Rješenja PONOVLJENOG završnog ispita iz Matematike 1
8.2.2006.

1. [2 boda] $f''(x) = e^{-x^2}(4x^3 - 6x) = 0 \Leftrightarrow x_1 = 0, x_{2,3} = +/- \frac{\sqrt{6}}{2}$
Na $(-\infty, -\frac{\sqrt{6}}{2}) \cup (0, \frac{\sqrt{6}}{2})$ funkcija f je konveksna, a na intervalu $(-\frac{\sqrt{6}}{2}, 0) \cup (\frac{\sqrt{6}}{2}, +\infty)$ funkcija f je konkavna.

2. [2 boda] $P_1 = xy,\ x \in (0,2)$ $\ P_1(x) = x(4-x^2) \Rightarrow P_1(x)' = 4 - 3x^2 = 0 \Leftrightarrow$
$x_{1,2} = +/- \frac{2\sqrt{3}}{3} \Rightarrow x = \frac{2\sqrt{3}}{3} \Rightarrow P = 2 \cdot P_1 = \frac{32\sqrt{3}}{9}$

3. [2 boda] $\lim\limits_{x\to 0+} = \lim\limits_{x \to 0-} = -\infty \Rightarrow x = 0$ je vertikalna asimptota
$k = \lim\limits_{x\to +/-\infty} \frac{f(x)}{x} = \frac{1}{2},\ l = \lim\limits_{x\to +/-\infty}(f(x) - \frac{1}{2}x) = 1 \Rightarrow\ y = \frac{1}{2}x + 1$ je kosa asimptota

4. [2 boda] Neka je f neprekinuta na $I = [a,b]$. Onda postoji $\xi \in \langle a, b\rangle$ takav da je $\int_a^b f(x)dx = (b-a)f(\xi)$.
Geometrijska interpretacija - ploština "krivocrtnog trapeza". Nedostaje slika!

5. [3 boda] $\int \frac{dx}{x^3 + 2x^2 + 2x} = \left(parcijalni\ razlomci\ A = \frac{1}{2},\ B = -\frac{1}{2},\ C = -1 \right) =$
$= \frac{1}{2}\int \frac{dx}{x} - \frac{1}{2}\int \frac{x+2}{(x+1)^2+1}dx = \frac{1}{2}ln|x| - \frac{1}{4}ln((x+1)^2+1) - \frac{1}{2}arctg(x+1) + C$

6. [2 boda] $\int x^2 sinx dx = |u = x^2, du = 2xdx, dv = sinxdx, v = -cosx| =$
$= -x^2cosx + 2\int xcosxdx = |u = x, du = dx, dv = cosxdx, v = sinx| =$
$= -x^2cosx + 2xsinx + 2cosx + C$

7. [3 boda] $\int_1^3 \sqrt{3 + 2x - x^2}dx = \int_1^3 \sqrt{4 - (x-1)^2}dx = |x - 1 = t, dx = dt| =$
$= \int_0^2 \sqrt{4 - t^2}dt = |t = 2sink, dt = 2coskdk| = \pi$

8. [2 boda] $P = \int_0^\infty (chx - shx)dx = ... = \int_0^\infty e^{-x}dx = 1$

9. [2 boda] $x = Rcost,\ y = Rsint,\ t \in [0, 2\pi] \Rightarrow s = \int_0^{2\pi} \sqrt{R^2sin^2t + R^2cos^2t}dt = 2R\pi$

10. [3 boda]
(a) $\sqrt[n]{z} = \sqrt[n]{rcis\varphi} = \sqrt[n]{r}cis(\frac{\varphi + 2k\pi}{n}),\ k = 0, 1, ..., n-1$
(b) $\sqrt[3]{z} = \sqrt[3]{rcis\varphi} = \sqrt[3]{r}cis(\frac{\varphi + 2k\pi}{3}),\ k = 0, 1, 2$ Nedostaje slika!
(c) $z_{1,2,3} = \sqrt[6]{2}cis(\frac{3\pi}{12} + \frac{2k\pi}{3}),\ k = 0, 1, 2$

11. [3 boda] $\alpha = 0$ (a) $\lambda = 2$ (b) $\begin{bmatrix} 0 \\ t \end{bmatrix}$

12. [3 boda] (a) $f'(x_0) := \lim\limits_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$

(b) $(fg)'(x) = \lim_{h\to 0}(\dfrac{f(x+h)-f(x)}{h}\cdot g(x+h)) + \lim_{h\to 0}(\dfrac{g(x+h)-g(x)}{h}\cdot f(x)) =$
$= f'(x)g(x) + f(x)g'(x)$
(c) $f'(x) = \dfrac{xarctgx}{\sqrt{x^2+1}} + \dfrac{\sqrt{x^2+1}}{x^2+1}$

13. [3 boda] (a) $\lim_n a_n = L \Leftrightarrow (\forall \varepsilon > 0)(\exists n_0 \in \mathbb{N})(\forall n \geq n_0)(|a_n - L| < \varepsilon)$
(b) $\forall \varepsilon > 0\ \exists n_0 \in \mathbb{N}$ takav da je $n_0 > \sqrt{\frac{1}{\varepsilon}}$. Tada $\forall n \geq n_0$ vrijedi $n > \sqrt{\frac{1}{\varepsilon}}$, tj. $\frac{1}{n^2} < \varepsilon$.
(c) $-\frac{1}{2}$

14. [3 boda] $\mathcal{D} = (0, +\infty)$ $\lim_{x\to 0+} f(x) = -\infty \Rightarrow x = 0$ je vertikalna asimptota
$k = \lim_{x\to\infty} \dfrac{f(x)}{x} = 0,\ l = \lim_{x\to\infty} f(x) = 0 \Rightarrow y = 0$ je horizontalna asimptota
$f'(x) = \frac{1-lnx}{x^2} = 0 \Leftrightarrow x = e \Rightarrow T(e, \frac{1}{e})$ je maksimum
Nedostaje slika!