

Rješenja završnog ispita iz Matematike 1
28.6.2006.

1. [2 boda] $k = \lim_{x \rightarrow \pm\infty} \frac{x \ln(e^2 + \frac{1}{x})}{x} = \lim_{x \rightarrow \pm\infty} \ln(e^2 + \frac{1}{x}) = 2$
 $l = \lim_{x \rightarrow \pm\infty} (x \ln(e^2 + \frac{1}{x}) - 2x) = \lim_{x \rightarrow \pm\infty} x(\ln(e^2 + \frac{1}{x}) - 2) = \lim_{x \rightarrow \pm\infty} \frac{\ln(e^2 + \frac{1}{x}) - 2}{\frac{1}{x}} \stackrel{L'H}{=} 0$
 $= \dots = \frac{1}{e^2}$

kosa asimptota: $y = 2x + \frac{1}{e^2}$

2. [2 boda] $f'(x) = 2\cos x \sin x - \cos x = 0 \Leftrightarrow \cos x = 0$ ili $\sin x = \frac{1}{2} \Rightarrow$
funkcija f u $x = \frac{\pi}{2}$ dostiže minimum, a maksimum dostiže na rubovima

3. [2 boda] $O = a + b + \sqrt{2} \Rightarrow O(b) = \sqrt{2 - b^2} + b + \sqrt{2} \Rightarrow O'(b) =$
 $= 1 - \frac{b}{\sqrt{2 - b^2}} = 0 \Leftrightarrow b = 1 \Rightarrow a = \sqrt{2 - 1^2} = 1 \Rightarrow O_{MAX} = 2 + \sqrt{2}$

4. [2 boda] parcijalna integracija: $u = \ln x, du = \frac{dx}{x},$
 $dv = (x^2 - 2x + 3)dx, v = \frac{x^3}{3} - x^2 + 3x \Rightarrow \int (x^2 - 2x + 3) \ln x dx =$
 $= (\frac{x^3}{3} - x^2 + 3x) \ln x - \int (\frac{x^3}{3} - x^2 + 3x) \frac{dx}{x} = (\frac{x^3}{3} - x^2 + 3x) \ln x - \frac{x^3}{9} +$
 $+\frac{x^2}{2} - 3x + C$

5. [2 boda] $\int_0^2 x^3 \sqrt{2x^2 + 1} dx = \left| \begin{array}{l} 2x^2 + 1 = t, \quad x^2 = \frac{t-1}{2} \\ 4x dx = dt \end{array} \right| = \int_1^9 \frac{t-1}{2} \sqrt{t} \frac{dt}{4} =$
 $\dots = \frac{298}{30}$

6. [2 boda] $\int \frac{x^3}{x^2 + x + 1} dx = \int \frac{x^3 + 1 - 1}{x^2 + x + 1} dx = \int (x - 1) dx +$
 $+\int \frac{dx}{(x + \frac{1}{2})^2 + \frac{3}{4}} = \frac{x^2}{2} - x + \frac{2\sqrt{3}}{3} \operatorname{arctg}(\frac{2x + 1}{\sqrt{3}}) + C$

7. [3 boda] Rješavamo prvo neodređeni integral;
supstitucija: $|1 + \sin x = t, \cos x dx = dt| \Rightarrow$
 $\int \frac{dt}{t^2(t-1)} \Rightarrow$ parcijalni razlomci $\Rightarrow \int \frac{-t-1}{t^2} dt + \int \frac{dt}{t-1} = \dots = \frac{2}{2 + \sqrt{2}} +$
 $+ \ln(\frac{3\sqrt{2}}{2 + \sqrt{2}}) - \frac{2}{3}$

8. [2 boda] $P = \int_0^2 \sqrt{x} dx + \int_2^3 \sqrt{6 - 2x} dx = \dots = \frac{4\sqrt{2}}{3} - \frac{\sqrt{6}}{3}$

9. [3 boda] horizontalna asimptota: $k = \lim_{x \rightarrow \infty} \frac{x-1}{x^2} = 0$,

$$l = \lim_{x \rightarrow \infty} \frac{x-1}{x} = 1 \Rightarrow y = 1$$

$$V = \pi \int_{-2}^{-1} \left(\frac{x-1}{x}\right)^2 dx - \pi \int_{-2}^{-1} dx = \dots = \frac{3\pi}{2} + 2\pi \ln 2 - \pi = \frac{\pi}{2} + 2\pi \ln 2$$

10. [3 boda] (a) $|z| = \frac{|\frac{1}{2} + i\frac{\sqrt{3}}{2}|^{22} \cdot |2 + i|^{44}}{|3 + 4i|^{23}} = \frac{1}{5}$

(b) $\varphi = \frac{2\pi}{3}$, $r = 1 \Rightarrow z = \left(\text{cis}\left(\frac{2\pi}{3}\right)\right)^{17} = \text{cis}\left(\frac{2 \cdot 17\pi}{3}\right) =$
 $= \text{cis}\frac{4\pi}{3} \Rightarrow \text{Rez} = \cos\frac{4\pi}{3} = -\frac{1}{2}$

(c) $z = x + iy \Rightarrow (x + iy)^2 = x - iy \Rightarrow x^2 - y^2 = x$, $2xy = -y \Rightarrow$
 $\Rightarrow y(2x + 1) = 0 \Rightarrow 1.$ slučaj: $y = 0, x = 0$ ili $y = 0, x = 1 \Rightarrow$
 $z_1 = 0, z_2 = 1$, 2. slučaj: $x = -\frac{1}{2}, y = \pm\frac{\sqrt{3}}{2} \Rightarrow z_3 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$,
 $z_4 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

11. [3 boda]

(a) Neka je $A \in M_n$. Matrica A' za koju je $A'A = AA' = I$ zove se inverzna matrica matrice A .

(b) $\det(AB) = \text{Binet} - \text{Cauchy} = \det A \cdot \det B = 12$

(c) $\det(A^{-1}B) = \frac{1}{6} \cdot 12 = 2$

12. [3 boda] Nedostaju slike.

13. [3 boda] (a) $f'(x) = (\cos x)' = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} =$
 $\lim_{h \rightarrow 0} \left(-\frac{2}{h} \sin\left(x + \frac{h}{2}\right) \sin\frac{h}{2}\right) = -\lim_{h \rightarrow 0} \sin\left(x + \frac{h}{2}\right) \frac{\sin\frac{h}{2}}{\frac{h}{2}} = -\sin x$

(b) $(f^{-1}(y))' = \frac{1}{f'(x)}$

(c) $y = \cos x \Rightarrow x = \arccos y; (\arccos y)' = \frac{1}{(\cos x)'} = \frac{1}{-\sin x} = \frac{1}{\sqrt{1-\cos^2 x}} =$
 $= -\frac{1}{\sqrt{1-y^2}}$

14. [3 boda] $D_f = \mathbb{R} \setminus \{1\}$ vertikalna asimptota: $\lim_{x \rightarrow 1^+} f(x) = +\infty$,

$\lim_{x \rightarrow 1^-} f(x) = 0 \Rightarrow x = 1$ je desna vert. asimptota;

$\lim_{x \rightarrow +\infty} f(x) = +\infty$, $\lim_{x \rightarrow -\infty} \frac{f(x)}{x} = 0$, $\lim_{x \rightarrow -\infty} f(x) = 0 \Rightarrow y = 0$ je lijeva

horizontalna asimptota; $f'(x) = e^{\frac{x^2}{x-1}} \cdot \frac{x^2-2x}{(x-1)^2} = 0 \Leftrightarrow x(x-2) = 0 \Leftrightarrow$

$x_1 = 0, x_2 = 2$ stacionarne točke; $T_1(0, 1)$ maksimum, $T_2(2, e^4)$ minimum

Nedostaje slika.