proposed in [27]–[30]. Methods for disturbance rejection, i.e. compensation for the sea current effects, have been proposed in [7], [31]–[39].

IV. Model predictive motion planning

A. MPC: A short introduction

Model predictive control (MPC) can be described as repeated solving of the optimal control problem (OCP) on a finite time horizon T_p in open loop, while taking into account system dynamics and constraints imposed on the states and the controls . Optimization is executed in each sampling time instant $t_i = iT_s$, $i = 0, 1, 2, \ldots$, where T_s is the sampling period. Estimated states at each time instant t_i are used as the initial conditions of the OCP. Even though the solution of the OCP are controls in the open loop, an implicit feedback is made with these initial conditions of the OCP. This way, the MPC is made more robust w.r.t. the measurement and model noise, but also disturbances. When the optimized controls are acquired, only a part of the duration T_c is applied to the system, and in the next sampling instant t_{i+1} the whole procedure is repeated. T_c is the so-called control horizon, and it is most often set as $T_c = T_s$. MPC can be, generally speaking, applied for nonlinear and linear, continuous and discrete systems, with hard and soft constraints [40].

The underlying concepts behind MPC are not novel. They include Hamilton-Jacobi-Bellman theory of dynamic programming [41], the maximum principle of optimality, and Kalman's observation that optimality does not imply stability [42]. MPC has been used since the 1970's in the areas of oil refining, petrochemicals and chemicals, but also pulp and paper, food processing, aerospace and automotive industries, gas, utility, furnaces, mining and metallurgy [40], [43]. The clock frequencies of processors are constantly increasing, thus enabling faster computation of the OCP even for complex systems. Therefore, MPC has long been used in the fields of industrial, field, aerospace, and underwater robotics.

The general mathematical formulation of the MPC problem goes as follows: Let us consider stabilizing a time-invariant (non)linear system described by

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)), \qquad \mathbf{x}(0) = \mathbf{x}_0 \tag{16}$$

w.r.t. the state and control constraints:

$$\mathbf{u}(t) \in \mathscr{U}, \qquad \forall t \geq 0 \tag{17}$$

$$\mathbf{x}(t) \in \mathscr{X}, \qquad \forall t \geq 0 \tag{18}$$

The solution of this problem is a control signal computed as a solution of the optimal control problem

$$\min_{\overline{\mathbf{u}}(\cdot)} J(\mathbf{x}(t_i), \overline{\mathbf{u}}(\cdot)) \tag{19}$$

w.r.t. the constraints

$$\dot{\overline{\mathbf{x}}} = \mathbf{f}(\overline{\mathbf{x}}(\tau), \overline{\mathbf{u}}(\tau)), \qquad \overline{\mathbf{x}}(t_i) = \mathbf{x}(t_i), \tag{20}$$

$$\overline{\mathbf{u}}(\tau) \in \mathscr{U}, \overline{\mathbf{x}}(\tau) \in \mathscr{X}, \qquad \tau \in [t_i, t_i + T_p], \tag{21}$$

$$\overline{\mathbf{x}}(t_i + T_p) \in \mathscr{E} \tag{22}$$

where $\mathscr{E}$ is called the terminal region, and (22) is called the terminal region constraint, which enforces system stability. The bar above the state and control vector symbols in (19)-(22) denotes prediction variables internal to the controller itself. This distinction between the real system's and its model's variables is necessary because their values will not be the same in the general case.

The cost function J which is being minimized on the prediction horizon $T_p \geqslant T_s \geq 0$ is usually given as

$$J(\mathbf{x}(t_i), \overline{\mathbf{u}}(\cdot)) := \int_{t_i}^{t_i+T_p} F(\overline{\mathbf{x}}(\tau), \overline{\mathbf{u}}(\tau))\,\mathrm{d}\tau + E(\overline{\mathbf{x}}(t_i + T_p)), \tag{23}$$

where function F is called the stage cost or Lagrange term, and function E is often called the terminal cost or Mayer term. The terminal cost E, i.e. the terminal region constraint (22), does not need to be included in the cost function. The mathematical formulation given here lacks mathematical rigour. It is given to describe the general concepts of the MPC without going into details.

Since the OCP described by (19)-(23) has to be solved on-line, this implies that the finite horizons have to be used. The shorter the prediction horizon is, the less complex and time consuming it is to solve the given OCP. However, if the finite prediction horizon is used, closed-loop control and system trajectories will differ from the predicted trajectories in open-loop, even if there is no model noise or measurement noise present [44]. Also, infinitely repeated finite horizon optimization in the sense of the receding horizon does not lead to the optimal solution on the infinite horizon [45]. A comprehensive overview of the MPC's stability analysis is given in [46].

B. MPC based motion planning

1) MPC-based motion planning on rough terrains: Planning mobile robot's motion along the path from some starting location to the desired goal location, while at the same time minimizing some cost e.g. roughness of the traversed terrain, can be done by using some of the classical path planning algorithms (Dijkstra, A* for known terrains, or D*, Field D*, D* Lite for unknown of partially known terrains, etc.). The resulting path, going from the start to the goal, usually has sharp turning points, since the above mentioned algorithms are grid-based. This way of generating the path does not consider the dynamics of the vehicle, its kinematic and/or kinetic model. Smoothing of these paths can lead to the loss of optimality, and even worse, the vehicle may hit an obstacle, roll-over, or slip, while following the smoothed path.

The success of the mission, i.e. vehicle traversing the rough terrain from the starting to the goal location, can be guaranteed if the path planner is designed in such a way that it gives the trajectory of the vehicle w.r.t the vehicle's model and state and/or control constraints (in the sense of avoiding obstacles, preventing slipping, and rolling-over), and the controls needed to execute that trajectory. MPC seems like an intuitive choice for tackling this problem.

One such MPC-based planner for mobile robots on known large-scale rough terrains is presented in [47]. It uses the interpolated roughness values, i.e. roughness function as the Lagrange term in the OCP, for locally planning the smoothest path. Also, the interpolation of the roughness-to-go map is used as the Mayer term in the formulated OCP. Roughness-to-go map represents a global planner along the smoothest path from the edge of the prediction horizon to the goal position. Roughness-to-go map has been calculated by using either the optimal Dijkstra algorithm, or the suboptimal Roughness based Navigational Function (RbNF) algorithm. Since RbNF algorithm cannot cope with arbitrarily large obstacles in order to give a near-optimal cost-to-go map, it was further improved in [48]. Wavefront Roughness based Navigational Function (WRbNF) algorithm presented in [48] is a near optimal cost-to-go map and it can also be used for large scale rough terrains with an arbitrary number and size of obstacles. The algorithm has an inherent parallel structure and the code can be parallelized, so it can be significantly faster than the Dijkstra algorithm when an adequate amount of computer resources is used. In [49], WRbNF algorithm was used as a terminal cost term in the MPC-based path planner for mobile robots on rough terrains.

Another solution, given in [50], is to use D* algorithm as a global planner for path planning on the unknown or partially known rough terrains. The D* cost is then used as the terminal cost in the MPC, while using the interpolated roughness values for local planning.

2) MPC-based motion planning in marine robotics: MPC has started being used in marine robotics field approximately from the 2000's onward. This is most probably due to the speed up of the computational power of computers, as well as vehicle on-board microcontrollers.

MPC scheme for path planning in a dynamically changeable environment with obstacles and sea currents is presented in [51]. Map of the environment is built using forward-looking sonar data, and D^* algorithm is used to generate waypoints, interpolated by using B-splines, for MPC to follow. Sampling based MPC (SBMPC) scheme with input sampling and obstacle avoidance is presented in [52]. MPC can be also used for safety motivated path planning, e.g. for ships at the open sea with high waves, in order to minimize the possibility of capsizing due to roll and pitch oscillations [53]. Strong currents have a great impact on energy consumption of the AUVs. MPC approach using A* algorithm for computing the path along the weakest current flows in the ocean in presented in [54]. A comprehensive overview of the MPC framework for path planning schemes is given in [55].

Even with the use of path planning algorithms, there remains the issue of tracking the path which was computed. This can pose a problem, especially in the case of unfeasible sharp turns between consecutive waypoints along the path. Path tracking is solved by MPC in a number of approaches. MPC scheme with a shrinking horizon (SHMPC) is presented in [56]. This approach shrinks the prediction horizon as the vehicle approaches the goal position, in order to relieve the computation burden of optimization solver. To reduce the posibility of capsizing in ship maneuvering, it is important to constrain roll and rudder turning rate, together with canceling the disturbances caused by high waves. MPC has been used to tackle this problem in [57], [58]. Piece-wise linear paths can be tracked very efficiently using LOS approach, which was integrated into MPC framework in [59], [60].

V. Informational gain guided MPC motion planning: A transition from land to sea

A. Definitions of informational gain metrics

Let $\mathcal{T}$ denote a sea floor terrain map which contains m rows and n columns of square patches p_{ij}, whose size is dependent on the resolution of the used sonar, and assume that the elevation map of the terrain $\mathcal{T}$ is not a-priori known. The patch corresponding to the vehicle location (x_v, y_v) and the goal position (x_g, y_g) are denoted as $\mathbf{p}_v$ and $\mathbf{p}_g$, respectively [48]. Informational gain in the context of exploring the sea floor can be defined as the roughness of the sea floor, i.e. the local variance of the elevation map sensed by the sonar.

Much of the roughness metrics research is related primarily to Mars rovers, and ground vehicles in general. In [61] the roughness metrics is based not only on the local variance of the terrain map in the spatial domain, but it also takes into account terrain's intrinsic properties in the frequency domain. Several spatial measures of the terrain's roughness are presented in [62].

In marine robotics, roughness is mostly related to the exploration of the sea floor relief complexitiy [63], and also in the research related to identifying benthic habitat types [64].

Roughness value $r_{i,j}$ represents the informational gain measure of a map patch $p_{i,j}$ (how interesting or informational that patch is) and it is calculated as a standard deviation of the elevation values of its neighboring map patches [65]. We define a set $\mathcal{N}_{i,j}$ of neighboring map patches of the patch $p_{i,j}$ as 8-connected neighborhood patches including the patch $p_{i,j}$ itself. Roughness value $r_{i,j}$ is defined by the relation:

$$r_{i,j} = \mu_{i,j} \sqrt{\mathrm{Var}(\mathcal{T}(\mathcal{N}_{i,j}))}, \tag{24}$$

where $\mu_{i,j} \in [0, 1]$ is a coefficient dependent on the terrain type of the map patch $p_{i,j}$, and Var denotes the variance operator.

Eq. (24) gives a non-zero values of the roughness for the elevation map patches which are on a perfectly flat sloped plane in 3D space. In order to remove the slope of the terrain out of the roughness measure, i.e. to remove the linear trend out of the sampled elevation values, it is possible to redefine roughness as the standard deviation of the residuum values $\rho_{i,j}$ w.r.t. the interpolated plane through the patches $\mathcal{N}_{i,j}$:

$$r_{i,j} = \mu_{i,j} \sqrt{\mathrm{Var}(\rho(\mathcal{N}_{i,j}))}. \tag{25}$$

Since the back propagation nature of the grid based D*-like algorithms, propagating roughness from the goal position would result in the vehicle's path along the least rough path to the goal. Considering that the main idea of the research is to navigate the vehicle along the way of maximum informational gain, i.e. roughness or minimum smoothness, it is necessary to make a conversion of the roughness into smoothness. This way, after propagation the smoothness costs in the path planning algorithm, a path which minimizes smoothness is computed, just as required. Smoothness can be defined as:

$$s_{i,j} = \begin{cases} 1, & \text{if } r_{i,j} = 0 \\ 1/r_{i,j}, & \text{otherwise.} \end{cases} \tag{26}$$

B. The proposed MPC frameworks

In this subsection, we discuss two possible frameworks of the informational gain guided MPC based motion planning for the AUV during sonar scanning missions.

1) *Characteristic waypoint following:* The first approach to informational gain guided MPC based motion planning for the AUV is pretty straightforward, see Fig. 3. It uses gathered sonar data to compute informational gain values of the scanned sea floor, and uses a D*-like (near-) optimal global path planner. This planner at its output gives a series of waypoints in the grid map for the AUV to follow. Since these paths often contain many sharp turns which the AUV cannot perform, it is necessary to smooth out the computed path, e.g. by using B-spline basis functions. The problem which remains with this smoothing process is that, in the presence of obstacles, the smoothed path can collide with them, even though the original path's waypoints perfectly avoid the obstacles.

After a global or local parametrization of the path with a smooth continuous function, MPC can be used for path following, as some sort of a local planner. Of course, MPC with AUV's kinematic model can be used for high-level control, giving the reference velocities to the low-level controllers, which in turn give forces and torques at their output. Another option is to include both kinematic and dynamic models into MPC, so that it directly optimizes forces and torques. The decision on which MPC model to use depends mainly on the fact whether the optimization execution times are still real-time.

2) *Informational gain guided MPC:* Another approach would be to integrate the cost-to-go values from the global planner directly into the MPC optimization framework, see Fig. 4. This can be done as in [49], [50], to use the global planner values as the terminal cost term E in (23). It is also possible to extend the local MPC planner F with the locally interpolated informational gain values. This way, the vehicle would move along the path which is maximizing the informational gain, both locally and globally.

Adding the terminal cost in a form of the interpolated values of the cost-to-go map, and/or adding the interpolated local informational gain in the optimization criterion of the OCP can cause the optimization problem to become more complex to solve. Of course, there should be a trade-off between the complexity which is imposed on the optimization solver, and the vehicle's performance improvement.

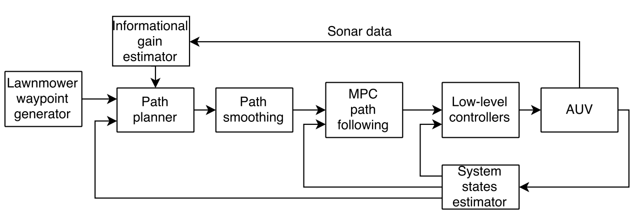

Fig. 3. Block diagram of the first proposed approach: Global path planner for computing the discrete path in the grid based map, which is then smoothed, and tracked with MPC path following control algorithm as a local planner.

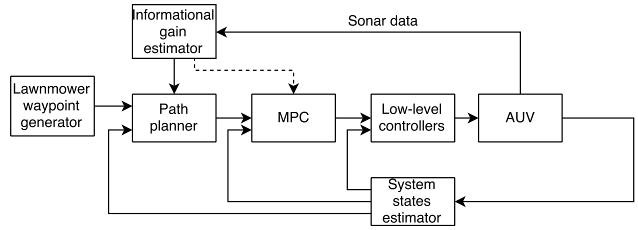

Fig. 4. Block diagram of the second proposed approach: Global path planner for computing the terminal cost in the MPC framework. Possible augmentation of the local MPC planner with the roughness values on the prediction horizon (dashed line).

VI. Preliminary results

The first step in our research was to make a modular MPC framework for lawnmower line following by an underactuated marine vehicle at a constant depth and in the presence of the constant sea current disturbance. This framework had to be fine tuned to work in real-time, in order to use it in the later experiments. ACADO-ROS framework has been designeddesigned, enabling high level speed control to be optimized within MPC, while using the existing low level controllers of forces and torques developed in [66]. The model which was used is the linearized model given in (14) and (15). The mentioned model has been extended with an additional state $\dot{d}_{int} = d$, which is an integral of the distance to the line. This additional state in the MPC scheme cancels the disturbance.

The simulation results are given in Fig. 5. A series of on-sea experiments were conducted during the beginning of October 2016 in Biograd na Moru, in order to validate the simulation results. Hybrid AUV/ROV robotic platform e-URoPe (e-Underwater Robotic Pet), developed at CNR-ISSIA (Genoa, Italy), has been used in ROV mode. Experimental results comparing the performance of the developed MPC framework and the approach developed in [66] are given in Fig. 6 and 7.

Both the simulation and the experimental results show good performance of the developed MPC line following scheme. Future work in this direction includes implementing the shortening horizon MPC scheme, together with LOS and/or ILOS approaches.

VII. Conclusion and Future work

As the seas and oceans get more and more explored, it is necessary to make some adaptive sampling schemes, which will optimize the way that the marine vehicles scan the sea bottom. One such scheme, presented in this paper, joins (near-) optimal path planners with MPC to get feasible paths for the marine vehicle w.r.t. its kinematics and state and/or control constraints. Previous work on MPC planners for the ground vehicles shows that it is possible to successfully join these two methods.

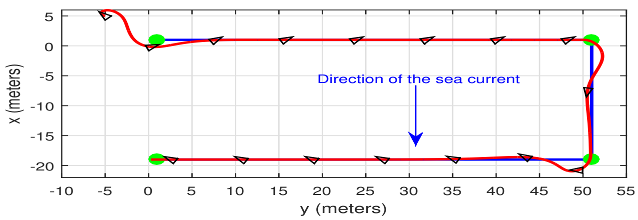

Fig. 5. Simulation results. Reference lawnmower lines at a constant depth (blue), defined by given waypoints (green markers). Vehicle's path while following the lines of the set lawnmower pattern (red). Vehicle's heading (black triangles).

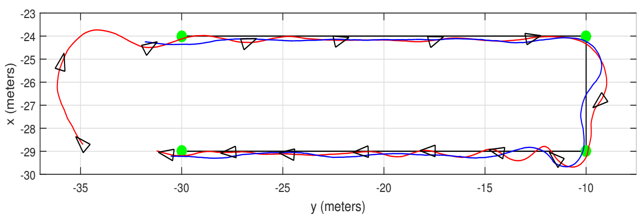

Fig. 6. Surface experiments. Reference lawnmower lines at a constant depth (black), defined by given waypoints (green markers). Vehicle's path while following the lines of the set lawnmower pattern: MPC controller (red), PID controller (blue). Heading of the vehicle controlled by MPC (black triangles).

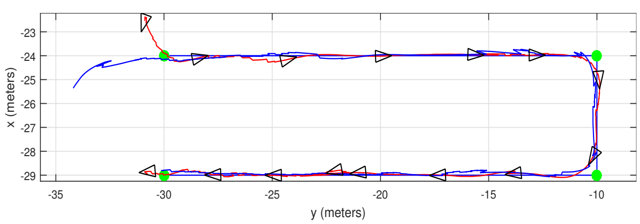

Fig. 7. Underwater experiments. Reference lawnmower lines at a constant depth (black), defined by given waypoints (green markers). Vehicles path while following the lines of the set lawnmower pattern: MPC controller (red), PID controller (blue). Heading of the vehicle controlled by MPC (black triangles).

Future work includes designing a plug-and-play framework which allows a combination of different informational gain definitions depending on the mission itself, but also different path planning algorithms, as well as different system models and MPC schemes.

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