BOOK OF ABSTRACTS

COMBINATORIAL CONSTRUCTIONS WORKSHOP

University of Zagreb, Croatia

June 27-29, 2022

HRZZ PMF FER Zagreb

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IMPRESSUM

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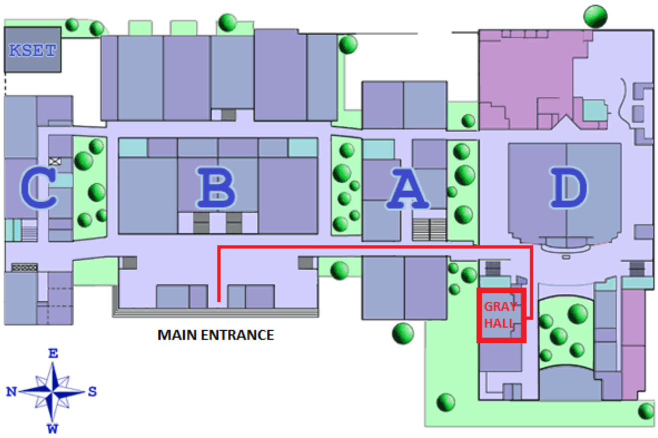
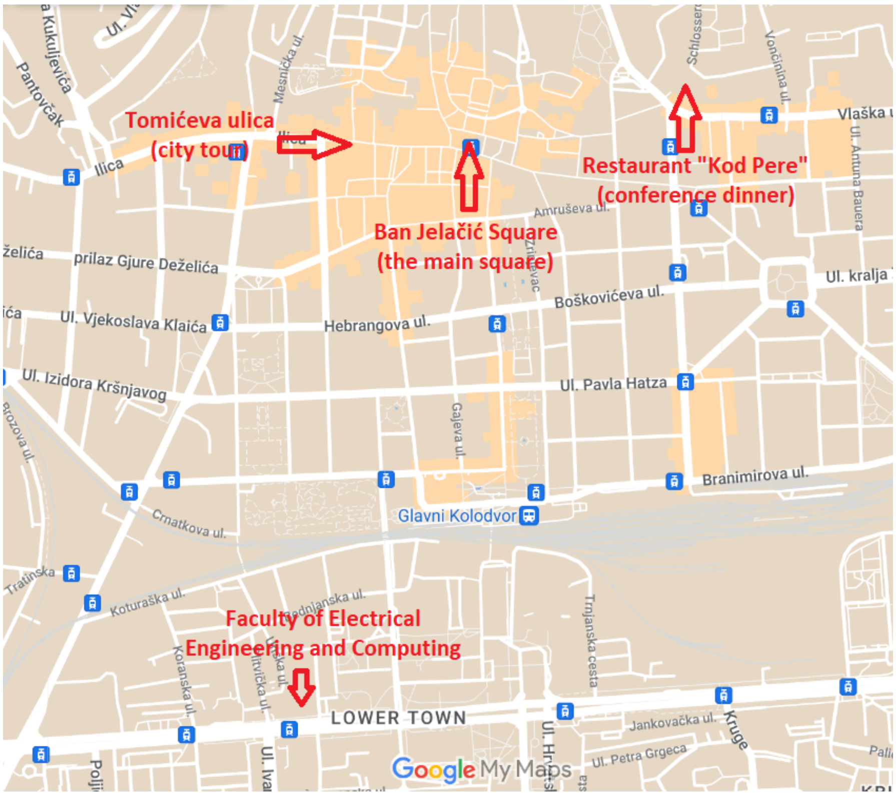
Workshop program

The workshop will take place at the **Faculty of Electrical Engineering and Computing**, Unska 3, Zagreb, Croatia. All talks will be in D building: **Gray Hall** (ground floor) and **D102** (1st floor).



The conference dinner will take place at the restaurant "**Kod Pere**" (Ulica Cvjetka Rubetića 25, Zagreb) on Tuesday at 20:00.



Monday, June 27^{th}

Time/Room	Gray Hall	D102
8:30	Registration	
9:25	Opening	
9:30	Andrea Švob, *Divisible design Cayley graphs and digraphs*	
10:30	Coffee break	
11:00	Gábor Péter Nagy, *Local modifications of 2-designs: theory and implementation* (Part 1)	
12:00	Break	
12:10	Marco Buratti, *On harmonious linear spaces*	Jelena Sedlar, *On the locally irregular edge colorings of graphs*
12:30	Francesca Merola, *Equitably 2-colourable cycle system*	Suzana Antunović, *Detecting communities under constraints in directed acyclic networks*
12:50	Simona Bonvicini, *Even cycle decompositions of 4-regular graphs*	Marija Maksimović, *New regular two-graphs on 38 and 42 vertices*
13:10	Lunch (Black Hall)	
14:30	Gábor Péter Nagy, *Local modifications of 2-designs: theory and implementation* (Part 2)	
15:30	Coffee break	
16:00	Petr Lisoněk, *Combinatorial structures arising from sharply 2- and 3-transitive groups*	Adrian Beker, *A generalisation of the Alon-Yuster 123 Theorem*
16:20	Alessandro Montinaro, *Flag-transitive, point-imprimitive symmetric 2-designs*	John R. Schmitt, *Higher Degree Davenport Constants Over Finite Commutative Rings*
16:40	Biserka Kolarec, *Equidistant walks to infinit*	Rudi Mrazović, *One-point concentration of the clique and chromatic numbers of the random Cayley graph on* $\mathbf{F}_2^n$
18:30	We walk together from the main entrance of FER (optional)	
19:00	**City tour** from starting from: Tomićeva ulica 4, Zagreb	
21:00	Free open-air concert in a park: Zagreb Classic. Nino Rota & Ennio Morricone: The most beautiful film music. Address: Trg kralja Tomislava (Main Train Station, 10 min walk from FER)	

Tuesday, June 28^{th}

Time/Room	Gray Hall	D102
9:30	Anita Pasotti, *Heffter arrays: origins, variants and applications*	
10:30	Coffee break	
11:00	Alfred Wassermann, *Construction of q-analogs of combinatorial objects with prescribed symmetries* (Part 1)	
12:00	Break	
12:10	Lorenzo Mella, *Constructions of globally simple relative non-zero sum Heffter arrays and biembedding*	Daniel Hawtin, *Neighbour-transitive codes in generalised quadrangles*
12:30	Simone Costa, *Biembeddings of Archdeacon type and their full automorphism group*	Matteo Mravić, *The search algorithm for extremal* $\mathbb{Z}_4$*-codes*
12:50	Mariusz Meszka, *Maximal partial Room square*	Dominik Hollý, *Constructing binary quasi-cyclic codes of index* 2 *from a pair of cyclic codes*
13:10	Lunch (Black Hall)	
14:30	Alfred Wassermann, *Construction of q-analogs of combinatorial objects with prescribed symmetries* (Part 2)	
15:30	Coffee break	
16:00	Ismael G. Yero, *On vertices belonging to all strong metric bases of a unicyclic graph*	Valentino Smaldore, *On a graph isomorphic to* $NO^+(6,2)$
16:20	Dorota Kuziak, *On the outer multiset dimension of graphs*	Silvia M.C. Pagani, *Ghostbusters in* $PG(2,q)$
16:40	J. Carlos Valenzuela-Tripodorok, *On the* $[k]$*-multiple Roman domination in graph*	Harald Gropp, *On orbital matrices, existence and enumeration*
19:30	We walk together from the main entrance of FER (optional)	
20:00	Conference dinner at restaurant "Kod Pere". Address: Ulica Cvjetka Rubetića 25, Zagreb	

Wednesday, June 29th

Time/Room	Gray Hall	D102
9:30	Valentina Pepe, *Some geometric aspects of extremal graphs*	
10:30	Coffee break	
11:00	Raúl Manuel Falcón Ganfornina: *Symmetries of partial Latin rectangles: Fundamentals and applications* (Part 1)	
12:00	Break	
12:10	Raúl Manuel Falcón Ganfornina: *Symmetries of partial Latin rectangles: Fundamentals and applications* (Part 2)	
13:10	Lunch (Gallery to the left of D1)	
14:30	Dean Crnković, *Switching for 2-designs and Hadamard matrices*	Vincenzo Pallozzi Lavorante, *A general construction of permutation polynomials*
14:50	Giovanni Falcone, *Extension theory for Steiner triple systems*	Marcella Takáts
15:10	Tommaso Traetta, *Row-sum matrices*	Tamás Héger, *Minimal codes, strong blocking sets and higgledy-piggledy lines*
15:30	Coffee break	
16:00	Morgan Rodgers, *Cameron–Liebler line classes admitting large cyclic groups*	Nikola Jedličková, *List Homomorphism Problems for Signed Graphs: Irreflexive graphs*
16:20	Maarten De Boeck, *Cameron-Liebler sets for hyperbolic quadrics*	Jan Bok, *List Homomorphism Problems for Signed Graphs: A Dichotomy for Trees*
16:40	Vedran Krčadinac, *Lacing designs in PAG*	
17:00	End of the workshop	
21:00	Free open-air concert: Zagreb Classic. Cameristi della Scala & Francesco Manara. Address: Trg kralja Tomislava (Main Train Station, 10 min walk from FER) ww.infozagreb.hr/dogadanja/zagreb-classic/cameristi-della-scala-francescomanara	

Invited speakers

Symmetries of partial Latin rectangles: Fundamentals and applications

Raúl Manuel Falcón Ganfornina
University of Seville (Spain)

Symmetries of a partial Latin rectangle are determined by its autoparatopism group, which arises from those permutations of its rows, columns and symbols preserving it, together with any interchange of roles among these three structural dimensions. In this talk, we will describe the fundamentals of these symmetries and delve into some of their different applications in cryptography and graph colouring games.

Local modifications of 2-designs: theory and implementation

Gábor P. Nagy
Budapest University of Technology (Hungary)

We call the triple $(\mathcal{P}, \mathcal{B}, I)$ an incidence structure, provided $\mathcal{P}$, $\mathcal{B}$ are disjoint sets and $I \subseteq \mathcal{P} \times \mathcal{B}$. We use geometric language and call the elements of $\mathcal{P}$ points, the elements of $\mathcal{B}$ blocks, and write $P\ I\ b$ instead of $(P, b) \in I$. The incidence structure is called simple, if each block can be identified with the set of points with which it is incident. The incidence graph (also called Levi graph) $\Gamma(\mathcal{X})$ of an incidence structure $\mathcal{X} = (\mathcal{P}, \mathcal{B}, I)$ has vertex set $V = \mathcal{P} \cup \mathcal{B}$ and edge set $\{\{P, b\} \mid PIb\}$. $\Gamma(\mathcal{X})$ is a bipartite graph with vertex color classes $\mathcal{P}$, $\mathcal{B}$. Automorphisms or isomorphism of incidence structures induce automorphisms of isomorphisms of the associated incidence graphs. The converse is also true when we require that vertex colors to be preserved. Hence, when classifying combinatorial incidence structures, many problems can be reduced to the isomorphism problem of simple graphs (GI).

We first give a brief survey on the theoretical aspects of the computational complexity of the graph isomorphism problem, including Babai's seminal result [1]. Then, we focus on the implementation issues of the problem. One of the most powerful and best known of these algorithms is due to Brendan McKay [3]. It is known that his algorithm has exponential running time on some inputs, but in general it performs exceptionally well. The algorithms of `nauty` and `BLISS` [2] are based on iterative refinement techniques, on equitable partitions, and on canonical labeling.

The second lecture will present methods that locally modify 2-designs and let the main parameters invariant. The switching method allows the modification of two columns of the incidence matrix. This turns out to be a special case of the paramodification method, which affects the columns of the incidence matrix that correspond to the points of a fixed block. We will present these methods in detail, and study them for specific classes of 2-designs: affine and projective planes, Steiner triples systems, and unitals [4]. Since we can construct an enormous amount of new unitals of order 4 in this way, we will also address the issues of classifying and storing combinatorial objects on the computer.

References

[1] L. Babai. Graph isomorphism in quasipolynomial time.

[2] T. Junttila and P. Kaski. Engineering an efficient canonical labeling tool for large and sparse graphs. In *Proceedings of the Meeting on Algorithm Engineering & Expermiments*, page 135–149, USA, 2007. Society for Industrial and Applied Mathematics.

[3] B. D. McKay and A. Piperno. Practical graph isomorphism, ii. *Journal of Symbolic Computation*, 60:94–112, 2014.

[4] D. Mezőfi and G. P. Nagy. New Steiner 2-designs from old ones by paramodifications. *Discrete Appl. Math.*, 288:114–122, 2021.

Heffter arrays:
origins, variants and applications

Anita Pasotti
University of Brescia (Italy)

The notion of a Heffter array has been introduced by Archdeacon [1]. Given a positive integer $v = 2nk + 1$, a *Heffter array* $\mathrm{H}(n;k)$ is an $n \times n$ partially filled array with entries in $\mathbb{Z}_v$ satisfying the following conditions: 1) each row and each column contains exactly k filled cells; 2) for every $x \in \mathbb{Z}_v \setminus \{0\}$, either x or $-x$ appears in the array; 3) the sum of the elements in every row and column is $0 \pmod{v}$. In this talk, besides presenting the most important existence results on this topic (see [2], [3] and [5]), I will propose recent variants [4] and generalizations [6] emphasizing their applications to difference families, (orthogonal) graph decompositions, and biembeddings.

References

[1] D.S. Archdeacon, *Heffter arrays and biembedding graphs on surfaces*, Electron. J. Combin., **22** #P1.74, 2015.

[2] D.S. Archdeacon, J.H. Dinitz, D.M. Donovan, E.S. Yazıcı, *Square integer Heffter arrays with empty cells*, Des. Codes Cryptogr. **77** (2015).

[3] N.J. Cavenagh, J. Dinitz, D. Donovan, E.S. Yazıcı, *The existence of square non-integer Heffter arrays*, Ars Math. Contemp. **17** (2019).

[4] S. Costa, S. Della Fiore, A. Pasotti, *Non-zero sum Heffter arrays and their applications*, to appear in Discrete Math..

[5] J.H. Dinitz, I.M. Wanless, *The existence of square integer Heffter arrays*, Ars Math. Contemp. **13** (2017).

[6] S. Costa, A. Pasotti, *On λ-fold relative Heffter arrays and biembedding multigraphs on surfaces*, Europ. J. Combin. **97** (2021).

Some geometric aspect of extremal graphs

Valentina Pepe
Sapienza University of Rome (Italy)

Extremal graphs typically are maximal or minimal with respect to some parameter, and such that they do not contain a local substructure such as a subgraph. Infinite families of such graphs are usually constructed by using finite fields and most of the known constructions are highly symmetric. Most of them can be easily associated to well-known substructures of a projective space over a finite field. The aim of this talk is essentially to enlighten the "geometric" picture behind some extremal graphs: that can be fascinating itself and furthermore it can also suggest new ways to tackle a problem. Some new results are also presented.

Divisible designs Cayley graphs and digraphs

Andrea Švob
University of Rijeka (Croatia)

Joint work with Dean Crnković and Hadi Kharaghani

In [4], Haemers, Kharaghani and Meulenberg have defined divisible design graphs as a generalization of (v, k, λ)-graphs. Divisible design digraphs, a directed graph version of divisible design graphs, were introduced in [1]. Let G be a group and S a subset of G not containing the identity element of the group, which will be denoted by e. The vertices of the Cayley digraph $Cay(G, S)$ are the elements of the group G, and its arcs are all the couples (g, gs) with $g \in G$ and $s \in S$. In this talk, we will present results on divisible design Cayley digraphs and give some constructions of such digraphs. Further, we will give some improvements on the study of divisible design Cayley graphs. The talk will be based on the studies presented in [2] and [3]. Finally, we will introduce a variation of directed Deza graphs and give connections between combinatorial structures presented in this talk.

References

1. D. Crnković, H. Kharaghani, Divisible design digraphs, in: Algebraic Design Theory and Hadamard Matrices, (C. J. Colbourn, Ed.), Springer Proc. Math. Stat., Springer, New York (2015).

2. D. Crnković, H. Kharaghani, A. Švob, Divisible design Cayley digraphs, Discrete Math. 343 (2020), 111784, 8 pages.

3. D. Crnković, A. Švob, New constructions of divisible design Cayley graphs, Graphs Combin. 38 (2022), 17, 8 pages.

4. W. H. Haemers, H. Kharaghani, M. Meulenberg, Divisible design graphs, J. Combin. Theory Ser. A 118 (2011), 978–992.

Construction of q-analogs of combinatorial objects with prescribed symmetries

Alfred Wassermann
University of Bayreuth (Germany)

The search for combinatorial objects like designs, error-correcting codes or incidence structures from finite geometry is limited very often by a phenomenon called “combinatorial explosion”. This describes the problem that the size of a search tree grows exponentially with the increase of parameters of the combinatorial structure. A successful approach to tame the combinatorial explosion to a certain extend is to restrict the search to objects with symmetry, i.e. to prescribe an automorphism group.

In the first part of this mini-course we will study the Kramer-Mesner method, which is one possible way to search for objects with prescribed symmetry. Up to now, it has been very successful in the search for combinatorial t-designs and many other combinatorial structures. In this method the search is reduced to solving a Diophantine linear system of equations. We will discuss available algorithms to solve such linear systems and especially highlight the connection to lattice algorithms.

In the second part, q-analogs of combinatorial structures are introduced and the search for those objects using the Kramer-Mesner approach is discussed. In particular, we will have a look at subspace designs, q-analogs of group divisible designs, MRD codes and designs in polar spaces.

Contributed talks

Detecting communities under constraints in directed acyclic networks

Suzana Antunović
Faculty of Civil Engineering, Architecture and Geodesy (Split, Croatia)

Joint work with Damir Vukičević

The study of networks, in the form of mathematical graph theory, is one of the fundamental pillars of discrete mathematics. Community detection in complex networks theory is an outstanding area of research with applications in many different branches of science. Many available algorithms for community detection in directed acyclic networks do not include analysis of the resulting set of communities, and those that do, mostly focus on factors like the number of communities and community stability, not on relations between communities. In this paper, we present an algorithm that, given the topological ordering of a directed acyclic network, produces an optimal division (in terms of modularity) for that ordering which allows the establishment of an ordering on the resulting set of communities. The algorithm is based on recursively placing of the vertices into appropriate communities, thus respecting the order of the vertices, and resulting in division with optimal modularity.

A generalisation of the Alon-Yuster 123 Theorem

Adrian Beker
University of Zagreb, Croatia

In their paper [1], Alon and Yuster proved, among other results, the following theorem: if X and Y are independent identically distributed (i.i.d. for short) random variables, then $\mathbb{P}(|X - Y| \leq 2) \leq 3\mathbb{P}(|X - Y| \leq 1)$, and the constant 3 is best possible. We prove a generalisation of this result, in the following form. Let (M, d) be a separable metric space and let $b > a > 0$ be real numbers. We define $C(M; a, b)$ to be the least constant $c \in [0, +\infty]$ such that $\mathbb{P}(d(X, Y) \leq b) \leq c\mathbb{P}(d(X, Y) \leq a)$ holds for all i.i.d. M-valued random elements X and Y. Given a finite connected graph G, we equip it with the graph distance and say that it (a, b)-*embeds into* M if there exists an injective b-Lipschitz map $f\colon G \to M$ whose image is strictly a-separated. Then $C(M; a, b)$ equals one plus the supremum of the eigenvalues of all graphs which (a, b)-embed into M. We also show how some of the original results of Alon and Yuster can be deduced from this framework and discuss some further applications.

References

1. N. Alon and R. Yuster, *The 123 theorem and its extensions*, J. Combin. Theory Ser. A 72, 322–331 (1995).

List Homomorphism Problems for Signed Graphs: A Dichotomy for Trees

Jan Bok
Charles University (Czech Republic)

Joint work with Richard Brewster, Pavol Hell,
Tomás Feder, and Nikola Jedličková

We consider homomorphisms of signed graphs from a computational perspective. In particular, we study the list homomorphism problem seeking a homomorphism of an input signed graph (G, σ), equipped with lists $L(v) \subseteq V(H), v \in V(G)$, of allowed images, to a fixed target signed graph (H, π). The complexity of the similar homomorphism problem without lists (corresponding to all lists being $L(v) = V(H)$) has been previously classified by Brewster and Siggers, but the list version remains open and appears difficult. We illustrate this difficulty by classifying the complexity of the problem when H is a tree (with possible loops). The tools we develop will be useful for classifications of other classes of signed graphs. The structure of the signed trees in the polynomial cases suggests that the class of general signed graphs for which the problems are polynomial may have nice structure, analogous to the so-called bi-arc graphs (which characterized the polynomial cases of list homomorphisms to unsigned graphs).

References

1. J. Bok, R. B. Brewster, T. Feder, N. Jedličková, and P. Hell: *List Homomorphism Problems for Signed Graphs*, submitted, 2021.

2. J. Bok, R. B. Brewster, T. Feder, N. Jedličková, and P. Hell: *List Homomorphism Problems for Signed Graphs*, In 45th International Symposium on Mathematical Foundations of Computer Science, MFCS 2020, volume 170 of Leibniz International Proceedings in Informatics (LIPIcs), pages 170:20:1–20:14, 2020.

Even cycle decompositions of 4-regular graphs

Simona Bonvicini
University of Modena and Reggio Emilia (Italy)

An even cycle decomposition (ECD) of an Eulerian graph is a partition of the edge set into even cycles. It is known that every 2-connected Eulerian graph containing no subgraph contractible to K_5 has an ECD [4]. Nevertheless, the results in [4] do not solve the problem for 4-regular graphs since almost all 4-regular graphs have a K_5-minor [2].

We study ECDs in 4-regular graphs satisfying the following additional condition. We color the even cycles so as two cycles sharing at least one vertex get distinct colors. If k is the minimum number of required colors, then we say that the ECD has index k. We are interested in ECDs with the smallest index k. For a 4-regular graph G, there exists an ECD of index $k = 2$ if and only if G is class 1, otherwise $k \geq 3$ for every ECD. We prove the existence of an ECD of index 3 for some infinite families of 4-regular graphs. The results give a new contribution to the problem on the existence of ECDs in 4-regular graphs and to other open problems that are known in the literature, see [1, 3].

References

1. A. Bonisoli, S. Bonvicini, Even cycles and even 2-factors in the line graph of a simple graph, Electron. J. Combin. 24 (2017) P4.15.

2. K. Markström, Complete minors in cubic graphs with few short cycles and random cubic graphs, Ars Combin. 70 (2004), 289–295.

3. K. Markström, Even cycle decompositions of 4-regular graphs and line graphs, Discrete Math. 312 (2012), 2676-2681.

4. C. Q. Zhang, On even circuit decompositions of Eulerian graphs, J. Graph Theory 18(1) (1994), 51–57.

On harmonious linear spaces

Marco Buratti
University of Perugia

Joint work with Dieter Jungnickel

We propose to say that a linear space is *harmonious* if it is resolvable and admits an automorphism group acting sharply transitively on the points and transitively on the parallel classes. In this talk I will present some constructions for harmonious linear spaces with a special focus on the following result: for any finite non-singleton subset K of $\mathbb{Z}^+$ there are infinitely many values of v for which there exists a harmonious linear space with v points whose line sizes are precisely the elements of K.

Cameron-Liebler sets for hyperbolic quadrics

Maarten De Boeck
University of Rijeka (Croatia)

Joint work with Jozefien D'haeseleer and Morgan Rodgers

In [1] Cameron and Liebler studied the orbits of the projective groups $\mathrm{PGL}(n+1, q)$. For this purpose they introduced line classes in the projective space $\mathrm{PG}(3, q)$ with a specific property, which afterwards were called Cameron-Liebler line classes. Many equivalent characterisations of these Cameron-Liebler classes are known, in particular they are intriguing sets of the Grassmann graph.

In the past decades the concept of Cameron-Liebler classes was generalised, and they were introduced for several combinatorial structures, in particular finite geometries. Amongst others (degree one) Cameron-Liebler classes of generators in polar spaces were discussed in [2] and [3].

In this talk I will present some recent constructions of Cameron-Liebler sets in hyperbolic quadrics and discuss classification results of Cameron-Liebler classes with a small parameter.

References

1. P. J. Cameron, R. A. Liebler, Tactical decompositions and orbits of projective groups, Linear Algebra Appl. 46:91–102, 1982.

2. M. De Boeck, J. D'haeseleer, Equivalent definitions for (degree one) Cameron-Liebler classes of generators in finite classical polar spaces, Discrete Math., 343(1): 111642, 13pp., 2019.

3. M. De Boeck, M. Rodgers, L. Storme, A. Švob, Cameron-Liebler sets of generators in finite classical polar spaces. J. Combin. Theory Ser. A 167:340–388, 2019.

Extension theory for Steiner triple systems

Giovanni Falcone
University of Palermo (Italy)

Joint work with Àgota Figula, Mario Galici

Despite the fact that the connections between Steiner triple systems (STS) and loops have been stressed since the beginning, a theory of extensions of STS has not been treated till now. We show that such an approach is not only possible, but fruitful, as we can both give simplified and improved proofs of standard results, and new constructions and characterizations.

For instance, it is well known that, as a consequence of the theorem of Veblen and Young, a STS $\mathcal{S}$ is the point-line design $\mathrm{PG}_1(d,2)$ of a projective geometry over $\mathrm{GF}(2)$ iff each point is a Veblen point, but we show that $\mathcal{S}$ is a $\mathrm{PG}_1(d,2)$ iff nearly a sixteenth of the points are Veblen points.

Steiner triple subsystems corresponding to *normal* subloops will be called normal subsystems, and STSs corresponding to *quotient* subloops will be called quotient systems.

We introduce extensions of a (normal sub-) STS $\mathcal{N}$ by a (quotient) STS $\mathcal{Q}$, and we show that, among the most general extensions of $\mathcal{N}$ by $\mathcal{Q}$, we find *Schreier extensions*, for which a very elegant description in terms of cocycles and coboundaries can be given.

Biembeddings of Archdeacon type and their full automorphism group

Simone Costa
University of Brescia (Italy)

Archdeacon, in his seminal paper "*Heffter arrays and biembedding graphs on surfaces*, Electron. J. Combin. **22** (2015) #P1.74.", defined the concept of Heffter array in order to provide explicit constructions of $\mathbb{Z}_v$-regular biembeddings of complete graphs K_v into orientable surfaces.

In this talk, we first introduce the *quasi*-Heffter arrays as a generalization of the concept of Heffer array and we show that, in this context, we can define a 2-colorable embedding of Archdeacon type of the complete multipartite graph $K_{\frac{v}{t}\times t}$ into an orientable surface. Then we will present a probabilistic result on their full automorphism groups: in particular we are able to prove that, almost always, this group is exactly $\mathbb{Z}_v$.

As an application, given a positive integer $t \not\equiv 0 \pmod 4$, we show that there are, for infinitely many pairs of v and k, at least $(1-o(1))\frac{(\frac{v-t}{2})!}{\phi(v)}$ non-isomorphic biembeddings of $K_{\frac{v}{t}\times t}$ whose face lengths are multiples of k. Moreover, in case $t=1$ and v prime, almost all these embeddings define faces that are all of the same length kv, i.e. we have a more than exponential number of non-isomorphic kv-gonal biembeddings of K_v.

Switching for 2-designs and Hadamard matrices

Dean Crnković
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Joint work with Andrea Švob

In this talk, we introduce a switching for 2-designs (see [2]). We apply this method to the symmetric $(64, 28, 12)$ designs constructed in [1]. Further, we show that this type of switching can be applied to any symmetric design related to a Bush-type Hadamard matrix. We apply the switching to the designs constructed in [3, 4, 5] and construct symmetric $(36, 15, 6)$ designs leading to new Bush-type Hadamard matrices of order 36, and symmetric $(100, 45, 20)$ designs yielding Bush-type Hadamard matrices of order 100.

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On vertices belonging to all strong metric bases of a unicyclic graph

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Joint work with Anni Hakanen, Ville Junnila and Tero Laihonen

For a connected graph G, a vertex $w \in V(G)$ *strongly resolves* two different vertices $u, v \in V(G)$ if $d_G(w,u) = d_G(w,v) + d_G(v,u)$ or $d_G(w,v) = d_G(w,u) + d_G(u,v)$. Equivalently, there is some shortest $w - u$ path that contains v or some shortest $w - v$ path containing u. A set $S \subset V(G)$ is a *strong resolving set* for G, if every two vertices of G are strongly resolved by some vertex of S. The cardinality of a smallest strong resolving set for G is called the *strong metric dimension* of G, denoted by $\dim_s(G)$. A *strong metric basis* of G is a strong resolving set of cardinality $\dim_s(G)$.

In this work we consider those vertices of a unicyclic graph G that belong to all strong metric bases of G, and call them as *strong basis forced vertices*. Specifically, we prove that there can be unicyclic graphs having as many strong basis forced vertices as we would require. We compute the number of strong basis forced vertices that have the unicyclic graphs whose unique cycle has even order. We also discuss the characterization of unicyclic graphs of odd order containing strong basis forced vertices.

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On orbital matrices, existence and enumeration

Harald Gropp
Germany

Orbital matrices were introduced already 35 years ago, but have not been studied very much since then. An orbital matrix is a generalization of the incidence matrix of a symmetric 2-design. It is a square matrix with non-negative integer entries with constant row and column sum such that $AA^t = (k + x - \lambda)I + \lambda J$. The existence problem of orbital matrices is discussed, especially for $\lambda \leq 3$. The theorem of Bruck-Ryser-Chowla holds also for orbital matrices. However, there remain a lot of cases where other techniques have to be used for deciding the existence or non-existence of such a matrix. Some special cases of parameters are considered and further research questions are adressed.

Neighbour-transitive codes in generalised quadrangles

Daniel Hawtin
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Joint work with Dean Crnković and Andrea Švob

A code C in an arbitrary graph Γ is a subset of the vertex set of Γ. The minimum distance δ of a code C is the smallest distance between a pair of distinct elements of C and the graph metric gives rise to the distance partition $\{C, C_1, \ldots, C_\rho\}$, where ρ is the maximum distance between any vertex of Γ and its nearest element in C. In this talk we consider the case where Γ is the point-line incidence graph of a generalised quadrangle $\mathcal{Q}$ and we say that C is a code in the generalised quadrangle $\mathcal{Q}$. Since the diameter of Γ is 4, both ρ and δ are at most 4. If $\delta = 4$ then C is a partial ovoid or partial spread of $\mathcal{Q}$, and if, additionally, $\rho = 2$ then C is an ovoid or a spread. A code C in $\mathcal{Q}$ is neighbour-transitive if its automorphism group acts transitively on each of the sets C and C_1. Our main results i) classify all neighbour-transitive codes admitting an insoluble group of automorphisms in thick classical generalised quadrangles that correspond to ovoids or spreads, and ii) give two infinite families and six sporadic examples of neighbour-transitive codes with minimum distance $\delta = 4$ in the classical generalised quadrangle $\mathsf{W}_3(q)$ that are not ovoids or spreads.

Minimal codes, strong blocking sets and higgledy-piggledy lines

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Joint work with Zoltán Lóránt Nagy

A codeword v of a linear code $\mathcal{C}$ is *minimal* if the support of v does not contain the support of any codeword other than its scalar multiples. The linear code $\mathcal{C}$ is a *minimal code* if all of its codewords are minimal.

The geometrical interpretaiton of minimal codes as point sets of a projective space are called strong (or cutting) blocking sets. A point set B of $\mathrm{PG}(n, q)$ is a *strong blocking set* if for every hyperplane H of $\mathrm{PG}(n, q)$, $H \cap B$ generates H; that is, $H \cap B$ contains n points in general position. Note that in $\mathrm{PG}(2, q)$, strong blocking sets are the same as double blocking sets.

A major problem regarding strong blocking sets is to find small examples. Formerly known constructions were either large (quadratic in n or superlinear in q) or required q to be large compared to n. In the talk, we review the idea of constructing strong blocking sets based on so-called higgledy-piggledy lines. Furthermore, we present a simple construction formed by a random set of higgledy-piggledy lines which works for all q and n, and whose size is linear in both n and q. Although the topic originates from coding theory, our focus will be on the geometrical point of view.

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Constructing binary quasi-cyclic codes of index 2 from a pair of cyclic codes

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The class of cyclic codes is frequently used in many areas of coding theory. Cyclic codes of given length n over a finite field F_q are well-classified as every code corresponds to a unique divisor of the polynomial $x^n - 1$ from $F_q[x]$. We will discuss a procedure for constructing the complete list of binary quasi-cyclic codes of index 2 (called half-cyclic codes) of length $2n$ from pairs of cyclic codes of length n. This procedure is based on the fact that projections of half-cyclic codes into their even/odd positions form a multiset whose unique elements form a cyclic code - in other words: Every half-cyclic codes is a subdirect product of a pair of cyclic codes. In the procedure, we apply methods of linear algebra to state conditions that need to be fulfilled for a pair of cyclic codes to be combined into a (non-empty) set of half-cyclic codes and to describe rules that lead to generating the complete and unique list of their combinations.

List Homomorphism Problems for Signed Graphs: Irreflexive graphs

Nikola Jedličková
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Joint work with Jan Bok, Richard Brewster, Pavol Hell, Tomás Feder

The complexity of the list homomorphism problem for signed graphs appears difficult to classify. Existing results focus on special classes of signed graphs, such as trees and reflexive signed graphs. Irreflexive signed graphs are the heart of the problem, and Kim and Siggers have recently conjectured a classification for these signed graphs. We focus on a special case of irreflexive signed graphs, namely those in which the unicoloured edges form a spanning path or cycle, and classify the complexity of list homomorphisms to these signed graphs. In particular, our results confirm the conjecture of Kim and Siggers for this class of signed graphs. We shall also briefly outline the recent progress towards a general complexity dichotomy.

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Equidistant walks to infinity

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One can consider arithmetical progressions as equidistant walks to infinity with the common difference as the length of the step. The main subjects of this research are progressions of odd and even numbers. There is a description of different equidistant walks over them given. Further, one can form grids of columns of progressions of odd numbers or even numbers or alternate columns of odd and even numbers. These grids are said to be equidistant if all walks in them are equidistant; we give a condition that ensures this. Furthermore, we define a concatenated product of odd numbers and even numbers to discuss equidistant walks over them.

Lacing designs in PAG

Vedran Krčadinac
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PAG is a package for constructing combinatorial objects with prescribed automorphism groups written for the computer algebra system GAP [1]. Pag is also an island in the Adriatic Sea renowned for its hand-made lacework. We will show a piece of Pag lace exhibiting dihedral symmetry and use it to get permutational representations of the dihedral group D_{16}. We will then demonstrate how to construct combinatorial designs with D_{16} as an automorphism group using the PAG package.

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On the outer multiset dimension of graphs

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Joint work with Sandi Klavžar and Ismael G. Yero

All graphs considered in this work are connected and of order at least 2. If u is a vertex of a graph G and $S \subseteq V(G)$, then the *multiset representation of* u *with respect to* S is

$$\mathrm{m}_G(u|S) = \{\!\{d_G(u,w_1),\ldots,d_G(u,w_t)\}\!\}\,,$$

where $\{\!\{\cdot\}\!\}$ limits a multiset. A set $S \subset V(G)$ is a *outer multiset resolving set* for G, if the multiset representations of vertices $u \notin S$ with respect to S are pairwise different. A multiset resolving set of the smallest possible cardinality is called an *outer multiset basis*, and the cardinality of an outer multiset basis is the *outer multiset dimension* of G, denoted by $\dim_{\rm ms}(G)$. We say that a graph G is *distance irregular* if for every two vertices $u, v \in V(G)$, the multisets $\mathrm{m}_G(u|V(G))$ and $\mathrm{m}_G(v|V(G))$ are different.

We have studied the outer multiset dimension of graphs in this work. Specifically, we have obtained the following contributions, among other ones.

- A graph G of order $n(G)$ satisfies $\dim_{\rm ms}(G) = n(G) - 1$ if and only if G is a regular graph with diameter at most two.
- We have characterized all the graphs having an outer multiset basis formed by two adjacent vertices.
- More in general, we have developed an algorithm which demonstrates that the problem of deciding whether a graph G satisfies $\dim_{\rm ms}(G) = 2$ can be polynomially done.
- We have studied the outer multiset dimension of lexicographic product graphs $G \circ H$, and have obtained that if G is a graph with $n(G) \geq 2$ and $H \in \{K_k, \overline{K_k}\}$, $k \geq 2$, then $\dim_{\rm ms}(G \circ H) \geq n(G)(k-1)$, and moreover, the equality holds if and only if G is distance irregular.
- We have also studied the outer multiset dimension of grid graphs $P_r \Box P_s$, and proved that $\dim_{\rm ms}(P_r \Box P_s) = 3$.

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Combinatorial structures arising from sharply 2- and 3-transitive groups

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Let G be a group acting on a set P. For $k \geq 1$ if for every two ordered k-tuples $(x_1, \ldots, x_k)$ and $(y_1, \ldots, y_k)$, each consisting of distinct elements of P, there exists a unique element $\pi \in G$ such that $\pi(x_1, \ldots, x_k) = (y_1, \ldots, y_k)$, then we say that the action of G on P is sharply k-transitive, abbreviated SkT. If we restrict attention to finite sets P then the cases $k = 2$ and $k = 3$ are of interest, because they lead to infinite families of groups that can be used to construct combinatorial structures with some distinguished properties.

Both classes of finite S2T and S3T groups were initially characterized by Zassenhaus in two papers (1936), and their study was further refined in subsequent decades. Zassenhaus showed that a finite S2T group consists of affine transformations of an algebraic structure known as nearfield. Drápal and Lisoněk (2020) constructed infinite families of maximally non-associative quasigroups using S2T groups acting on Dixon's nearfields; the very existence of such quasigroups was an open problem prior to that work.

In this talk we further report new results employing finite S3T groups. Zassenhaus showed that in this case the group consists of rational transformations of two possible types. For both types the group acts on the projective line $\mathrm{PG}(1, q)$ over $\mathbb{F}_q$. We use these S3T groups to construct a set of $q - 2$ Latin cubes of order $q + 1$, where q is a prime power; when q is an odd square we construct two such sets. We study properties of these Latin cubes.

New regular two-graphs on 38 and 42 vertices

Marija Maksimović
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Joint work with Sanja Rukavina

We will talk about the construction of regular two-graphs with 38 and 42 vertices from strongly regular graphs. We give a classification of strongly regular graphs with parameters (41,20,9,10) that have a nontrivial automorphism, and enumerate all regular two-graphs with 38 and 42 vertices that have at least one descendant whose full automorphism group is nontrivial.

Constructions of globally simple relative non-zero sum Heffter arrays and biembeddings

Lorenzo Mella
University of Modena (Italy)

Joint work with Anita Pasotti

In [1] Costa, Della Fiore and Pasotti introduced a class of partially filled arrays with entries in a cyclic group, called *non-zero sum Heffter arrays*. Whenever the partial sums of every column (from top to bottom) and of every row (from left to right) of an array are all distinct and non-zero, the array is called *globally simple*. From a globally simple (relative) non-zero sum Heffter array one can obtain two cyclic orthogonal path decompositions of the complete (multipartite) graph.

In [2,3] we give direct constructions of square globally simple relative non-zero sum Heffter arrays, denoted as $NH_t(n;k)$, completely solving the existence problem: (1) for every prime $n = k$ and every admissible t; (2) for every odd $n = k$ and any divisor t of n; (3) for every integer n and every $t = k$ smaller than n. In this talk, we present these results, showing also their relations with biembeddings of graphs on orientable surfaces.

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Equitably 2-colourable cycle systems

Francesca Merola
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Joint work with Andrea Burgess

An ℓ-cycle decomposition of a graph G is said to be *equitably c-colourable* if there is a c-vertex-colouring of G such that each colour is represented (approximately) an equal number of times on each cycle: more precisely, we ask that in each cycle C of the decomposition, each colour appears on $\lfloor \ell/c \rfloor$ or $\lceil \ell/c \rceil$ of the vertices of C. In this talk, we consider the case $c = 2$ and present some new results on the existence of 2-colourable ℓ-cycle systems.

Maximal partial Room squares

Mariusz Meszka
AGH University of Science and Technology, Kraków, Poland

Joint work with Alexander Rosa

A *partial Room square* of order n and side $n-1$ on an n-element set S is an $(n-1)\times(n-1)$ array F satisfying the following properties:
(1) every cell of F is either empty or contains an unordered pair of symbols from S,
(2) every symbol of S occurs at most once in each row and at most once in each column of F,
(3) every unordered pair of symbols of S occurs in at most one cell of F.
A partial Room square is maximal if no further pair of elements can be placed into any unoccupied cell without violating the conditions that define a partial Room square.

The aim is to determine the spectrum of volumes of maximal partial Room squares of even order n, where the volume means the number of occupied cells.

Flag-transitive, point-imprimitive symmetric 2-designs

Alessandro Montinaro
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Let $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ be a symmetric 2-(v, k, λ) design admitting a flag-transitive, point-imprimitive automorphism group G that leaves invariant a non-trivial partition Σ of $\mathcal{P}$. C. E. Praeger and S. Zhou [4] have shown that, there is a constant k_0 such that, for each $B \in \mathcal{B}$ and $\Delta \in \Sigma$, the size of $|B \cap \Delta|$ is either 0 or k_0. In this talk, which is based on the result contained in [2], we show that, if $k > \lambda(\lambda - 3)/2$ and $k_0 \geq 3$, $\mathcal{D}$ is isomorphic to one of the flag-transitive, point-imprimitive symmetric 2-designs with parameters $(45, 12, 3)$ or $(96, 20, 4)$ classified in [3] and in [1] respectively.

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The search algorithm for extremal $\mathbb{Z}_4$-codes

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Joint work with Sanja Rukavina

A $\mathbb{Z}_4$-code C of length n is a $\mathbb{Z}_4$ sub-module of $\mathbb{Z}_4^n$. With respect to the standard inner product modulo 4, the dual code $C^\perp$ of the $\mathbb{Z}_4$-code C is defined. The code C is self-dual if $C = C^\perp$. There are two binary codes associated with a $\mathbb{Z}_4$-code C called a residue code and a torsion code. These two codes are a starting point in the construction of self-dual $\mathbb{Z}_4$-codes by the method given in [1]. For $\mathbb{Z}_4$-codes, the Euclidean weight of codeword x is defined by $n_1(x) + 4n_2(x) + n_3(x)$, where $n_i(x)$ is the number of components of x which are equal to i. A $\mathbb{Z}_4$-code C of length n is said to be extremal if its minimal Euclidean weight is $8\left\lfloor \frac{n}{24} \right\rfloor + 8$. In this talk, we will discuss an algorithm that improves the search for extremal self-dual $\mathbb{Z}_4$-codes which we used to obtain some new extremal codes of lengths 32 and 40.

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One-point concentration of the clique and chromatic numbers of the random Cayley graph on $\mathbf{F}_2^n$

Rudi Mrazović
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We show that the clique number of the Cayley graph on $\mathbf{F}_2^n$ generated by a random subset is, with high probability, between $\frac{1}{2}n\log n$ and $1.01n\log n$. Moreover, we prove that for n in a set of density 1, the clique number is actually concentrated on a single value. As a simple consequence of these results, we also prove a one-point concentration result for the chromatic number, thus proving a $\mathbf{F}_2^n$ analogue of the famous conjecture by Bollobás and giving almost the complete answer to a question by Green.

Ghostbusters in PG$(2, q)$

Silvia M.C. Pagani
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Joint work with M.L. Della Vedova (UniBG) and S. Pianta (UniCatt)

The *power sum polynomial* of a (multi-)subset S of PG(n, q) is a homogeneous polynomial in $n + 1$ variables, defined as the sum of the $(q - 1)$-th powers of the Rédei factors associated to the points of S. Differently from the Rédei polynomial, a same power sum polynomial may be obtained from more than one set. Indeed, it turns out that two multi-subsets having the same power sum polynomial "differ", in the multiset sum sense, by a multi-subset whose associated power sum polynomial is the null one. Such multisets are called *ghosts*, in analogy with the corresponding objects in discrete tomography.

In this talk we present some results about ghosts in PG$(2, q)$. We investigate the space of ghosts, compute its dimension and characterize some classes. Moreover, we explicitly enumerate ghosts for planes of small order.

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A general construction of permutation polynomials

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Joint work with Xiang-Dong Hou (University of South Florida)

Let r be a positive integer, $h(X) \in \mathbb{F}_{q^2}[X]$, and μ_{q+1} be the subgroup of order $q+1$ of $\mathbb{F}_{q^2}^*$. It is well known that $X^r h(X^{q-1})$ permutes $\mathbb{F}_{q^2}$ if and only if $\gcd(r, q-1) = 1$ and $X^r h(X)^{q-1}$ permutes μ_{q+1}. There are many ad hoc constructions of permutation polynomials of $\mathbb{F}_{q^2}$ of this type such that $h(X)^{q-1}$ induces monomial functions on the cosets of a subgroup of μ_{q+1}. We give a general construction that can generate, through an algorithm, *all* permutation polynomials of $\mathbb{F}_{q^2}$ with this property, including many which are not known previously. The construction is illustrated explicitly for permutation binomials and trinomials.

Cameron–Liebler line classes admitting large cyclic groups

Morgan Rodgers
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Cameron–Liebler line classes are special sets of lines in $\mathrm{PG}(3,q)$ with nice intersection properties; they were originally described in [1] in relation to line orbits of projective groups. We can think of them as extremal sets of lines having as many intersections as possible, or they can alternately be characterized as sets sharing a constant number of lines with every spread of the space. Such a set must contain precisely $x(q^2+q+1)$ lines for some integer x, called the *parameter* of the line class.

There has been a lot of recent interest in Cameron–Liebler line classes, though there are only a small number of known families of examples. After giving some basic definitions, we will look at some recent examples constructed as unions of line orbits from a cyclic group of size q^2+q+1. These examples are especially notable since they also give Cameron–Lieber line classes of the affine space $\mathrm{AG}(3,q)$.

This talk describes joint work appearing in [2] and [3].

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Higher Degree Davenport Constants Over Finite Commutative Rings

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Joint work with Yair Caro and Benjamin Girard

We generalize the notion of Davenport constants to a 'higher degree' and obtain various lower and upper bounds, which are sometimes exact as is the case for certain finite commutative rings of prime power cardinality. Two simple examples that capture the essence of these higher degree Davenport constants are the following. 1) Suppose $n = 2^k$, then every sequence of integers S of length $2n$ contains a subsequence S' of length at least two such that $\sum_{a_i,a_j \in S'} a_i a_j \equiv 0 \pmod{n}$ and the bound is sharp. 2) Suppose $n \equiv 1 \pmod{2}$, then every sequence of integers S of length $2n-1$ contains a subsequence S' of length at least two such that $\sum_{a_i,a_j \in S'} a_i a_j \equiv 0 \pmod{n}$. These examples illustrate that if a sequence of elements from a finite commutative ring is long enough, certain symmetric expressions have to vanish on the elements of a subsequence.

On the locally irregular edge colorings of graphs

Jelena Sedlar
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Joint work with Riste Škrekovski

A locally irregular graph is a graph in which any two neighboring vertices have distinct degree. An edge coloring of a graph G is said to be locally irregular if each of the colors induces a locally irregular subgraph of G. A graph G is colorable if it admits a locally irregular edge coloring. We consider the smallest number of colors required for a locally irregular edge coloring of cacti and comment it in the light of the conjectured upper bound for such number.

On a graph isomorphic to $NO^+(6,2)$

Valentino Smaldore
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Joint work with Federico Romaniello

Tangent graphs of quadrics are a well-known item in spectral graph theory. The graph $NO^+(2m,2)$ consists on all the $2^{2m-1} - 2^{m-1}$ non-singular point respect a non-degenerate quadratic form of type *+1* in $PG(2m-1,q)$, where two vertices are adjacent if the points are on a line tangent to the quadric $Q^+(2m-1,2)$. Such graph is strongly regular with parameters (v,k,λ,μ) where:

$$v = 2^{2m-1} - 2^{m-1},$$

$$k = 2^{2m-2} - 1,$$

$$\lambda = 2^{2m-3} - 2,$$

$$\mu = 2^{2m-3} + 2^{m-2}.$$

Now, let $\mathcal{V}_2^4 = \{(a^2,b^2,c^2,ab,ac,bc)|(a,b,c) \in PG(2,q)\} \subseteq PG(5,q)$ be the *Veronese surface*. The union of all *conic planes* in $PG(5,q)$, i.e. all planes meeting $\mathcal{V}_2^4$ in a conic, is the hypersurface $\mathcal{M}_4^3$, that has $(q^2+q+1)(q^2+1)$ points, and so $|Q^+(5,q)| = |\mathcal{M}_4^3|$. Moreover, the hypersurface $\mathcal{M}_4^3$ has the same intersections numbers of the *Klein Quadric* with hyperplanes of $PG(5,q)$. A natural question should be to define the graph $N\mathcal{M}_4^3$ considering the points not on $\mathcal{M}_4^3$, adjacent if the line through them meets $\mathcal{M}_4^3$ in exactly one point. In this talk we will show that, in the case $q = 2$, the graph $N\mathcal{M}_4^3$ is strongly regular, and isomorphic to the tangent graph $NO^+(6,2)$.

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Row-sum matrices

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Joint work with A. Burgess, P. Danziger, A. Pastine

Let Γ be an additive group, $S \subset \Gamma$, and let Σ be an $|S|$-list of elements of Γ. A *row-sum matrix* $RSM_\Gamma(S, g; \Sigma)$ is an $|S| \times g$ matrix whose $g \geq 2$ columns are permutations of S and such that the list of (left-to-right) row-sums is Σ. Denoting by $\omega : \Gamma \to \mathbb{N}$ the map where $\omega(g)$ represents the order of g, we say that a list L of positive divisors of $|\Gamma|$ is g*-realizable* over Γ, if there exists an $RSM_\Gamma(S, g; \Sigma)$ such that $\omega(\Sigma) = L$.

Row-sum matrices have been extensively used to factorize Cayley subgraphs of blown-up cycles. Indeed, if $L = \{\ell_1, \ldots, \ell_s\}$ is g-realizable over Γ, then $C_g[S] = Cay[\mathbb{Z}_g \times \Gamma : \{1\} \times S]$ has a 2-factorization $\mathcal{F} = \{F_1, \ldots, F_s\}$ where each F_i is $g\ell_i$-uniform, that is, the vertex-disjoint union of $g\ell_i$-cycles. These partial decompositions are very useful to factorize complete (equipartite) graphs into a prescribed set of uniform 2-factors, a problem that has been constructively solved only when all the 2-factors are pairwise isomorphic.

In this talk, we present some existence results on row-sum matrices, and show that working over a non-abelian group Γ turns out to be helpful whenever we need to g-realize a list L of positive integers with distinct parities.

On the $[k]$-multiple Roman domination in graphs

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Roman domination in graphs is a very well-known topic in graph theory. As a variation of the classic domination problem, the main goal is to search for a dominating set of vertices in the graph that satisfies some additional condition. One can see the Roman domination as a defensive strategy of the vertices of the graph in which self-defended vertices are labelled 1, and any undefended vertex, labelled with a 0, must have a strong neighbour, with a 2 label. In some sense, the stronger vertices are able to 'send' a unit to defend an unprotected one, without leaving their own spot undefended.

Formally, a Roman dominating function is a function $f : V(G) \to \{0, 1, 2\}$, in such a way that any vertex with $f(u) = 0$ must be adjacent to, at least, a vertex w with $f(w) = 2$.

In this work, we initiate the study of a new domination parameter and its properties. Prescribed a positive integer $k \geq 2$, we consider a vertex labelling given by a function $f : V(G) \to \{0, 1, \ldots, k+1\}$ such that $f(N[v]) \geq |AN(v)| + k$ for all $v \in V(G)$, where $N[v]$ is the closed neighbourhood of v and $AN(v)$ is the set formed by the active neighbours of vertex v, that is to say, the neighbours of v having a positive label by f.

By analogy with the interpretation of Roman domination as a defensive strategy, we could say that in this case we must be able to guarantee that each vertex of the graph can be defended by k units, which are located in it or in its neighbourhood, without leaving any of its active neighbours unprotected.

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