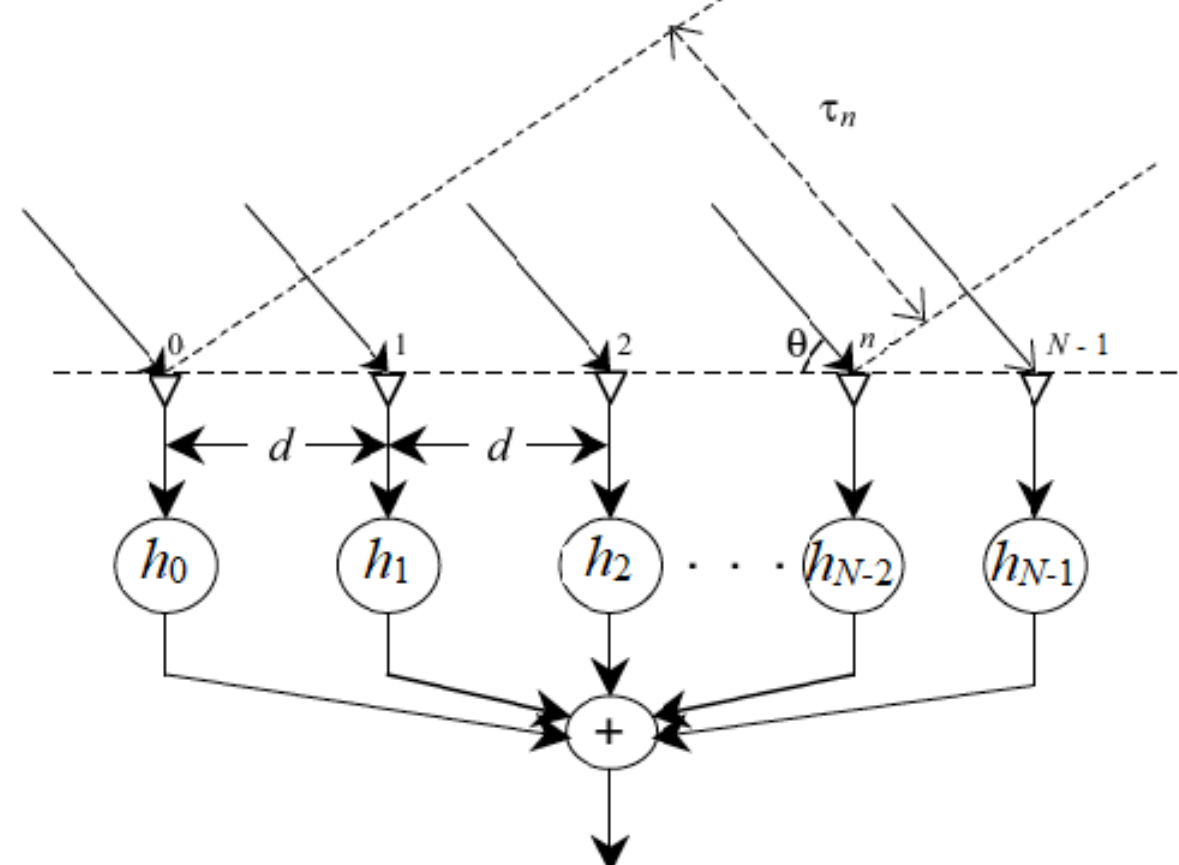


1. Introduction

Beamforming based on antenna arrays plays a significant role in the fifth generation of wireless communication systems. Many today's research interests are focused on the robust synthesis of radiation patterns with high performances.

Linear antenna array with isotropic elements, single temporal frequency and equally spaced elements is **spatial FIR filter**.



Radiation pattern

$$H(\omega) = \sum_{n=0}^{N-1} h_n e^{-j\omega n}$$

$$\omega = \frac{2\pi}{\lambda} d \cos \theta$$

2. Problem formulation

Develop analytical and optimization methods for the synthesis of narrow radiation patterns with

- high **beam efficiency (BE)**
- controllable **sidelobe level (SLL)**
- low and controllable **dynamic range ratio (DRR)**

$$BE = \frac{\int_{-\pi}^{\pi} H^2(\omega) d\omega}{\int_{-\pi}^{\pi} H^2(\omega) d\omega}$$

$$DRR = \frac{\max_{0 \leq k \leq N-1} \{|h_k|\}}{\min_{0 \leq k \leq N-1} \{|h_k|\}}$$

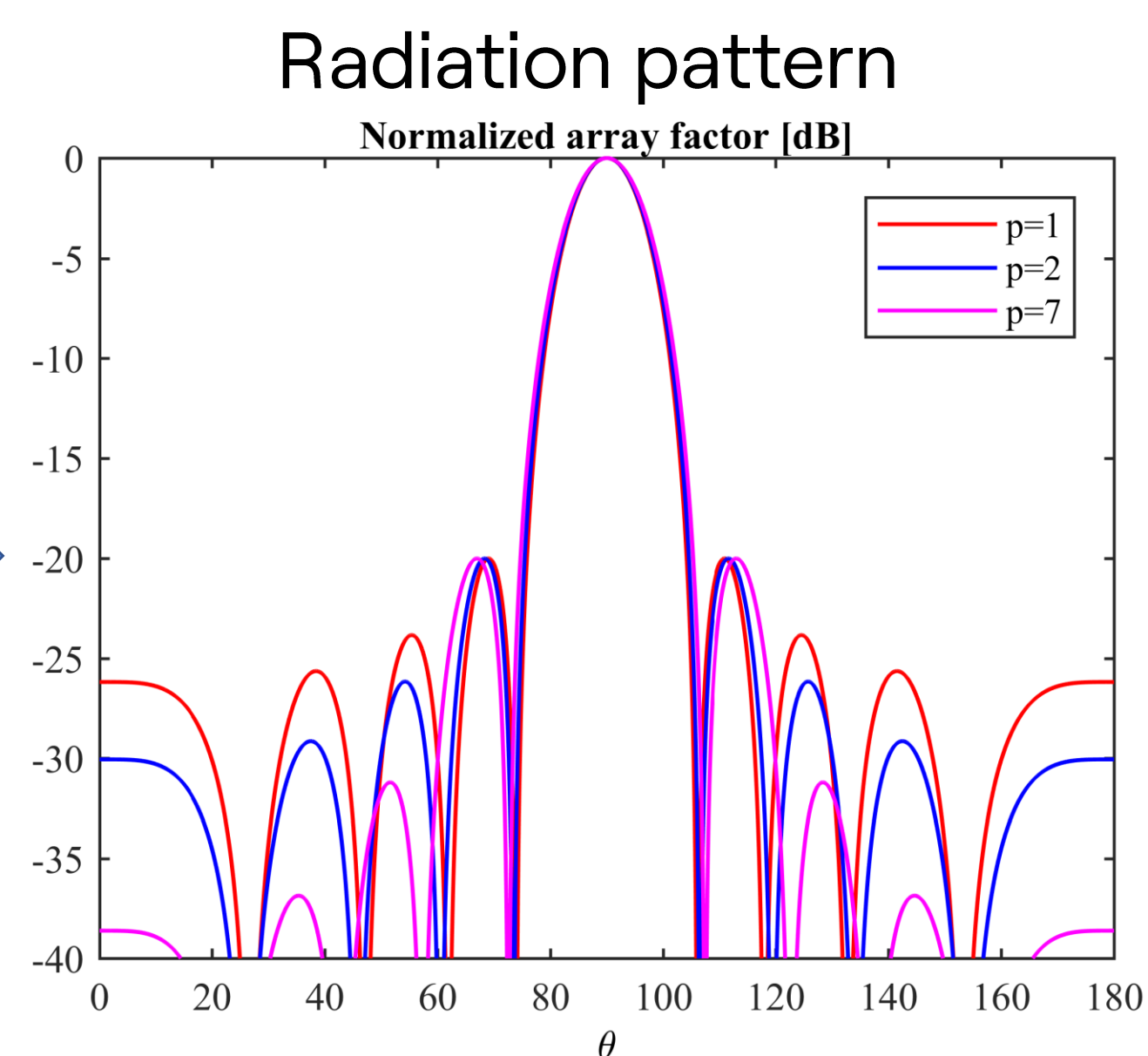
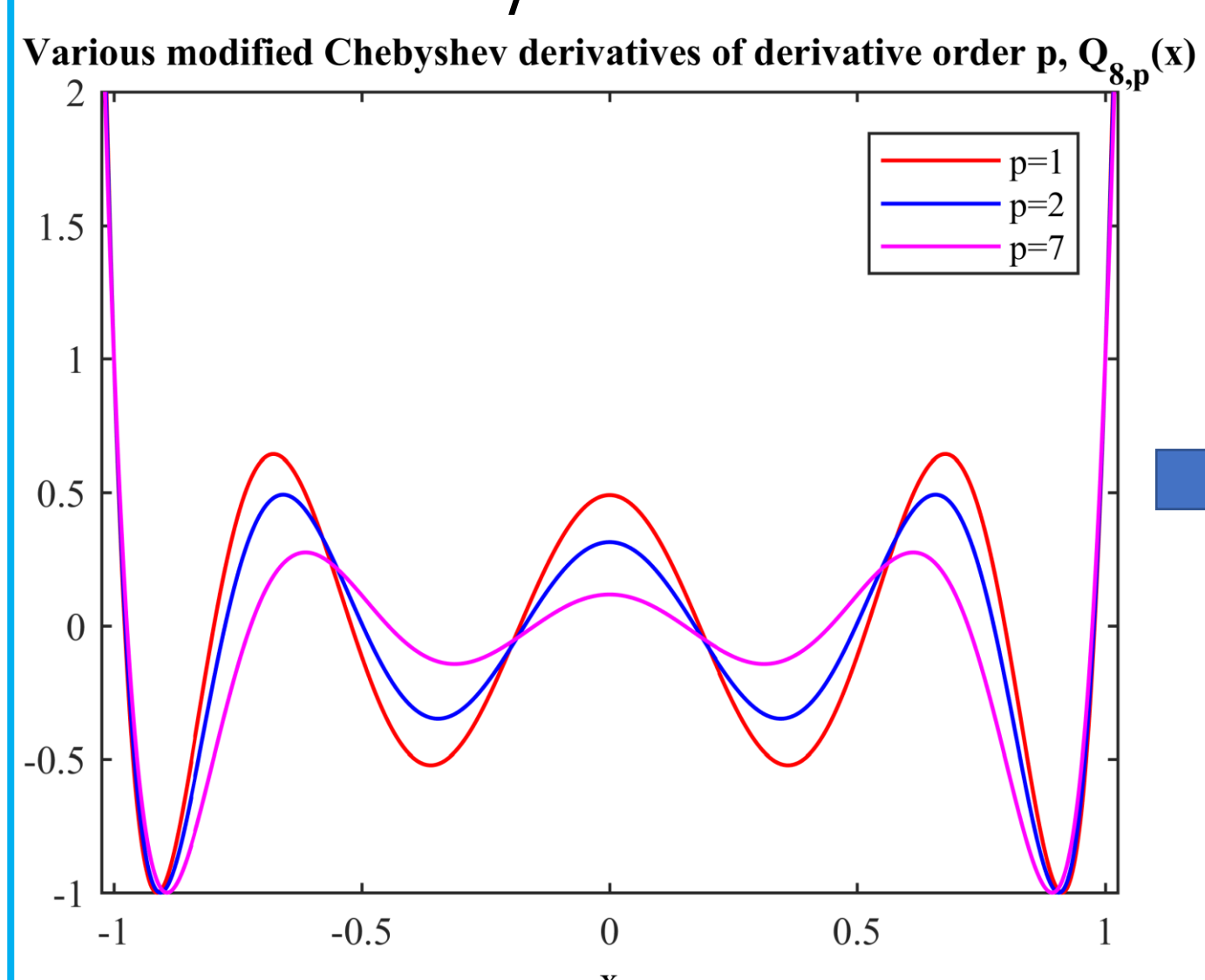
3. Methodology

Analytical method: polynomial approximation

- Forms radiation pattern by a given polynomial

$$H(u) = Q_{N-1}(x_0 \cos u)$$

- Ensures sidelobe level control



Optimization method: sidelobe power minimization

Sidelobe power

$$\varepsilon_2(\mathbf{a}, \theta_s) = \sqrt{4\pi \int_{\sin(\theta_s)}^1 |f(\mathbf{a}, \omega)|^2 d\omega}$$

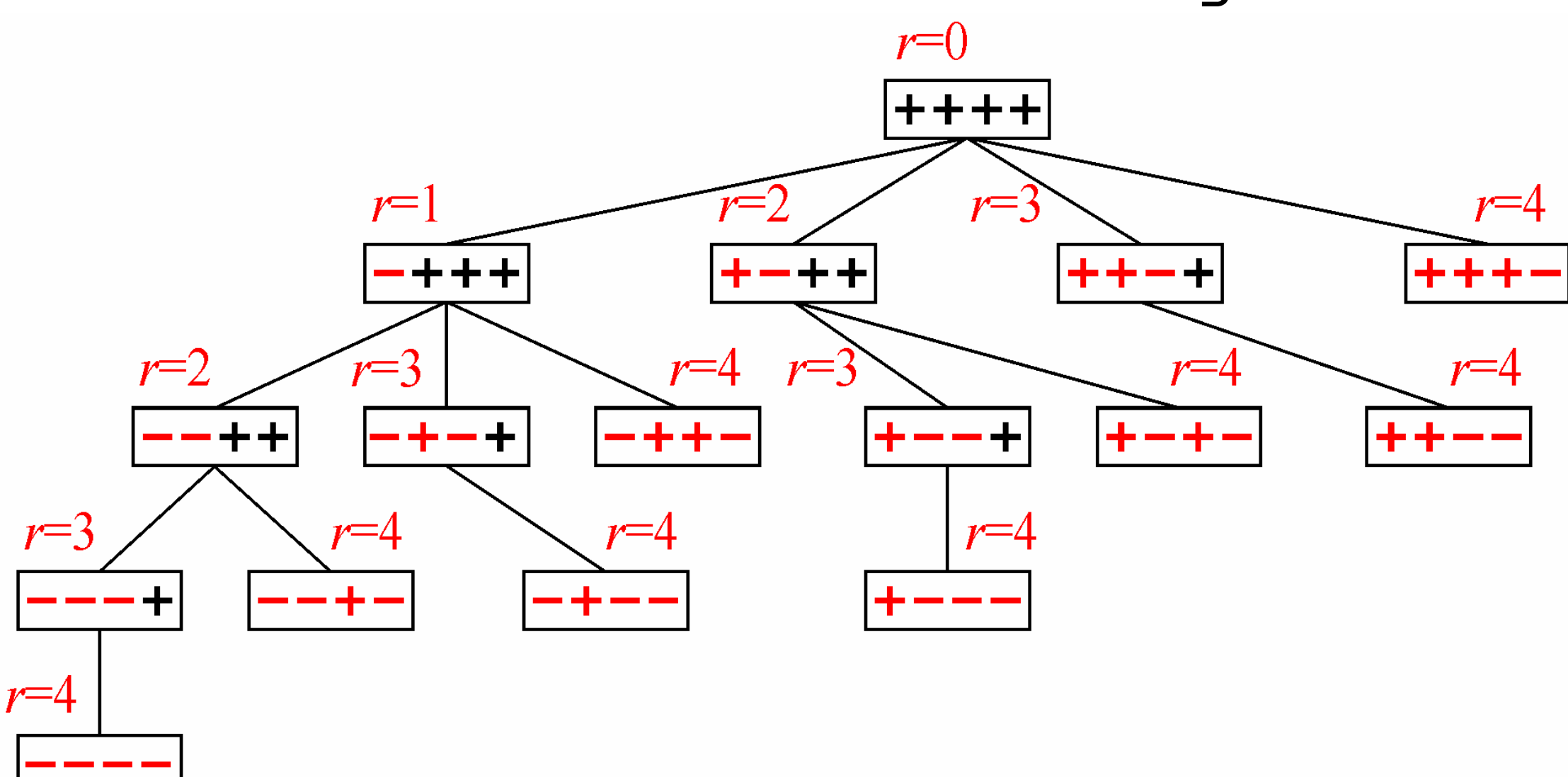
DRR constraint

$$\frac{\max_{1 \leq k \leq N} \{|a_k|\}}{\min_{1 \leq k \leq N} \{|a_k|\}} \leq D$$

Optimization problem

$$\begin{aligned} &\text{minimize}_{\mathbf{a}, t} \quad \varepsilon(\mathbf{a}, \theta_s) \\ &\text{subject to} \quad \sum_{k=1}^N a_k = 1 \\ &\quad |a_k| \leq Dt, \quad k = 1, 2, \dots, N \\ &\quad |a_k| \geq t, \quad k = 1, 2, \dots, N \\ &\quad t \geq 0 \end{aligned}$$

Global solution: Branch and bound algorithm



4. Results

- Pascal polynomial brings **radiation pattern with high beam efficiency**
- Chebyshev derivative order p and Gegenbauer's free polynomial parameter α bring **good trade-off between DRR and beam efficiency**

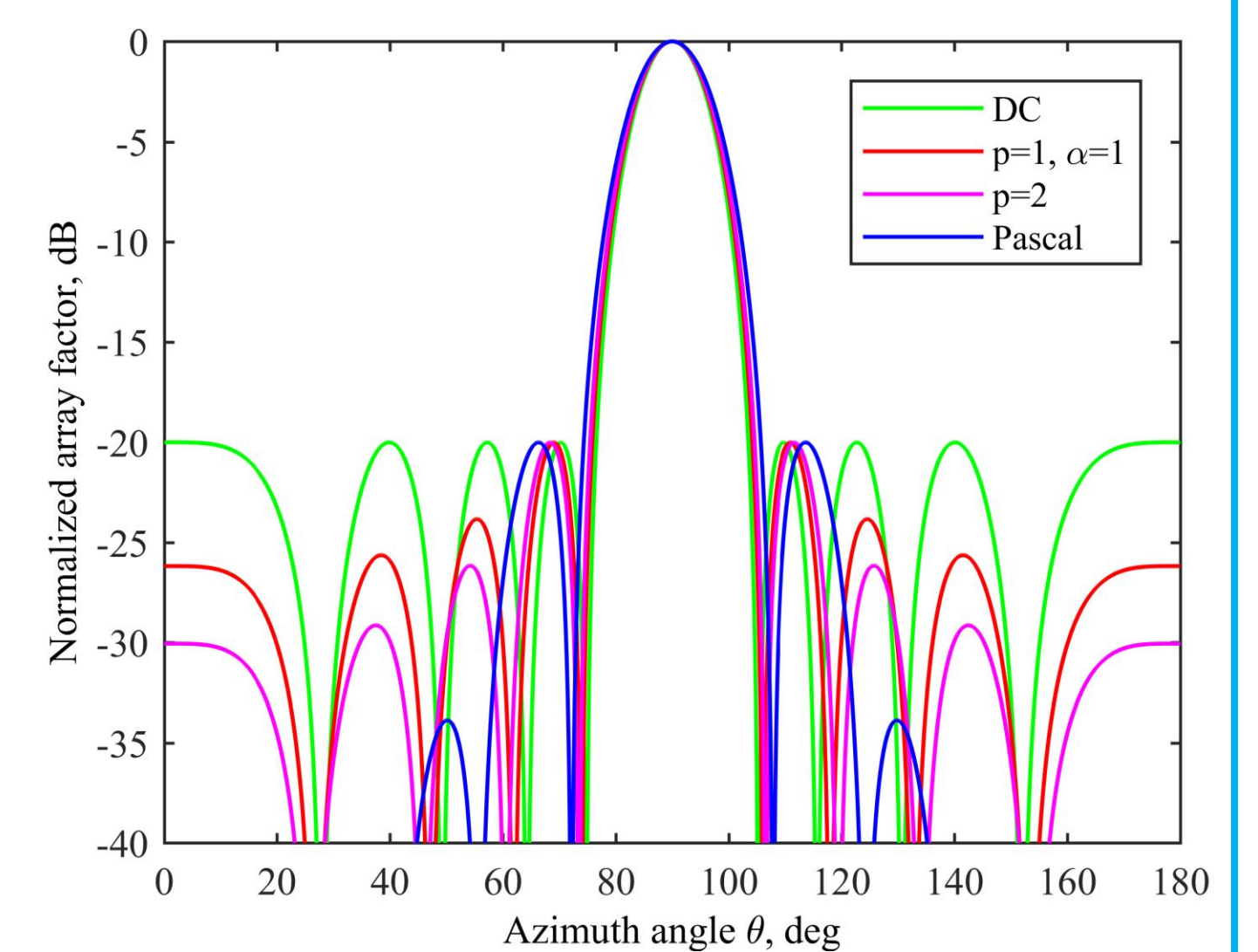


Figure of merit	Dolph Chebyshev	$p=1$ ($\alpha=1$)	$p=2$	Pascal
Beam efficiency, %	96.80	98.47	98.81	99.04
Dynamic range ratio	1.66	2.27	2.70	4.35

- Sidelobe power minimization with constrained DRR: **brings high improvement in DRR and small loss of efficiency comparing to the BE optimum (DPSS)**

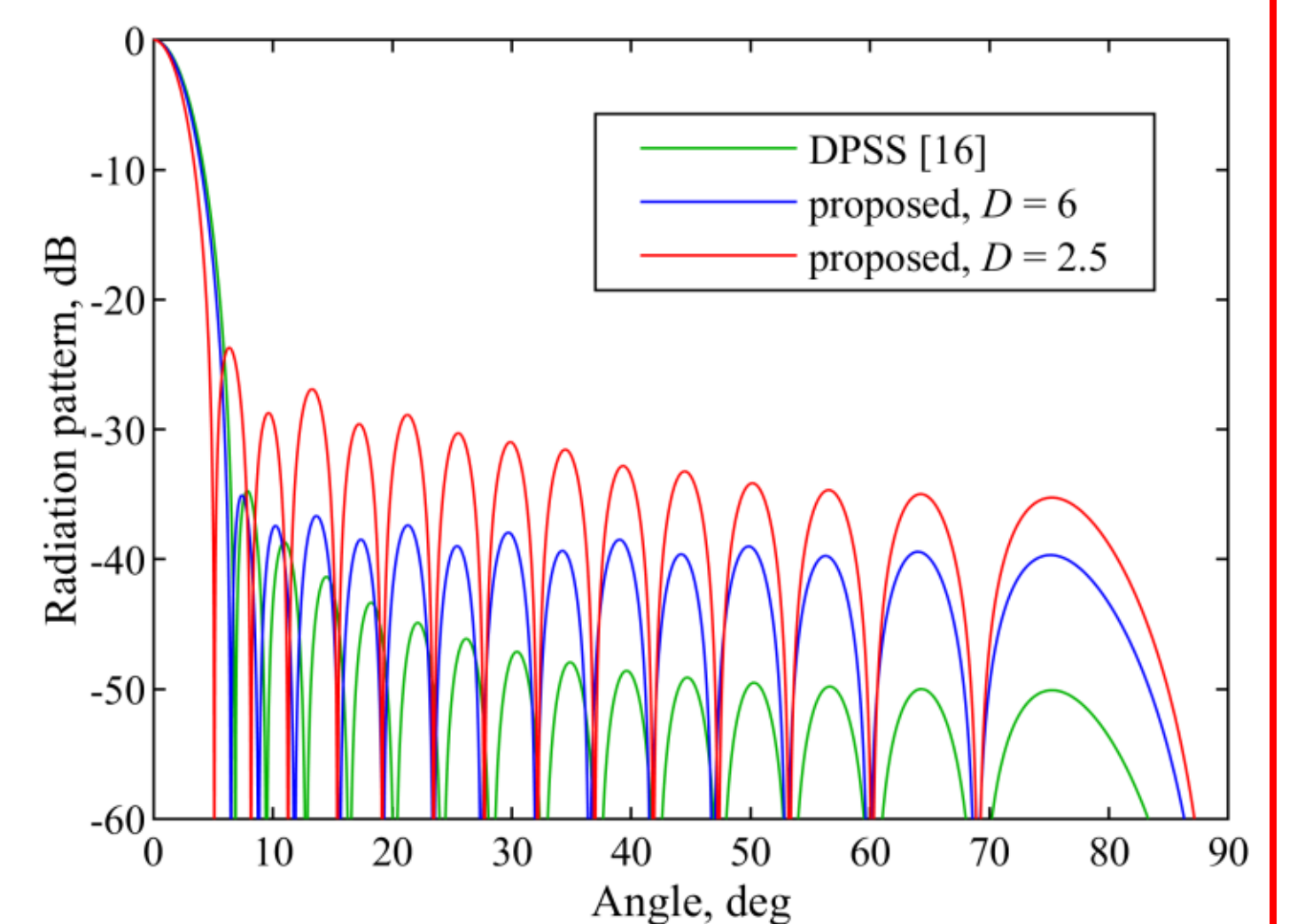
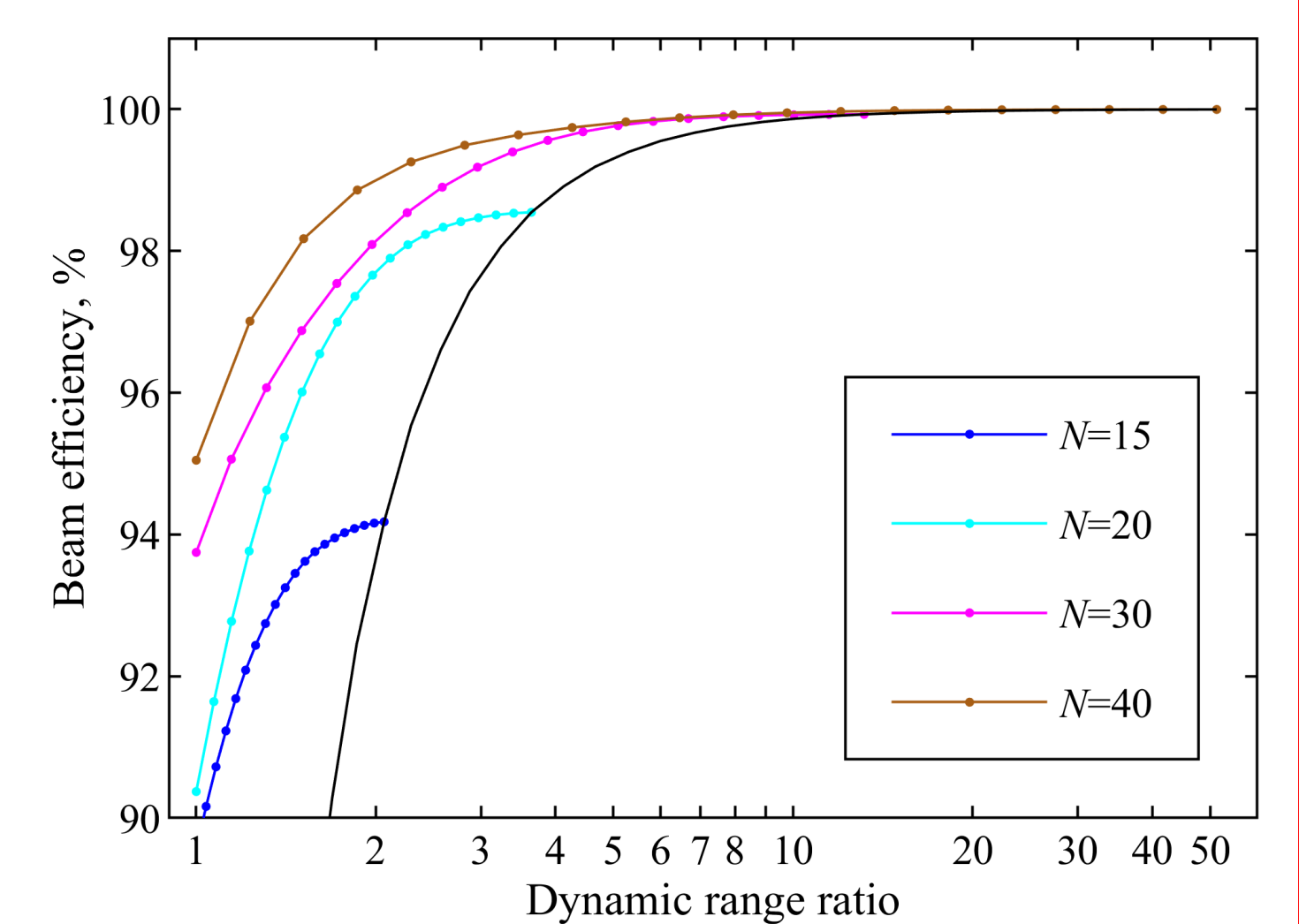


Figure of merit	DPSS	Proposed, $D=6$	Proposed, $D=2.5$
Beam efficiency, %	99.9	99.8	98.8
Dynamic range ratio	13.5	6.0	2.5

- Increase in DRR is accompanied with increase in beam efficiency
- Opens possibility to **design arrays with specified beam efficiency and minimum DRR**



DPSS		Proposed	
BE_{DPSS} %	DRR_{DPSS}	BE_{opt} %	DRR_{opt}
99.93	14.67	99.90	8.06
		99.00	2.71

5. Conclusion

Narrow radiation patterns with high beam efficiency are synthesized by using non-equiripple polynomials with rapid decrease towards the origin, or by minimization of sidelobe power. The polynomial approximation ensures sidelobe level control, while sidelobe power minimization ensures global design of antenna arrays with controllable dynamic range ratio.

6. Acknowledgments

This paper was supported in part by Croatian Science Foundation under the project IP-2019-04-4189 and in part by Ericsson Nikola Tesla d.d. and Faculty of Electrical Engineering and Computing under the project ILTERA.