Rješenja 1. međuispita iz Matematike 1

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1. [2 boda] a)$\neg A \equiv (\exists x_1 \in D(f))(\exists x_2 \in D(f))(x_1 < x_2 \wedge f(x_1) \geq f(x_2))$

b) za $f(x) = \sin x$ $\neg A$ je točan sud; f nije strogo rastuća funkcija;

npr. $\sin \frac{\pi}{2} > \sin \pi$.

2. [2 boda] $z^6 = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^4 = (cis\frac{\pi}{3})^4 = cis\frac{4\pi}{3};$

$z_{1,..,6} = cis(\frac{2\pi}{9} + \frac{k\pi}{3}),\ k = 0, 1, 2, 3, 4, 5$

3. [2 boda] Zrcaljenjem s obzirom na pravac $y = 1$ dobivamo krivulju $y = 2 - x^3$. Moramo naći inverznu funkciju funkcije $g(x) = 2 - x^3$, a to je funkcija $g^{-1}(x) = \sqrt[3]{2-x}$, pa je dakle tražena krivulja $y = \sqrt[3]{2-x}$.

4. [2 boda] a) $D(f) = \mathbb{R},\ Im(f) = \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$

b) $D(f) = \langle -\frac{1}{2}, \frac{1}{2} \rangle,\ Im(f) = \mathbb{R}$

5. [2 boda] a) $\mathbf{A}$ je regularna matrica, tj. postoji matrica $\mathbf{A}^{-1}$ takva da je $\mathbf{A} \cdot \mathbf{A}^{-1} = \mathbf{I}$. Po Binet-Cauchyevom teoremu je $\det \mathbf{A} \cdot \det \mathbf{A}^{-1} =$ $= \det \mathbf{I} = 1$, pa mora biti $\det \mathbf{A} \neq 0$.

b) $(\mathbf{A} \cdot \mathbf{B}) \cdot (\mathbf{B}^{-1} \cdot \mathbf{A}^{-1}) = \mathbf{A} \cdot (\mathbf{B} \cdot \mathbf{B}^{-1})\mathbf{A}^{-1} = \mathbf{A} \cdot \mathbf{A}^{-1} = \mathbf{I}$,

$(\mathbf{B}^{-1} \cdot \mathbf{A}^{-1}) \cdot (\mathbf{A} \cdot \mathbf{B}) = \mathbf{B}^{-1} \cdot (\mathbf{A}^{-1} \cdot \mathbf{A})\mathbf{B} = \mathbf{B}^{-1} \cdot \mathbf{B} = \mathbf{I}$.

6. [2 boda] a) $\mathbf{X}^{-1} = \mathbf{A}^{-1} \cdot \mathbf{B} \cdot \mathbf{A} \cdot \mathbf{B}^{-1} \Rightarrow \mathbf{X} = (\mathbf{A}^{-1} \cdot \mathbf{B} \cdot \mathbf{A} \cdot \mathbf{B}^{-1})^{-1} \Rightarrow$ $\Rightarrow \mathbf{X} = \mathbf{B} \cdot \mathbf{A}^{-1} \cdot \mathbf{B}^{-1} \cdot \mathbf{A}$

b) $\mathbf{X} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

7. [2 boda] $\Delta = -\begin{vmatrix} 8 & 8 & 8 & 8 \\ 2 & 2 & 8 & 4 \\ 4 & 4 & 8 & 4 \\ 2 & 1 & 8 & 4 \end{vmatrix} = -64 \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 4 & 2 \\ 1 & 1 & 2 & 1 \\ 2 & 1 & 8 & 4 \end{vmatrix} =$

$= -64 \cdot \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 6 & 2 \end{vmatrix}$ =po 1.stupcu= $-64 \cdot \begin{vmatrix} 0 & 3 & 1 \\ 0 & 1 & 0 \\ -1 & 6 & 2 \end{vmatrix} =$

$=$ po 1.stupcu $= -64 \cdot \begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} = -64$

8. [3 boda] 1. NAČIN:

$\overrightarrow{\mathbf{b}} = \lambda_1 \overrightarrow{\mathbf{a_1}} + \lambda_2 \overrightarrow{\mathbf{a_2}} \Rightarrow \begin{bmatrix} 7 \\ -2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 2\lambda_1 + \lambda_2 \\ 3\lambda_1 + 4\lambda_2 \\ 5\lambda_1 + 3\lambda_2 \end{bmatrix} \Rightarrow$

$\Rightarrow \lambda_1 = 6, \ \lambda_2 = -5, \ \lambda = 15$

2. NAČIN:

Vektori $\overrightarrow{\mathbf{a_1}}, \overrightarrow{\mathbf{a_2}}, \overrightarrow{\mathbf{b}}$ moraju biti zavisni, pa λ mora biti takav da rang matrice $\begin{bmatrix} 2 & 3 & 5 \\ 1 & 4 & 3 \\ 7 & -2 & \lambda \end{bmatrix}$ bude manji od 3 $\Rightarrow \begin{bmatrix} 2 & 3 & 5 \\ 1 & 4 & 3 \\ 7 & -2 & \lambda \end{bmatrix} \sim .... \sim \begin{bmatrix} 1 & 4 & 3 \\ 0 & 5 & 1 \\ 0 & 0 & \lambda - 15 \end{bmatrix} \Rightarrow$

$\Rightarrow \lambda = 15$

ili iz $\begin{vmatrix} 2 & 3 & 5 \\ 1 & 4 & 3 \\ 7 & -2 & \lambda \end{vmatrix} = 0$ dobivamo $\lambda = 15$.

9. [3 boda] a) $A\overrightarrow{\mathbf{v}} = \lambda\overrightarrow{\mathbf{v}},\ \overrightarrow{\mathbf{v}} \neq \overrightarrow{\mathbf{0}} \Rightarrow (\mathbf{A}-\lambda\mathbf{I})\overrightarrow{\mathbf{v}} = \overrightarrow{\mathbf{0}},\ \overrightarrow{\mathbf{v}} \neq \overrightarrow{\mathbf{0}} \Rightarrow \mathbf{A}-\lambda\mathbf{I}$ nije regularna matrica $\Rightarrow \det(\mathbf{A}-\lambda\mathbf{I}) = \mathbf{0}$.

b) $\lambda_1 = 2,\ \lambda_2 = 3,\ \lambda_3 = 4,\quad (\mathbf{A}-\mathbf{2I})\overrightarrow{\mathbf{v}} = \overrightarrow{\mathbf{0}} \Rightarrow$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$\Rightarrow x_1 = -x_3, x_1 = x_2, \overrightarrow{\mathbf{v}} = \alpha \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix},\ \alpha \in \mathbb{R} \setminus \{\mathbf{0}\}.$$

U rješenjima 2. i 4. zadatka nedostaju slike, nužne za ostvarenje maksimalnog broja bodova.