**Rješenja zadataka za dodatnu vježbu (gradivo 8.knjižice):**

1. $f(x) = \sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x^3}$.
   $f'(x) = (x^{\frac{1}{2}})' + (x^{\frac{1}{3}})' + (x^{\frac{3}{4}})' = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{3}x^{-\frac{2}{3}} + \frac{3}{4}x^{-\frac{1}{4}} =$
   $= \frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}} + \frac{3}{4\sqrt[4]{x}}$.

2. $f(x) = x^2 \cdot \mathrm{e}^x$.
   $f'(x) = (x^2)' \cdot \mathrm{e}^x + x^2 \cdot (\mathrm{e}^x)' = 2x \cdot \mathrm{e}^x + x^2 \cdot \mathrm{e}^x$.

3. $f(x) = \frac{\ln x}{x}$.
   $f'(x) = \frac{(\ln x)' \cdot x - \ln x \cdot x'}{x^2} = \frac{1 - \ln x}{x^2}$.

4. $f(x) = x \cdot \mathrm{e}^x \cdot \sin x$.
   $f'(x) = x' \cdot \mathrm{e}^x \cdot \sin x + x \cdot (\mathrm{e}^x)' \cdot \sin x + x \cdot \mathrm{e}^x \cdot (\sin x)'$
   $= \mathrm{e}^x \cdot \sin x + x \cdot \mathrm{e}^x \cdot \sin x + x \cdot \mathrm{e}^x \cdot \cos x$.

5. $f(x) = \frac{x \cdot \cos x}{x^2+1}$.
   $f'(x) = \frac{(x \cdot \cos x)'(x^2+1) - x \cdot \cos x (x^2+1)'}{(x^2+1)^2}$
   $= \frac{(x' \cdot \cos x + x \cdot (\cos x)')(x^2+1) - x \cdot \cos x \cdot 2x}{(x^2+1)^2} = \frac{(\cos x - x \cdot \sin x)(x^2+1) - 2x^2 \cdot \cos x}{(x^2+1)^2}$.

6. $f(x) = (x^2+1)^{10}$.
   $f'(x) = 10(x^2+1)^9 \cdot (x^2+1)' = 10(x^2+1)^9 \cdot 2x$.

7. $f(x) = \mathrm{th}(4x)$.
   $f'(x) = \frac{1}{ch^2(4x)}(4x)' = \frac{4}{ch^2(4x)}$.

8. $f(x) = \arcsin(\frac{1}{x})$.
   $f'(x) = \frac{1}{\sqrt{1-(\frac{1}{x})^2}} \cdot (\frac{1}{x})' = \frac{1}{\sqrt{1-(\frac{1}{x})^2}} \cdot (-\frac{1}{x^2})$.

9. $f(x) = \mathrm{cth}\frac{x}{2x+1}$.
   $f'(x) = -\frac{1}{sh^2\frac{x}{2x+1}} \cdot \left(\frac{x}{2x+1}\right)' = -\frac{1}{sh^2\frac{x}{2x+1}} \cdot \frac{2x+1-2x}{(2x+1)^2}$
   $= -\frac{1}{sh^2\frac{x}{2x+1}} \cdot \frac{1}{(2x+1)^2}$

10. $f(x) = \mathrm{tg}^3(2x)$.
    $f'(x) = 3\mathrm{tg}^2(2x) \cdot (\mathrm{tg}(2x))' = 3\mathrm{tg}^2(2x) \cdot \frac{1}{\cos^2(2x)} \cdot (2x)' = 6\mathrm{tg}^2(2x) \cdot \frac{1}{\cos^2(2x)}$.

11. $f(x) = e^{-\sin(2x)}$.
    $f'(x) = e^{-\sin(2x)} \cdot (-\sin(2x))' = e^{-\sin(2x)} \cdot (-\cos(2x)) \cdot (2x)' =$
    $= 2e^{-\sin(2x)} \cdot (-\cos(2x))$.

12. $f(x) = x^2 \cdot 2^{-x}$.
    $f'(x) = (x^2)' \cdot 2^{-x} + x^2 \cdot (2^{-x})' = 2x \cdot 2^{-x} + x^2 \cdot 2^{-x} \ln 2 \cdot (-x)' =$
    $= 2x \cdot 2^{-x} - x^2 \cdot 2^{-x} \ln 2$.

13. $f(x) = \sqrt{x} \cdot \mathrm{arctg}\frac{1}{x}$.
    $f'(x) = (\sqrt{x})' \cdot \mathrm{arctg}\frac{1}{x} + \sqrt{x} \cdot (\mathrm{arctg}\frac{1}{x})' = \frac{1}{2\sqrt{x}} \cdot \mathrm{arctg}\frac{1}{x} + \sqrt{x} \cdot \frac{1}{1+\frac{1}{x^2}} \cdot (\frac{1}{x})' =$
    $= \frac{1}{2\sqrt{x}} \cdot \mathrm{arctg}\frac{1}{x} + \sqrt{x} \cdot \frac{1}{1+\frac{1}{x^2}} \cdot (-\frac{1}{x^2}) = \frac{1}{2\sqrt{x}} \cdot \mathrm{arctg}\frac{1}{x} - \frac{\sqrt{x}}{x^2+1}$.

14. $f(x) = \frac{x}{\sqrt{x^2+1}}$.

$f'(x) = \frac{x' \cdot \sqrt{x^2+1} - x \cdot (\sqrt{x^2+1})'}{x^2+1} = \frac{\sqrt{x^2+1} - x \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x}{x^2+1} =$

$= \frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{x^2+1} = \frac{1}{\sqrt{(x^2+1)^3}}$.

15. $f(x) = \frac{\mathrm{ctg}(3x)}{x}$.

$f'(x) = \frac{(\mathrm{ctg}(3x))' x - \mathrm{ctg}(3x) \cdot x'}{x^2} = \frac{-\frac{1}{\sin^2(3x)} \cdot (3x)' \cdot x - \mathrm{ctg}(3x)}{x^2}$

$= \frac{-\frac{3x}{\sin^2(3x)} - \mathrm{ctg}(3x)}{x^2}$.

16. $f(x) = x^3 \cdot \sin^2(4x)$.

$f'(x) = (x^3)' \cdot \sin^2(4x) + x^3 \cdot (\sin^2(4x))' =$

$= 3x^2 \cdot \sin^2(4x) + x^3 \cdot 2\sin(4x) \cdot (\sin(4x))' =$

$= 3x^2 \cdot \sin^2(4x) + x^3 \cdot 2\sin(4x) \cdot \cos(4x) \cdot (4x)' =$

$= 3x^2 \cdot \sin^2(4x) + 8x^3 \cdot \sin(4x) \cdot \cos(4x) =$

$= 3x^2 \cdot \sin^2(4x) + 4x^3 \cdot \sin(8x)$.

17. $f(x) = x \cdot \ln(x \cdot \mathrm{ch}x)$.

$f'(x) = x' \cdot \ln(x \cdot \mathrm{ch}x) + x \cdot (\ln(x \cdot \mathrm{ch}x))' =$

$= \ln(x \cdot \mathrm{ch}x) + x \cdot \frac{1}{x \cdot \mathrm{ch}x}(x \cdot \mathrm{ch}x)' =$

$= \ln(x \cdot \mathrm{ch}x) + x \cdot \frac{1}{x \cdot \mathrm{ch}x}(x' \cdot \mathrm{ch}x + x \cdot (\mathrm{ch}x)') =$

$= \ln(x \cdot \mathrm{ch}x) + \frac{1}{\mathrm{ch}x}(\mathrm{ch}x + x \cdot \mathrm{sh}x)$.

18. $f(x) = x^3 \cdot \mathrm{arsh}(e^{-2x})$.

$f'(x) = (x^3)' \cdot \mathrm{arsh}(e^{-2x}) + x^3 \cdot (\mathrm{arsh}(e^{-2x}))' =$

$= 3x^2 \cdot \mathrm{arsh}(e^{-2x}) + x^3 \cdot \frac{1}{\sqrt{1+(e^{-2x})^2}} \cdot (e^{-2x})' =$

$= 3x^2 \cdot \mathrm{arsh}(e^{-2x}) + x^3 \cdot \frac{1}{\sqrt{1+(e^{-2x})^2}} \cdot e^{-2x} \cdot (-2x)' =$

$= 3x^2 \cdot \mathrm{arsh}(e^{-2x}) - \frac{2x^4 \cdot e^{-2x}}{\sqrt{1+(e^{-2x})^2}}$.

19. $f(x) = e^{-x} \cdot \cos^3(2x)$.

$f'(x) = (e^{-x})' \cdot \cos^3(2x) + e^{-x} \cdot (\cos^3(2x))' =$

$= -e^{-x} \cdot \cos^3(2x) + e^{-x} \cdot 3\cos^2(2x)(-\sin(2x)) \cdot 2 =$

$= -e^{-x} \cdot \cos^3(2x) - 6e^{-x} \cdot \cos^2(2x) \cdot \sin(2x)$.

20. $f(x) = \frac{x \cdot e^{-x}}{\sqrt{x^2+x+1}}$.

$f'(x) = \frac{(e^{-x} - x \cdot e^{-x})\sqrt{x^2+x+1} - x \cdot e^{-x} \frac{2x+1}{2\sqrt{x^2+x+1}}}{x^2+x+1}$.

21. $f(x) = (\sqrt{x})^x$.

$f'(x) = (\mathrm{e}^{x \ln \sqrt{x}})' = \mathrm{e}^{x \ln \sqrt{x}} \cdot (\ln \sqrt{x} + x \frac{1}{\sqrt{x}} \frac{1}{2\sqrt{x}})$

$= (\sqrt{x})^x \cdot (\ln \sqrt{x} + \frac{1}{2})$.

22. $f(x)=x^x+(\frac{2x+1}{2x-1})^x$.

$f'(x)=(\mathrm{e}^{x\ln x})'+(\mathrm{e}^{x\ln\frac{2x+1}{2x-1}})'=\mathrm{e}^{x\ln x}(x\ln x)'+\mathrm{e}^{x\ln\frac{2x+1}{2x-1}}(x\ln\frac{2x+1}{2x-1})'=$

$=x^x(\ln x+x\cdot\frac{1}{x})+(\frac{2x+1}{2x-1})^x(\ln\frac{2x+1}{2x-1}+x\frac{2x-1}{2x+1}\frac{2(2x-1)-2(2x+1)}{(2x-1)^2})=$

$=x^x(\ln x+1)+(\frac{2x+1}{2x-1})^x(\ln\frac{2x+1}{2x-1}-\frac{4x}{4x^2-1})$.

23. $f(x)=arth\frac{1}{x}$.

$f'(x)=\frac{1}{1-\frac{1}{x^2}}(-\frac{1}{x^2})=\frac{1}{x^2-1}$.

$f''(x)=-\frac{2x}{(x^2-1)^2}$.

24. $f(x)=\cos(\mathrm{arctg}x)$.

$f'(x)=-\frac{\sin(arctgx)}{1+x^2}$.

$f''(x)=-\frac{\frac{\cos(arctgx)}{1+x^2}(1+x^2)-2x\sin(arctgx)}{(1+x^2)^2}=-\frac{\cos(arctgx)-2x\sin(arctgx)}{(1+x^2)^2}$.

25. $f(x)=\ln(ax+b)$.

$f^{(n)}(x)=\frac{(-1)^{n+1}\cdot a^n\cdot(n-1)!}{(ax+b)^n}, n\geq 2$. (dokazati indukcijom)

26. $x^4y+xy^4=2$.

$4x^3y+x^4y'+y^4+x\cdot 4y^3y'=0$.

$y'=-\frac{4x^3y+y^4}{x^4+4xy^3}$.

27. $arctg\left(\frac{x}{y}\right)=\ln(\sqrt{x^2+y^2})$.

$y'=\frac{x-y}{x+y}$.

$y''=\frac{(1-y')(x+y)-(x-y)(1+y')}{(x+y)^2}=2\frac{y^2+2xy-x^2}{(x+y)^3}$.

28. $x^5+y^5=2xy$.

$y'|_T=-1$.

29. $x=t-\sin t,\quad y=1-\cos t$.

$y'=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\sin t}{1-\cos t}$.

$y''=\frac{\frac{d(y')}{dt}}{\frac{dx}{dt}}=\frac{\frac{\sin t\sin t-\cos t(1-\cos t)}{(1-\cos t)^2}}{1-\cos t}=\frac{1}{(1-\cos t)^2}$.

$y'''=\frac{\frac{d(y'')}{dt}}{\frac{dx}{dt}}=\frac{\frac{-2\sin t}{(1-\cos t)^3}}{1-\cos t}=\frac{-2\sin t}{(1-\cos t)^4}$.

30. $f'(x)=\frac{2}{\cos^2(2x)}$, $f'(0)=2$.

Jednadžba tangente je $y=2x$.

31. Točka sjecišta s osi $x$ ima apscisu $x=\frac{1}{3}$.

$f'(x)=\frac{3}{3x}=\frac{1}{x}$, $f'(\frac{1}{3}=3$.

Jednadžba tangente je $y=3(x-\frac{1}{3})$, tj. $y=3x-1$.

32. $14x-13y+12=0$.

33. $y-\frac{1}{8}=-\sqrt{3}(x-\frac{3\sqrt{3}}{8})$.

34. $13x+14y-41=0$.

35. $40^o36'$.

36. $y - \frac{\sqrt{19}}{2} = \frac{1}{2}(x - \frac{\sqrt{3}}{2})$.

37. $y = \frac{\sqrt{3}}{\pi} \sin(\pi x)$.

38. $y = \frac{3}{4}x^2 - 1$.

39. $a = \frac{27}{256}$.

40. Apscisa dirališne točke tangente je 2.