

Rješenja 4. kpz - grupe 2, 4, 6, 8, 10 - B

1. ZADATAK

"Gornja" funkcija je $y = 6 - x^2$, a "donja" funkcija je $y = |x| + 1$. Dva vrha pravokutnika se nalaze na grafu gornje funkcije, a druga dva na grafu donje funkcije. Ti vrhovi su redom $A(x, 6 - x^2)$, $B(x, 6 - x^2)$, $C(x, x + 1)$,

$D(-x, x + 1)$; $x > 0$.

Površina pravokutnika je

$$P = (x_B - x_A)(y_B - y_C) = 2x(6 - x^2 - x - 1) = 10x - 2x^2 - 2x^3$$

$$P' = 10 - 4x - 6x^2 = 0$$

$$3x^2 + 2x - 5 = 0$$

$$x_1 = 1, x_2 = -\frac{5}{3} < 0$$

$$P'' = -4 - 12x, P'(\frac{1}{2}) = -16 < 0$$

$$P_{max} = 2(5 - 1 - 1) = 6$$

2. ZADATAK

$$\int \frac{x}{\sqrt{x^4+x^2+1}} dx = (\text{supstitucija } t = x^2, dt = 2x dx) = \frac{1}{2} \int \frac{dt}{\sqrt{t^2+t+1}} = \frac{1}{2} \int \frac{dt}{\sqrt{(t+\frac{1}{2})^2+\frac{3}{4}}} =$$
$$\frac{1}{2} \frac{2}{\sqrt{3}} \int \frac{dt}{\sqrt{(\frac{2}{\sqrt{3}}(t+\frac{1}{2}))^2+1}} = \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{2} Arsh(\frac{2}{\sqrt{3}}(t+\frac{1}{2})) + C = \frac{1}{2} Arsh(\frac{2x^2+1}{\sqrt{3}}) + C$$

3. ZADATAK

$$\int_0^1 \frac{x^2+1}{e^x} dx = \int_0^1 (x^2+1)e^{-x} dx =$$

$$(u = x^2 + 1, \quad dv = e^{-x})$$

$$du = 2x dx, \quad v = -e^{-x}$$

$$= -((x^2+1)e^{-x})|_0^1 + 2 \int_0^1 x e^{-x} dx =$$

$$(u = x, \quad dv = e^{-x})$$

$$du = dx, \quad v = -e^{-x}$$

$$= \frac{2}{e} + 1 + 2(-x e^{-x})|_0^1 + 2 \int_0^1 e^{-x} dx = 1 - \frac{4}{e} - 2e^{-x}|_0^1 = 3 - \frac{6}{e}$$