

Rješenja završnog ispita iz Matematike 1

22.1.2010.

1. [3 boda] $O = 4x^2 + 6xh = 10 \Rightarrow h = \frac{5}{3x} - \frac{2x}{3}, V = 2x^2h \Rightarrow$
 $V = 2x^2(\frac{5}{3x} - \frac{2x}{3}) = \frac{10x}{3} - \frac{4x^3}{3} \Rightarrow V' = \frac{10}{3} - 4x^2 = 0 \Rightarrow x^2 = \frac{5}{6} \Rightarrow$
 $x = \sqrt{\frac{5}{6}} \Rightarrow V = \frac{5}{3}(\frac{5}{3}\sqrt{\frac{6}{5}} - \frac{2}{3}\sqrt{\frac{5}{6}})$

2. [4 boda] $\lim_{x \rightarrow 2^+} x e^{\frac{1}{x-2}} = +\infty, \lim_{x \rightarrow 2^-} x e^{\frac{1}{x-2}} = 0, x = 2$ v.a.,
 $k = 1, l = \lim_{x \rightarrow \pm\infty} x(e^{\frac{1}{x-2}} - 1) = 1,$
 $y = x + 1$ k.a., $f'(x) = e^{\frac{1}{x-2}} \frac{x^2 - 5x + 4}{(x-2)^2} \Rightarrow x_1 = 1, x_2 = 4$ lokalni max, min.

3. [2 boda] a) $F(x) = \int x \cos x dx = x \sin x - \int \sin x dx = x \sin x +$
 $\cos x + C,$ b) $F(0) = 0 \Rightarrow C = -1 \Rightarrow F(x) = x \sin x + \cos x - 1$

4. [2 boda] $I_1 + I_2 = \int \frac{\arctg x}{x^2+1} dx + \int \frac{x}{x^2+1} dx \Rightarrow$
 $I_1 = |\text{supst.}: \arctg x = t| = \frac{\arctg^2 x}{2} + C_1, I_2 = |\text{supst.}: x^2 + 1 = t| =$
 $= \frac{1}{2} \ln(x^2 + 1) + C_2,$ ukupno rješenje: $I = I_1 + I_2 + C$

5. [3 boda] parcijalni razlomci $I = \int \frac{A}{x-1} dx + \int \frac{Bx+C}{x^2+4} dx \Rightarrow$
 $A = -\frac{6}{5}, B = -\frac{6}{5}, C = \frac{4}{5} \Rightarrow I = \frac{6}{5} \ln|x-1| - \frac{3}{5} \ln(x^2+4) + \frac{2}{5} \arctg(\frac{x}{2}) + C$

6. [2 boda] $F(x) = \int \frac{dx}{\sqrt{1+2x-x^2}} = \int \frac{dx}{\sqrt{2-(x-1)^2}} = |\text{supst.}: x-1 = t| =$
 $= \arcsin \frac{x-1}{\sqrt{2}} + C$

7. [2 boda] $|\cos x \cdot 3^{-x^2}| = |\cos x| \cdot 3^{-x^2} \leq 3^{-x^2} \Rightarrow$ dokazujemo konver-
genciju integrala \Rightarrow za $x \geq 1$ je $3^{-x^2} \leq 3^{-x} \Rightarrow \int_1^{+\infty} 3^{-x^2} dx \leq \int_1^{+\infty} 3^{-x} dx =$
 $= \frac{3^{-x}}{\ln 3} \Big|_1^{+\infty} = \lim_{B \rightarrow \infty} (\frac{3^{-B}}{\ln 3} + \frac{3^{-1}}{\ln 3}) = \frac{1}{3 \ln 3} < +\infty;$ int. kvg.

8. [2 boda] $P = 2 \int_0^{\frac{1}{\sqrt{a}}} (1 - ax^2) dx = 2(x - a\frac{x^3}{3}) \Big|_0^{\frac{1}{\sqrt{a}}} = 1 \Rightarrow a = \frac{16}{9}$

9. [2 boda] $z^6 = \text{cis}\pi \Rightarrow z_{0,\dots,5} = \text{cis}(\frac{\pi}{6} + \frac{k\pi}{3}), k = 0, 1, 2, 3, 4, 5$

10. [3 boda] a) $A\vec{v} = \lambda\vec{v}, \vec{v} \neq \vec{0} \Rightarrow (A - \lambda I)\vec{v} = \vec{0}, \vec{v} \neq \vec{0} \Rightarrow$
 $A - \lambda I$ nije regularna matrica, jer homogeni sustav ima netrivialno rješenje

$$\Rightarrow \det(\mathbf{A} - \lambda \mathbf{I}) = 0$$

$$\text{b) } \lambda_1 = 0, \lambda_2 = -2, \lambda_3 = 16, (\mathbf{A} - 16\mathbf{I})\vec{\mathbf{v}} = \vec{\mathbf{0}} \Rightarrow$$

$$\Rightarrow -9\mathbf{x}_1 + 9\mathbf{x}_3 = \mathbf{0}, -16\mathbf{x}_2 = \mathbf{0} \Rightarrow \mathbf{x}_1 = \mathbf{x}_3, \mathbf{x}_2 = \mathbf{0} \Rightarrow$$

$$\vec{\mathbf{v}}_3 = \alpha \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \alpha \in \mathbb{R} \setminus \{0\}$$

$$\mathbf{11. [2 \text{ boda}] } \lim_{x \rightarrow 0^+} \frac{x+1}{3x^2+x} \operatorname{arctg} x = \lim_{x \rightarrow 0^+} \frac{x+1}{3x+1} \frac{\operatorname{arctg} x}{x} = 1,$$

$$\lim_{x \rightarrow 0^-} a(1 + \sin x) = a \Rightarrow a = 1$$

$$\mathbf{12. [3 \text{ boda}] } \quad \text{a) } (\sin x)' = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h} = \lim_{h \rightarrow 0} \frac{2}{h} \sin \frac{h}{2} \cos(x + \frac{h}{2}) =$$

$$= \cos x \quad \text{b) } y = A \sin(\frac{\pi}{2}x), y' = A \frac{\pi}{2} \cos(\frac{\pi}{2}x) \Rightarrow y'(0) = \operatorname{tg} 30^\circ = \frac{\sqrt{3}}{3} =$$

$$= A \frac{\pi}{2} \cos 0 \Rightarrow A = \frac{2\sqrt{3}}{3\pi} \Rightarrow \text{jedn. sinusoid: } y = \frac{2\sqrt{3}}{3\pi} \sin(\frac{\pi}{2}x)$$

$$\mathbf{13. [3 \text{ boda}] } \quad P(x) = 2x(1-y) = \frac{2x}{x^2+1}, P'(x) = \frac{2-2x^2}{(x^2+1)^2} = 0 \Rightarrow x = 1 \Rightarrow$$

$$P''(1) = -\frac{1}{2} < 0 \text{ pa je } x = 1 \text{ max. i } P(1) = 1$$

$$\mathbf{14. [2 \text{ boda}] } \quad \text{parabola: } y = -4x(x-2), \text{ pravac: } y = 4x \Rightarrow$$

$$P = \int_0^1 (-4x^2 + 8x - 4x) dx = \frac{2}{3}$$