Rješenja ponovljenog 2. međuispita iz Matematike 1

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1. [2 boda] dokaz mat. ind. da je niz omeđen i monoton $\Rightarrow$ niz je konvergentan i $L = \frac{1}{2}$

2. [2 boda] a)podijelimo brojnik i nazivnik s x^2 i dobivamo: $\frac{2}{3}$, b) racionaliziramo razlomak, podijelimo brojnik i nazivnik s x i dobivamo: $\frac{2}{3}$

3. [2 boda] $\lim_{x\to 0} \frac{\mathrm{tg}^2(3x)}{x\cdot\ln(1+2x)} = \lim_{x\to 0} \frac{9x^2}{2x^2} = \frac{9}{2}$

4. [2 boda] $f'(x) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h} = \lim_{h\to 0} \frac{(x_0+h)^{\frac{1}{3}} - x_0^{\frac{1}{3}}}{h}$ = nadopunom brojnika do razlike kubova dobivamo $= \frac{1}{3} \cdot x_0^{-\frac{2}{3}}$

5. [2 boda] $(f^{-1} \circ f)(x) = x \Rightarrow (f^{-1} \circ f)'(x) = 1 \Rightarrow (f^{-1})'(f(x)) \cdot f'(x) = 1 \Rightarrow f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$

$(\mathrm{arctg}x)' = \frac{1}{\mathrm{tg}'(\mathrm{arctg}x)} = \frac{1}{\mathrm{tg}^2(\mathrm{arctg}x)+1} = \frac{1}{x^2+1}$

6. [3 boda] $f(x) = 2x + \frac{2}{3}x^3,\ R_3(x) = \frac{x^4}{4}(-\frac{1}{(1+x_1)^4}) + \frac{1}{(1-x_1)^4}$

7. [2 boda] $f'(x) = (\frac{2}{1+x^2} - \frac{\ln(1+x^2)}{x^2}) \cdot (1+x^2)^{\frac{1}{x}}$

8. [3 boda] $y' = \frac{-2xy-y^3}{x^2+3xy^2} \Rightarrow y'(1) = -\frac{3}{4}; y'' = -\frac{2y+4xy'+6y^2y'+6xyy'^2}{x^2+3xy^2} \Rightarrow y''(1) = \frac{17}{32}$

9. [2 boda] supst.: $\lim_{x\to\frac{\pi}{2}} \frac{\frac{1}{\cos^2(5x)}\cdot 5}{\frac{1}{\cos^2(3x)}\cdot 3} = L'H = \frac{5}{3} \cdot (\frac{3}{5})^2 = \frac{3}{5}$