Rješenja ponovljenog završnog ispita iz Matematike 1

2.2.2010.

1. [3 boda] pravokutni trokut unutar elipse, vrhovi u točkama $(x_1, y_1), (x_1, y_2), (x_2, y_2) \Rightarrow y_1 = \sqrt{1 - \frac{x_1^2}{2}}, y_2 = -\sqrt{1 - \frac{x_1^2}{2}} \Rightarrow x_2 = -x_1 \Rightarrow P = x_1 \cdot \sqrt{1 - \frac{x_1^2}{2}} \Rightarrow P_{max} = \frac{1}{\sqrt{2}}$

2. [3 boda] $D_f = \mathbb{R} \setminus \{-1\}$, $x = -1$ v.a., $y = x - 1$ k.a., $x = -2$ je max., $x = 0$ je min.

3. [2 boda] supst.: $x = \sin t \Rightarrow I = \int_0^1 e^t(1 - t^2)dt \Rightarrow$ parc. int.: $u = 1 - t^2, dv = e^t dt \Rightarrow I = -1 + 2\int_0^1 te^t dt \Rightarrow$ parc. int.: $u = t, dv = e^t dt \Rightarrow I = 1$

4. [2 boda] supst.: $x = \text{tg}t \Rightarrow I = \frac{\pi}{18}$

5. [3 boda] supst.: $t = \ln x \Rightarrow I \frac{1}{4} \ln|\ln x| - \frac{1}{8} \ln|\ln^2 x + 4| + C$

6. [2 boda] ograda: $\frac{1}{\sqrt[3]{x^4+1}} \leq x^{-\frac{4}{3}} \Rightarrow I = \int_1^{+\infty} x^{-\frac{4}{3}} = 3$, int. kvg.

7. [3 boda] parc. int.: $u = e^{-2x}, dv = \sin x dx \Rightarrow I = \frac{1}{3} - \frac{2}{3}\int_0^{+\infty} e^{-2x}\cos(3x)dx \Rightarrow$ parc. int.: $u = e^{-2x}, dv = \cos(3x)dx \Rightarrow I = \frac{1}{3} - \frac{4}{9}I \Rightarrow I = \frac{3}{13}$

8. [2 boda] $P(x) = r^2\pi = (R^2 - x^2)\pi \Rightarrow V = \int_{-R}^{R} P(x)dx = ... = \frac{4}{3}R^3\pi$

9. [3 boda] $z_{0,1,2} = 2\text{cis}(\frac{\pi}{6} + \frac{2k\pi}{3}), k = 0, 1, 2;$ $z_{3,4,5} = \text{cis}(\frac{\pi}{2} + \frac{2k\pi}{3}), k = 0, 1, 2$

10. [2 boda] $\mathbf{X} = \mathbf{B} \cdot \mathbf{A} = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 4 & 0 \\ 1 & -5 & 14 \end{pmatrix}$

11. [2 boda] $\lim_{x\to 0} \frac{\text{tg}^2(3x)}{x \cdot \ln(1+2x)} = \lim_{x\to 0} \frac{9x^2}{2x^2} = \frac{9}{2}$

12. [3 boda] $f(x) = 2x + \frac{2}{3}x^3$, $R_3(x) = \frac{x^4}{4}(-\frac{1}{(1+x_1)^4}) + \frac{1}{(1-x_1)^4}$

13. [3 boda] $f(\frac{1}{2}) = 13$ max., $f(0) = 9\sqrt[3]{3}$, $f(5) = 13$, $f(\frac{9}{2}) = 9$min.

14. [2 boda] $I = \int_{-1}^{1}(-y^2 + 2 - y^2)dy = ... = \frac{8}{3}$