Tensor Voting for Line Structure Enhancement

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1. Introduction

Tensor voting is an information propagation mechanism based on tensor representation of data. We use tensor voting for enhancing line segments and for connecting broken lines in an image. Our idea is to implement this algorithm for improving blood vessel detection in eye fundus photographs.

2. Method

Our algorithm consists of two main steps:

1. Calculating *Hessian* matrix and representing each pixel by a tensor
2. Tensor voting

Hessian matrix is a square matrix of second-order partial derivatives of a function. It describes the local curvature of a function of many variables. For a 2D image $f(x,y)$ *Hessian* matrix is a 2x2 matrix with these elements:

$$H[f(x,y)] = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} \quad (1)$$

We use a 2x2 *Hessian* matrix as a second order tensor that can be decomposed as in the following equation:

$$T = \lambda_1 e_1 e_1^T + \lambda_2 e_2 e_2^T = (\lambda_1 - \lambda_2)\, e_1 e_1^T + \lambda_2 (e_1 e_1^T + e_2 e_2^T) \quad (2)$$

where λ_i are the eigenvalues in decreasing order and e_i are the corresponding eigenvectors. See Table 1. for how oriented and un-oriented inputs are encoded.

Table 1.: Decomposition of a 2D second order tensor into its *stick* and *ball* components

Input	Second order tensor	Eigenvalues	Quadratic form
n, oriented	Stick tensor	$\lambda_1 = 1, \lambda_2 = 0$	$\begin{bmatrix} n_1^2 & n_1 n_2 \\ n_1 n_2 & n_2^2 \end{bmatrix}$
un-oriented	Ball tensor	$\lambda_1 = \lambda_2 = 1$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The *stick tensor* indicates an elementary curve token with e_1 as its curve normal. The *ball tensor* corresponds to a perceptual structure which has no preference of orientation or to a location where multiple orientations coexist. The size of the stick component $(\lambda_1 - \lambda_2)$ indicates curve saliency.
Based on the above, an elementary curve with normal *n* is represented by a stick tensor parallel to the normal, while an un-oriented token is represented by a ball tensor.

Tensor voting is done by casting votes from token to token and accumulating them by tensor addition. The votes cast by the stick component are multiplied by $\lambda_1 - \lambda_2$ and those of the ball component by λ_2. The strength of the votes attenuates:

1. with distance – distant tokens have smaller influence
2. with increased curvature of the hypothesized structure - straight continuations are preferable to curved ones.

To control the strength of the votes voting is done inside of a voting field which contains a tensor at every position that is the vote cast there by a unitary stick tensor located at the origin of the field. The shape of the field in 2D can be seen in Fig. 1.a.

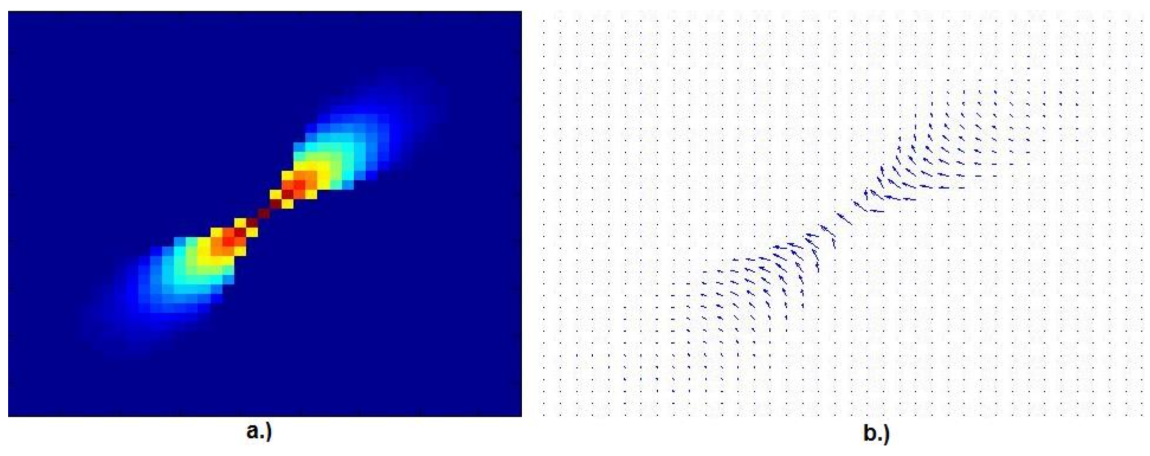

Figure 1.: Voting field: (a) shape and attenuation with distance; (b) tensor array

Depicted at every position is the eigenvector corresponding to the largest eigenvalue of the second order tensor contained there (Fig. 1.b.). To compute a vote cast by an arbitrary stick tensor, we need to align the field with the orientation of the voter and multiply the saliency of the vote that coincides with the receiver by the saliency of the arbitrary stick tensor.

4. Results

Some of the results can be seen in Fig. 2. Firstly we tested the tensor voting algorithm on a broken line to see will the algorithm be able to connect parts of the line. We drew a broken line and used that image because we lacked proper data for this kind of testing. This image and the result of tensor voting can be seen in Fig. 2.a. We used the same image but added *Gaussian* noise to better simulate real data. This result can be seen in Fig. 2.b.
After testing on these images we continued testing the algorithm on parts of eye fundus images. Figures 2.c. – 2.f. show real eye fundus data and tensor voting results.

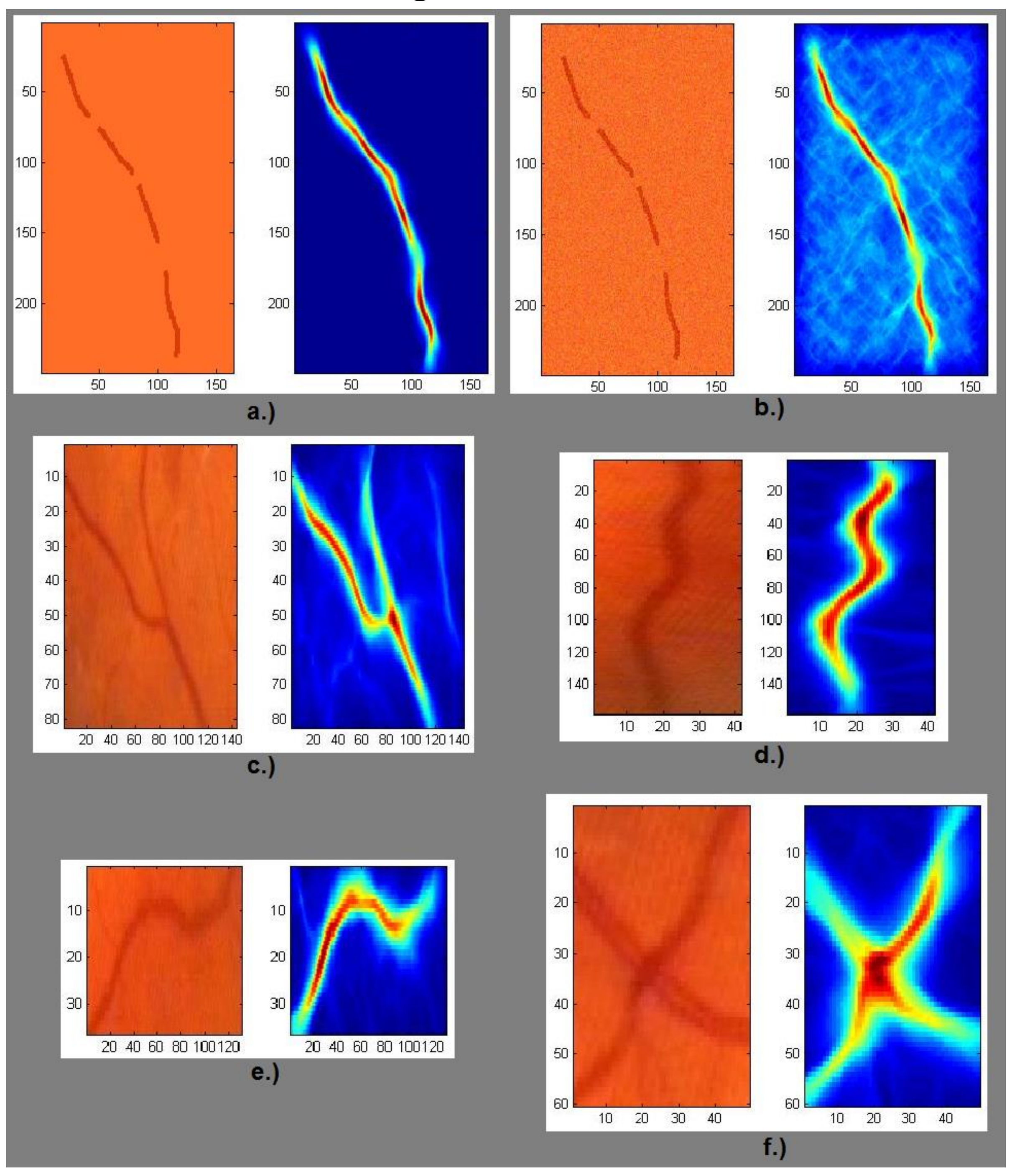

Figure 2.: Photographs used for testing with corresponding results of tensor voting on artificial (a,b) and eye fundus photographs (c-f)

5. Conclusion

Any tensor can be decomposed into the basis components (stick and ball in 2D) according to its eigensystem. Then, the corresponding voting fields can be aligned with each component. Our approach using *Hessian* matrix for tensor representation of the data gives promising results but further work and research is needed and will be done on this subject.