

AUDITORNE VJEŽBE – 11. TJEDAN

- (1) Linearan i vremenski nepromjenljiv kontinuirani sustav zadan je diferencijalnom jednačbom

$$y''(t) + y'(t) - 6y(t) = u'(t) - 5u(t).$$

Korištenjem Laplaceove transformacije, odredite:

- (a) odziv sustava na pobudu $u(t) = e^{3t}\mu(t)$ uz početne uvjete $y(0^-) = 1$ i $y'(0^-) = 2$;
 - (b) odziv mirnog sustava na pobudu $u(t) = e^{2t}\mu(t)$;
 - (c) odziv mirnog sustava na pobudu $u(t) = e^{5t}\mu(t)$; te
 - (d) impulsni odziv mirnog sustava.
- (2) Linearan i vremenski nepromjenljiv diskretni sustav zadan je diferencijalnom jednačbom

$$6y[n] - y[n-1] - y[n-2] = 5u[n] - u[n-1].$$

Korištenjem Z-transformacije, odredite:

- (a) odziv sustava na pobudu $u[n] = \left[\frac{1}{3}\right]^n \mu[n]$ uz početne uvjete $y[-1] = 1$ i $y[-2] = 2$;
- (b) odziv mirnog sustava na pobudu $u[n] = \left[\frac{1}{2}\right]^n \mu[n]$;
- (c) odziv mirnog sustava na pobudu $u[n] = \left[\frac{1}{5}\right]^n \mu[n]$; te
- (d) impulsni odziv mirnog sustava.

RJEŠENJA

(1)

$$y''(t) \quad \circ \text{---} \bullet \quad s^2 Y(s) - sy(0^-) - y'(0^-)$$

$$y'(t) \quad \circ \text{---} \bullet \quad sY(s) - y(0^-)$$

$$y(t) \quad \circ \text{---} \bullet \quad Y(s)$$

$$u'(t) \quad \circ \text{---} \bullet \quad sU(s) - u(0^-)$$

$$u(t) \quad \circ \text{---} \bullet \quad U(s)$$

$$\left(s^2 Y(s) - sy(0^-) - y'(0^-) \right) + \left(sY(s) - y(0^-) \right) + 6Y(s) = \left(sU(s) - u(0^-) \right) - 5U(s)$$

$$Y(s) = \frac{s-5}{s^2+s-6} \cdot U(s) + \frac{s+3}{s^2+s-6}$$

(a)

$$U(s) = \frac{1}{s-3}$$

$$Y(s) = \frac{s-5}{(s-2)(s+3)(s-3)} + \frac{s+3}{(s-2)(s+3)}$$

$$\frac{s-5}{(s-2)(s+3)(s-3)} = \frac{c_1}{s-2} + \frac{c_2}{s+3} + \frac{c_3}{s-3} \quad \Big| \cdot (s-2)$$

$$\frac{s-5}{(s+3)(s-3)} = c_1 + \frac{c_2}{s+3} \cdot (s-2) + \frac{c_3}{s-3} \cdot (s-2) \quad \Big| \lim_{s \rightarrow 2}$$

$$\frac{3}{5} = c_1 + 0 + 0$$

⋮

$$Y(s) = \frac{3/5}{s-2} + \frac{-4/15}{s+3} + \frac{-1/3}{s-3} + \frac{1}{s-2}$$

$$y(t) = \underbrace{\frac{3}{5}e^{2t}\mu(t) - \frac{4}{15}e^{-3t}\mu(t) - \frac{1}{3}e^{3t}\mu(t)}_{\text{odziv mirnog sustava}} + \underbrace{e^{2t}\mu(t)}_{\text{odziv nepobuđenog sustava}}$$

(b)

$$U(s) = \frac{1}{s-2}$$

$$Y(s) = \frac{s-5}{(s-2)^2(s+3)}$$

$$\frac{s-5}{(s-2)^2(s+3)} = \frac{c_1}{(s-2)^2} + \frac{c_2}{s-2} + \frac{c_3}{s+3} \quad \Big| \cdot (s-2)^2$$

$$\frac{s-5}{s+3} = c_1 + c_2 \cdot (s-2) + \frac{c_3}{s-3} \cdot (s-2)^2 \quad \Big| \lim_{s \rightarrow 2}$$

$$-\frac{3}{5} = c_1 + 0 + 0$$

$$\frac{s-5}{s+3} = c_1 + c_2 \cdot (s-2) + \frac{c_3}{s-3} \cdot (s-2)^2 \quad \Big| \frac{d}{ds}$$

$$\frac{(s+3) - (s+5)}{(s+3)^2} = 0 + c_2 + \left(\frac{c_3}{(s-3)^2} + 2 \cdot \frac{c_3}{s-3} \right) \cdot (s-2) \quad \Big| \lim_{s \rightarrow 2}$$

$$\frac{8}{25} = 0 + c_2 + 0$$

$$\vdots$$

$$Y(s) = \frac{-3/5}{(s-2)^2} + \frac{8/25}{s-2} + \frac{-8/25}{s+3}$$

$$y(t) = -\frac{3}{5}te^{2t}\mu(t) + \frac{8}{25}e^{2t}\mu(t) - \frac{8}{25}e^{-3t}\mu(t)$$

(partikularno rješenje $-\frac{3}{5}te^{2t}$ je drugačijeg oblika u odnosu na pobudu)

(c)

$$U(s) = \frac{1}{s-5}$$

$$Y(s) = \frac{1}{(s-2)(s+3)}$$

$$\vdots$$

$$Y(s) = \frac{6/5}{s-2} + \frac{-1/5}{s+3}$$

$$y(t) = \frac{6}{5}e^{2t}\mu(t) - \frac{1}{5}e^{-3t}\mu(t)$$

(partikularno rješenje je 0)

(d)

$$U(s) = 1$$

$$Y(s) = \frac{s-5}{(s-2)(s+3)} = H(s)$$

$$\vdots$$

$$H(s) = \frac{-3/5}{s-2} + \frac{8/5}{s+3}$$

$$h(t) = -\frac{3}{5}e^{2t}\mu(t) + \frac{8}{5}e^{-3t}\mu(t)$$

(2)

$$y[n] \quad \circ \text{---} \bullet \quad Y(z)$$

$$u[n] \quad \circ \text{---} \bullet \quad U(z)$$

$$y[n-1] \quad \circ \text{---} \bullet \quad z^{-1}Y(z) + y[-1]$$

$$u[n-1] \quad \circ \text{---} \bullet \quad z^{-1}U(z) + u[-1]$$

$$y[n-2] \quad \circ \text{---} \bullet \quad z^{-2}Y(z) + z^{-1}y[-1] + y[-2]$$

$$6Y(z) - \left(z^{-1}Y(z) + y[-1] \right) - \left(z^{-2}Y(z) + z^{-1}y[-1] + y[-2] \right) = 5U(z) - \left(z^{-1}U(z) + u[-1] \right)$$

$$Y(z) = \frac{5 - z^{-1}}{6 - z^{-1} - z^{-2}} \cdot U(z) + \frac{3 + z^{-1}}{6 - z^{-1} - z^{-2}}$$

(a)

$$U(z) = \frac{1}{1 - \frac{1}{3}z^{-1}} = \frac{3}{3 - z^{-1}}$$

$$Y(z) = \frac{5 - z^{-1}}{(2 - z^{-1})(3 + z^{-1})} \cdot \frac{3}{3 - z^{-1}} + \frac{3 + z^{-1}}{(2 - z^{-1})(3 + z^{-1})}$$

(rastav na parcijalne razlomke radi se kao i kod Laplaceove transformacije)

⋮

$$\begin{aligned} Y(z) &= \frac{9/5}{2 - z^{-1}} + \frac{4/5}{3 + z^{-1}} + \frac{-1}{3 - z^{-1}} + \frac{1}{2 - z^{-1}} \\ &= \frac{9/10}{1 - \frac{1}{2}z^{-1}} + \frac{4/15}{1 + \frac{1}{3}z^{-1}} + \frac{-1/3}{1 - \frac{1}{3}z^{-1}} + \frac{1/2}{1 - \frac{1}{2}z^{-1}} \\ y[n] &= \underbrace{\frac{9}{10} \left[\frac{1}{2} \right]^n \mu[n] + \frac{4}{15} \left[-\frac{1}{3} \right]^n \mu[n] - \frac{1}{3} \left[\frac{1}{3} \right]^n \mu[n]}_{\text{odziv mirnog sustava}} + \underbrace{\frac{1}{2} \left[\frac{1}{2} \right]^n \mu[n]}_{\text{odziv nepobuđenog sustava}} \end{aligned}$$

(b)

$$\begin{aligned} U(z) &= \frac{1}{1 - \frac{1}{2}z^{-1}} = \frac{2}{2 - z^{-1}} \\ Y(z) &= \frac{5 - z^{-1}}{(2 - z^{-1})(3 + z^{-1})} \cdot \frac{2}{2 - z^{-1}} \\ &\vdots \end{aligned}$$

$$\begin{aligned} Y(z) &= \frac{6/5}{(2 - z^{-1})^2} + \frac{16/25}{2 - z^{-1}} + \frac{16/25}{3 + z^{-1}} \\ &= \frac{6/20}{(1 - \frac{1}{2}z^{-1})^2} + \frac{8/25}{1 - \frac{1}{2}z^{-1}} + \frac{16/75}{1 + \frac{1}{3}z^{-1}} \\ y[n] &= \frac{6}{20} [n+1] \left[\frac{1}{2} \right]^n \mu[n] + \frac{8}{25} \left[\frac{1}{2} \right]^n \mu[n] + \frac{16}{75} \left[-\frac{1}{3} \right]^n \mu[n] \end{aligned}$$

(partikularno rješenje $\frac{6}{20} [n+1] \left[\frac{1}{2} \right]^n \mu[n]$ je drugačijeg oblika u odnosu na pobudu)

(c)

$$\begin{aligned} U(z) &= \frac{1}{1 - \frac{1}{5}z^{-1}} = \frac{5}{5 - z^{-1}} \\ Y(z) &= \frac{5 - z^{-1}}{(2 - z^{-1})(3 + z^{-1})} \cdot \frac{5}{5 - z^{-1}} \\ &\vdots \end{aligned}$$

$$\begin{aligned} Y(z) &= \frac{1}{2 - z^{-1}} + \frac{1}{3 + z^{-1}} \\ &= \frac{1/2}{1 - \frac{1}{2}z^{-1}} + \frac{1/3}{1 + \frac{1}{3}z^{-1}} \\ y[n] &= \frac{1}{2} \left[\frac{1}{2} \right]^n \mu[n] + \frac{1}{3} \left[-\frac{1}{3} \right]^n \mu[n] \end{aligned}$$

(partikularno rješenje je nula)

(d)

$$\begin{aligned}U(z) &= 1 \\Y(z) &= \frac{5 - z^{-1}}{(2 - z^{-1})(3 + z^{-1})} = H(z) \\&\vdots \\H(z) &= \frac{3/5}{2 - z^{-1}} + \frac{8/5}{3 + z^{-1}} \\&= \frac{3/10}{1 - \frac{1}{2}z^{-1}} + \frac{8/15}{1 + \frac{1}{3}z^{-1}} \\h[n] &= \frac{3}{10} \left[\frac{1}{2} \right]^n \mu[n] + \frac{8}{15} \left[-\frac{1}{3} \right]^n \mu[n]\end{aligned}$$