Rješenja završnog ispita iz Matematike 1
1.2.2006.

1. [2 boda] $f'(x) = \frac{1}{2\sqrt{x}x}\left(1 - ln\sqrt{x}\right) = 0 \Leftrightarrow 1 - ln\sqrt{x} = 0 \Leftrightarrow x = e^2$ maks.
Na intervalu $(0, e^2)$ funkcija f raste, a na intervalu $(e^2, +\infty)$ funkcija f pada.

2. [2 boda] $P_1 = xy, \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y^2 = b^2\left(1 - \frac{x^2}{a^2}\right) \Rightarrow$
$P_1^2(x) = x^2y^2 = x^2b^2\left(1 - \frac{x^2}{a^2}\right) = -\frac{b^2}{a^2}x^4 + b^2x^2$
$\frac{dP_1^2}{dx} = 2b^2x - 4\frac{b^2}{a^2}x^3 = x(2b^2 - 4\frac{b^2}{a^2}x^2) = 0 \Leftrightarrow x^2 = \frac{a^2}{2} \Rightarrow x = +/- \frac{a}{\sqrt{2}},$
$y = +/- \frac{b}{\sqrt{2}} \Rightarrow P_1 = \frac{ab}{2} \Rightarrow P = 4 \cdot P_1 = 2ab$

3. [2 boda] $y'(x) = e^{-x}(1 - x), \quad y''(x) = e^{-x}(x - 2) = 0 \Leftrightarrow x = 2 \Rightarrow T(2, \frac{2}{e^2})$
$n....e^2y - e^4x - 2 = 2e^4 = 0$

4. [3 boda] $\mathcal{D}= \mathbb{R} \setminus \{-1\}$
$\lim_{x\to-1-} \frac{e^x}{1+x} = -\infty, \ \lim_{x\to-1+} \frac{e^x}{1+x} = +\infty$
$\lim_{x\to+\infty} \frac{e^x}{x^2+x} = L'H = +\infty$
$\lim_{x\to-\infty} \frac{e^x}{x^2+x} = L'H = 0 \Rightarrow y = 0$ je lijeva horizontalna asimptota
$f'(x) = 0 \Leftrightarrow \frac{e^x(1+x) - e^x}{(1+x)^2} = 0 \Leftrightarrow x = 0 \ T(0,1)$ minimum
Nedostaje slika!

5. [2 boda] $\Phi(x) = \int_a^x f(t)dt \Rightarrow \lim_{h\to0} \frac{\Phi(x+h) - \Phi(x)}{h} = \lim_{h\to0} \frac{\int_x^{x+h} f(t)dt}{h} =$
$= \lim_{h\to0} \frac{(x+h-x)f(\eta)}{h} = f(x)$, za $\eta \in (x, x+h)$

6. [3 boda]
Parcijalni razlomci: $\frac{1}{x^3+1} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \Rightarrow A = \frac{1}{3}, \ B = -\frac{1}{3}, \ C = \frac{2}{3}$
$\int \frac{dx}{x^3+1} = \frac{1}{3}\int \frac{dx}{x+1} - \frac{1}{3}\int \frac{x-2}{x^2-x+1}dx = \frac{1}{3}ln|x+1| - \frac{1}{3}\int \frac{x-2}{(x-\frac{1}{2})+\frac{3}{4}}dx =$
$= \frac{1}{3}ln|x+1| - \frac{1}{3}\int \frac{t-\frac{3}{2}}{t^2+\frac{3}{4}}dt = \frac{1}{3}ln|x+1| - \frac{1}{6}ln|x^2-x+1| - \frac{1}{\sqrt{3}}arctg\left(\frac{2(x-1)}{\sqrt{3}}\right) + C$

7. [2 boda] Supstitucija: $|1 + shx = t, chxdx = dt| \Rightarrow$
$\int t^9 chx\frac{dt}{chx} = \frac{t^{10}}{10} + C = \frac{(1+shx)^{10}}{10} + C$

8. [2 boda] Supstitucija: $|x = 2sint, dx = 2costdt| \Rightarrow$
$\int_0^{\sqrt{3}} \sqrt{4-x^2}dx = 2\int_0^{\frac{\pi}{3}} |cost| \cdot 2costdt = 4\int_0^{\frac{\pi}{3}} cos^2tdt = 2\int_0^{\frac{\pi}{3}} (1+cos2t)dt =$
$= |2t = k, 2dt = dk| = \frac{2\pi}{3} + sink|_0^{\frac{2\pi}{3}} = \frac{2\pi}{3} + \frac{\sqrt{3}}{2}$

9. [2 boda] $P = \int_{-1}^{1} (3 - x^2 - 2x^2)dx = 6 - 2 = 4$

10. [3 boda]
(a) $z = a + ib,\ |z| := \sqrt{a^2 + b^2}$
(b) $z_1 = a + ib,\ z_2 = c + id \Rightarrow \sqrt{(ad + bc)^2 + (ac - bd)^2} = \sqrt{(a^2 + b^2) \cdot (c^2 + d^2)} \Rightarrow$
$\Rightarrow |z_1 \cdot z_2| = |z_1| \cdot |z_2|$
(c) Nedostaje slika!

11. [3 boda] (a) $x = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ (b) $r = 3$

12. [3 boda] (a) $f'(x_0) := \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$
(b) $\left(\frac{1}{x^2}\right)' = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = -\lim_{h \to 0} \frac{h \cdot (2x + h)}{h \cdot x^2 (x + h)^2} = -\frac{2}{x^3}$
(c) $f'(1) = -2 \quad t.....y - 1 = -2(x - 1)$

13. [3 boda] (a) Desna kosa asimptota funkcije f je pravac $y = kx + l$ za koji vrijedi $\lim_{x \to \infty} (f(x) - kx - l) = 0$
(b) Ako postoji limes $\lim_{x \to \infty} (f(x) - kx - l)$, onda je $\lim_{x \to \infty} \frac{f(x) - kx - l}{x} =$
$= \lim_{x \to \infty} \frac{f(x)}{x} - k - 0 = 0 \Rightarrow k = \lim_{x \to \infty} \frac{f(x)}{x}$
(c) $k = \lim_{x \to \infty} \frac{2x^2 + 1}{x^2 - x} = 2, \quad l = \lim_{x \to \infty} \left(\frac{2x^2 + 1}{x - 1} - 2x\right) = \lim_{x \to \infty} \frac{2x + 1}{x - 1} = 2 \Rightarrow$
$y = 2x + 2$ desna kosa asimptota

14. [3 boda] (a) $u = f(x), v = g(x) \Rightarrow (uv)' = u'v + uv' \Rightarrow$ nakon int. $\Rightarrow uv = \int u'v + \int uv'$
(b) $\int lnxdx = |u = lnx, du = \frac{dx}{x}| = xlnx - x + C$
(c) $P = \int_1^{e^2} lnxdx = xlnx|_1^{e^2} - x|_1^{e^2} = e^2 + 1$