Hierarchical wind farm control for power/load optimization

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Abstract—The paper describes the hierarchical control concept for wind farm power/load optimization with emphasis on compensation of disturbances. The control concept was developed for the EU-FP7 project Distributed Control of Large-Scale Offshore Wind Farms – Aeolus. The Aeolus project approaches the wind farm control problem by considering wind turbines in a wind farm as individual power actuators affected by constraints. It is assumed that the demanded wind farm power production is given in advance, i.e., the power reference value for the whole farm is provided by the Transmission System Operator (TSO). The supervisory farm controller then distributes the power demand among wind turbines based on the dynamic model of wind field inside the wind farm. The objective of the distribution is to minimize the cumulative load of all wind turbines in the wind farm, while respecting the system constraints and fulfilling the TSO demands. The presented control concept consists of two levels: i) the high level that acts on a slower time scale and computes the optimal distribution of production and loads, and ii) the low level of control that acts on the faster time scale and ensures the tracking of the optimal distribution under disturbances. In this paper the focus is on the control design for the low level of control. To minimize the computational effort the control design procedure replaces the online optimization with the off-line optimization approach based on the principles of multi-parametric programming.

Keywords: wind farm control, Model Predictive Control, multi-parametric programming

I. INTRODUCTION – WIND FARM CONTROL PROBLEM

The large penetration of wind energy in power systems in recent years has led to the requirement that large wind turbines and particularly wind farms contribute to regulation and stabilisation of the power system, see e.g. [1]. In order to accomplish most of the required tasks the regulation of wind farm power production is required [1], [2]. In this paper the wind farm control concept is described that dynamically distributes the active power demand provided by the TSO between the wind turbines in the wind farm in such a manner that the overall loading and wear of the wind turbines in the wind farm are minimized.

The optimization of wind farm behavior is accomplished by employment of the wind field model that describes the dynamic development of wakes inside a wind farm [3], [4]. From a control perspective, the wakes impose coupling between individual wind turbines – a wind turbine standing in a wake created by an upwind turbine experiences reduced wind speed and increased turbulence, which influence its loading and power production potential. The motivation for proposed wind farm optimization lies in the fact that wake effects generated by a wind turbine can be influenced by changing its power demand (a wind farm control variable). Combination of the wind field model and models of wind turbines yields an overall dynamic wind farm model suitable for optimization. Since this system involves many constraints, which are related mostly to wind turbine operation, and the goal is the optimization of the production, a natural choice for control design framework is the Model Predictive Control (MPC). The wind field model enables the prediction of propagation of large disturbances and thus dynamical optimization of wind farm behavior.

A centralized approach to the optimization of large wind farm operation is an extremely complex control problem. Namely, the system in scope is best described as a coupled, constrained multi-input multiple-output model whose order grows very fast with the number of wind turbines in the wind farm. The wind turbine and especially the wind field are highly nonlinear systems. Furthermore, the system is subjected to large number of disturbances due to random nature of wind, and/or possible wind turbine malfunctions that may prevent or restrict its operation. Finally, wind farm model inherently comprises processes acting on very different time scales: the mechanical part of a typical megawatt scale wind turbine with local speed and power controller has dominant dynamics in the time scale of 1 second, while the typical propagation time of wind between two rows of wind turbines can be significantly longer than 10 seconds.

One of the primary demands on the wind farm controller set in the Aeolus project is the scalability of the control algorithms to arbitrarily large wind farms. The approach reported in this paper altogether is aimed for application in large wind farms in which the wind turbines are laid out in multiple rows. In such wind farms a large number of wind turbines are affected by wakes all the time, which should be taken into account by wind farm control system.

For such wind farms computation of a centralized model
predictive controller might be computationally intractable for practical, real-time applications. As an illustrative example consider the wind farm that comprises 16 rows of wind turbines separated by 200 meters. For wind speed of 15 m/s the prediction horizon should ideally be around 200 seconds in order to capture the wind propagation through entire farm and thereby compute the globally optimal control. With 1 second sampling time that would mean the computation of optimization problem with hundreds of optimization variables with prediction horizon of 200 samples in less than 1 second. To circumvent the need to solve such a complex optimization problems in this paper the hierarchical supervisory control concept adapted to the application at hand is proposed.

II. HIERARCHICAL APPROACH TO CENTRALIZED WIND FARM CONTROL

The wind farm dynamics are effectively decoupled through different time scales, [3]. A two level hierarchical control concept is proposed that utilizes this fact as a separation principle.

The high level of control – the nominal supervisory controller – considers the propagation of mean wind stream through the wind farm and therefore accounts for the coupling that emerges due to wakes. This level of control derives the overall optimal wind farm operating point – a production and a measure of loading is attributed to each wind turbine in the wind farm. The sampling time of this level can be adjusted to the size of wind farm and to separations of wind turbines inside a wind farm. If the separations between wind turbines are large the sampling time can be increased. The expected sampling time for supervisory level of wind farm controller is around 10 s [5].

The low level of control works at faster sampling rate and adjusts the operating point, computed by the high level controller, to disturbances that occur on faster time scale, e.g. sudden shut-downs of wind turbines and the influence of local wind gusts (i.e. wind gusts that are not predicted by the wind flow model). An implicit assumption is that the overall wind field in a wind farm and the short time scale disturbances are to a large extent decoupled. The local occurrence of a wind gust or a shut-down of a single wind turbine do not influence the propagation of the wind field significantly. Therefore, the distribution of production and loading computed by the nominal controller is still very close to the optimal point. Hence, the low level controller does not need the knowledge of the entire wind field. Its objective is to remain as close as possible to the optimal production and loading distribution set by the nominal controller and at the same to deliver the power required by the TSO and respect any newly imposed constraints.

The proposed control scheme is a two level hierarchical control concept with one-directional communication from high to low level. The high level receives information (measurements) at slower sampling rates and the low level on faster sampling rates. The controllers use the system models that are in accordance to the sampling rate they use – the low level controller models the wind turbine in more details and disregards the wind field model, while the high level controller uses a very simple wind turbine model and models the wind field in detail. Both controllers are discrete-time optimal controllers. The control structure, depicted in Fig. 1, is essentially the concept for hierarchical MPC for multi time scale systems described in [6].

In this paper the control design solution for low level wind farm control is presented. Its main objective is to redistribute (reconfigure) the power reference between wind turbines in case of disturbances. In Aeolus project this level of control is entitled reconfigurable control or disturbance rejection. The high level controller is referred to as supervisory or nominal controller.

III. WIND TURBINE AS A WIND FARM ACTUATOR

The aim of the Aeolus project the aim is to develop the wind farm control algorithms that is superposed to wind turbine control system. The wind farm control algorithm must provide the appropriate power reference signal for each wind turbine in the farm. The wind turbine control algorithm is considered fixed and known. The wind farm controller sends the power reference to the wind turbine controller.

In this paper the aeroelastic model of a wind turbine developed at National Renewable Energy Laboratory (NREL) is used, which is described in [7]. It is a model of a conventional variable-speed pitch-controlled wind turbine. The wind turbine speed and power production are controlled by blade-pitch and generator-torque controllers that are designed to work simultaneously and independently. The generator torque control loop is extremely fast because the system is manipulated at electrical level. The blade pitch control loop uses the mechanical actuator and therefore has significantly larger time constants.

![Fig. 2. Wind turbine control regions](image-url)
square of the wind speed in order to obtain the optimal tip-speed ratio that maximizes the power capture [8]. This type of torque controller implicitly provides the negative feedback to the generator speed.

When wind speed becomes larger than nominal wind speed the wind turbine is able to produce the nominal power and it enters the control region 3. In this region the pitch controller becomes active. The increase in blade-pitch angle reduces the aerodynamical torque that wind turbine captures from the wind and thereby slows down the wind turbine. In order to maintain the aerodynamical torque that wind turbine captures from the wind becomes active. The increase in blade-pitch angle reduces the generator speed.

In transition regions $1 \frac{1}{2}$ and $2 \frac{1}{2}$ the generator torque reference is linearly dependant on the generator speed. The region $1 \frac{1}{2}$ is active when rotational speed required to obtain the optimal tip-speed ratio is smaller than the minimum rotational speed at which wind turbine can remain grid connected. Similarly, the region $2 \frac{1}{2}$ is active when the required rotational speed for the optimal tip-speed ratio is higher than the generator nominal speed.

This control concept is common in industrial practice and is well described in literature, see e.g. [8].

For the implementation of the wind farm control system with objectives described in Sec. I and II the wind turbine controller needs to be able to receive an external power reference $P_{\text{ref}}$ from wind farm controller and to use it instead of its preset power reference that equals wind turbine nominal power. It is achieved by replacing the wind turbine nominal power, which is constant in the conventional wind turbine controller, with a control signal that will be designated as power reference, $P_{\text{ref}}$. The power reference is then the upper limit on the power production. Introduction of external varying power reference transforms the torque characteristics shown in Fig.2 into family of curves since each power reference value corresponds to different torque characteristics. Lowering the power reference widens the control region 3 and narrows the control region $2 \frac{1}{2}$. If the reference power is low enough, the control region $2 \frac{1}{2}$ does not exist and the region 2 narrows.

Since the wind farm control in the Aeolus project uses power reference as the only control variable, the natural requirement is that the wind turbine actually produces its demanded power. To ensure this, one has to constrain the demanded power at time $t$, $P_{\text{ref}}(t)$, to the amount of power that a wind turbine can produce:

$$P_{\text{min}} \leq P_{\text{ref}}(t) \leq \min\left(P_{\text{max}}(t), P_{\text{nom}}\right),$$

where $P_{\text{min}}$ denotes the minimal power that a wind turbine can produce at any given time. Depending on the type of the wind turbine $P_{\text{min}}$ can be constant but can also be a function of time since minimal power that a wind turbine can produce can depend on the wind speed [9]. In this paper the minimal power is used as a constant, but all results can easily be extended to accommodate variable minimal power. $P_{\text{nom}}$ denotes the nominal power of the wind turbine. A variable $P_{\text{max}}(t)$ is maximal available power at time $t$. This variable depends strongly on the wind speed that varies in time [8]:

$$P_{\text{max}}(t) = \frac{1}{2} \rho R^2 \pi v(t)^3 C_p^{\text{max}},$$

where $v(t)$ is the wind speed at time $t$, $\rho$ denotes the air density, $R$ is the wind turbine rotor radius and $C_p^{\text{max}}$ is the maximum value of the power coefficient $C_p$ that is equal to the ratio between wind turbine power and available wind power.

If the constraint (1) is satisfied at time $t$ in stationary conditions the equality $P = P_{\text{ref}}$ holds with $P$ being wind turbine power. The constraint (1) also corresponds to the static condition that ensures that the wind turbine controller is in region 3. In dynamic conditions some region switching should be expected. This is due to the fact that the switching governor is based on the measured generator speed [7] – the condition for switching from region $2 \frac{1}{2}$ (or 2) to region 3 is that the demanded torque multiplied by measured generator speed exceeds the demanded power. Due to turbine inertia and low-pass filtering of the generator speed, the measured generator speed can be lower than its static value that corresponds to the available wind power at that operating point. Therefore, the region for power optimization will be active.

In this paper the switching of regions is not analyzed, it is assumed that all wind turbines in the wind farm operate in control region 3. This scenario is in fact very common in practice and also the most interesting scenario because it is the governing scenario at high winds – for winds faster than the nominal wind speed it is the only possible scenario. Also, the aim of the approach is to optimally distribute the power production. This task implicitly requires the existence of certain power reserve in the system – if the demanded wind farm power requires that all wind turbines maximize their power there is no room for optimization of the distribution.

A. Control design model of power controlled wind turbine

The wind farm controller as defined in the Aeolus project is a controller that is superposed to the wind turbine controller. Therefore, it can not compete with wind turbine controller in active regulation of wind turbine fast states like for instance tower motion or shaft torsion. From wind farm point of view a wind turbine is described by its rotational speed and pitch angle, which influence the power production and axial induction factor that in turn determines the wake development [10].

For the reconfigurable control design a simplified model of the NREL wind turbine in control region 3 is derived. The very fast dynamics related to the electrical systems are neglected. Also, the fast oscillations present in the system – the shaft torsion and the tower movement – are not modeled. These states are to fast to be actively controlled at wind farm level. Their efficient control requires decoupling between wind turbine actuators, which is impossible to achieve with power reference as only control variable. The dominant dynamics that is relevant for compensation of wind farm disturbances comes from wind turbine inertia and the blade pitch control system.

For control design the general static model is derived and the linearized model in a general operating point was obtained.
In the control region 3 the torque controller characteristic is described by the equation:

\[ T_{g}^{\text{ref}} = \frac{P_{\text{ref}}}{\omega_{g} \mu}, \quad (3) \]

where \( T_{g}^{\text{ref}} \) is the generator torque reference and \( \mu \) denotes the generator efficiency. The very fast dynamics of the frequency converter and the generator is neglected and the generator torque characteristic is linearized around the operating point \( (P_{\text{ref}0}, \Omega_{g0}) \) to obtain:

\[ T_{g}(s) = \frac{1}{\Omega_{g0} \mu} P_{\text{ref}}(s) - \frac{P_{\text{ref}0}}{\Omega_{g0}^2 \mu} \Omega_{g}(s); \quad P(s) = P_{\text{ref}}(s), \quad (4) \]

In control region 3 the generator speed is regulated towards the nominal value \( \omega \), which is a constant. The regulation is performed in closed-loop with a PI gain-scheduled controller which acts upon the difference between the filtered measurement of the generator speed and the nominal generator speed. The feedback function from measured generator speed to pitch angle reference is:

\[ \beta_{\text{ref}}(z) = \frac{K_{p} + K_{I} T_{s}}{K_{\text{corr}}(P_{\text{ref}}, \beta_{h}, z^{-1})}, \quad (5) \]

where \( T_{s} \) denotes the sampling time, \( K_{p} \) and \( K_{I} \) are proportional and integral gains respectively, and \( K_{\text{corr}} \) denotes the correction factor that is a function in \( P_{\text{ref}} \) and the pitch angle reference from the preceding time sample. In order to obtain a continuous-time linear time-invariant wind turbine model (necessary for the reconfigurable controller design) the transfer function (5) was transferred to continuous time domain and the feedback through the gain-scheduling of the correction factor was disregarded:

\[ \frac{\beta_{\text{ref}}(s)}{\Omega_{g}^{\text{meas}}(s)} = \frac{K_{p} + K_{I} s}{K_{\text{corr}}(P_{\text{ref}}, \beta_{h})}, \quad (6) \]

The low-pass filtered measurement of the generator speed is modeled as a first order linear model with time constant \( T_{\text{g}} \) that lumps the effects of the measurement device and subsequent filtering:

\[ \Omega_{g}^{\text{meas}}(s) = \frac{1}{1 + T_{\text{g}} s} \Omega_{g}(s). \quad (7) \]

The transmission system is modeled as a shaft with lumped inertia. The fast dynamics related to the shaft elasticity are thus omitted. The following linear relations are obtained:

\[ \Omega_{g}(s) = \frac{1}{J_{s} s} (T_{i}(s) - n_{gb} T_{g}(s)), \quad (8) \]

\[ \Omega_{g}(s) = n_{gb} \Omega_{b}(s), \quad (9) \]

where \( n_{gb} \) is the gearbox ratio.

The linear expression for rotor torque \( T_{r} \) is obtained by linearizing the nonlinear equation:

\[ T_{i} = \frac{1}{2} \rho R^{3} \pi v^{2} C_{Q}(\lambda, \beta), \quad (10) \]

where \( C_{Q}(\lambda, \beta) \) is the torque coefficient. The obtained expression has a form:

\[ T_{i}(s) = K_{v} V(s) + K_{\omega} \Omega(s) + K_{\beta} \beta(s), \quad (11) \]

where \( K_{v}, K_{\omega}, \) and \( K_{\beta} \) are linearization coefficients.

When equations (4), (8), (9) and (11) are combined a third order state-space model is obtained with the state vector defined as \( x = [\beta \quad \omega \quad \omega_{g}^{\text{meas}}]^{T} \), an input matrix as \( u = [P_{\text{ref}}]^{T} \) and disturbance vector \( d = [v]^{T} \). The state-space model is defined by differential equation:

\[ \dot{x}(t) = Ax(t) + Bu(t) + Bd(t). \quad (12) \]

The model outputs are produced power and loading measures. A loading measure that in fact reflects the cumulative loading of the wind turbine mechanical and electrical components can adopt various forms depending on the control and optimization criteria. In this paper the tower bending moment and of the shaft moment are used as the measures of loading, as suggested in [10]. The aim is to reduce the cyclic stresses that cause wind turbine fatigue.

The tower bending moment is caused by several effects: the thrust force that acts on the rotor, the tower drag force and the force due to eccentricity of the nacelle [10]. Since in the available wind turbine model neither the tower drag nor the nacelle eccentricity were not modeled, for now those components are disregarded. The bending moment \( M_{t} \) due to thrust force \( F_{t} \) is given by:

\[ M_{t} = H \cdot F_{t} = H \cdot \frac{1}{2} \rho R^{2} \pi C_{t}(\lambda, \beta) v^{2}, \quad (13) \]

where \( H \) is the tower height and \( C_{t}(\lambda, \beta) \) is the thrust coefficient. The equation (13) was linearized to obtain a relation:

\[ M_{t}(s) = K_{v} M_{v} V(s) + K_{\omega} M_{\omega} \Omega(s) + K_{\beta} M_{\beta} \beta(s), \quad (14) \]

where \( K_{v} M_{v}, K_{\omega} M_{\omega}, \) and \( K_{\beta} M_{\beta} \) are linearization coefficients.

The shaft moment \( M_{\text{shaft}} \) can be modeled as described in e.g. [11]. After disregarding the shaft elasticity, is modeled as:

\[ M_{\text{shaft}}(s) = \frac{n_{gb} J_{r}}{J_{r} + n_{gb} J_{g}} T_{g}(s) + \frac{n_{gb}^{2} J_{g}}{J_{r} + n_{gb}^{2} J_{g}} T_{r}(s). \quad (15) \]

If the output vector is defined as \( y = [P \quad M_{t} \quad M_{\text{shaft}}]^{T} \) then, following from (15) and (11), the outputs can be written in state-space form:

\[ y(t) = C x(t) + D u(t) + D_{d} d(t). \quad (16) \]

IV. RECONFIGURABLE CONTROL CONCEPT

The main prerequisite for the reconfigurable control is to be able to solve a large optimization problem at a fast sampling rate. Therefore, the intended approach was to transfer as much as possible computational effort for solving the optimization problem off-line.

As it was explained in Sec. II, the reconfigurable controller receives the set-points for individual wind turbines from the supervisory controller. The local objective (local – related to a single wind turbine), which a reconfigurable controller needs to accomplish, is to keep the wind turbine outputs and
states close to their set-points, while respecting newly imposed constrains (such as e.g. smaller maximal production due to low wind). This problem is regarded as local wind farm control problem for reconfigurable control. It is solved off-line by employing multi-parametric programming.

Additionally, the reconfiguration has to ensure that the wind farm production follows the demand set by the TSO. This requirement can not be expressed by a local objective. Therefore, this objective will be pursued on-line by observing solutions to the local control problem as wind turbine costs.

### A. Local wind farm control problem

The requirement that a wind turbine stays close to the reference provided by the supervisory controller and respect the imposed constrains can be cast as a Constrained Finite-Time Optimal Control problem (CFTOC) (a tracking formulation):

$$
\min_{U} \ J(x_0, x_{ref}, U_{ref}, U) := (Y - Y_{ref})'Q_y(Y - Y_{ref}) + 
+(X - X_{ref})'Q_x(X - X_{ref}) + (U - U_{ref})'R(U - U_{ref})
$$

subject to

$$
\begin{align*}
X &= Ax_0 + BU + BS_d + F, \\
Y &= Cx_0 + DU + Ds_d + G, \\
E'U + E'x_0 &\leq E,
\end{align*}
$$

where:

- $Y = [y_1 \ldots y_{N-1}]$, $X = [x_1 \ldots x_{N-1}]$, $U = [u_1 \ldots u_{N-1}]$,
- $D = [d_0 d_1 \ldots d_{N-1}]$ are vectors of predicted outputs, states, inputs and disturbances respectively; $Y_{ref} = [y_{ref,1} \ldots y_{ref,N-1}]$, $X_{ref} = [x_{ref,1} \ldots x_{ref,N-1}], U_{ref} = [u_{ref,1} \ldots u_{ref,N-1}]$ are vectors of output, state and input references respectively; $Q_y, Q_x, R$ are the weight matrices such that $Q_y = (Q_y)' \succeq 0$, $Q_x = (Q_x)' \succeq 0$, $Q_u = (Q_u)' > 0$, $Q^{\infty} \succeq 0$; $x_0$ is the initial system state; $E_U, E_s, E$ define system constraints and $N$ is prediction horizon. $A, B, D_s, F, C, D, D_d, G$ are matrices that describe the system evolution that can be obtained from simulation models.

Solving (17) ensures that the offsets from the set-points provided by supervisory controller are minimized in the entire prediction horizon while the imposed constrains are respected.

In this optimization program one can define a parameter that lumps together all the variables that change in time:

$$
\theta = [x_0^t \ u^t \ x_{ref}^t \ y_{ref}^t \ u_{max}^t]'.
$$

By introducing $\theta$ one can restate (17) in the following form:

$$
J^*(\theta) = \frac{1}{2} \theta' G \theta + \min_{U} \ \frac{1}{2} U' H U + U' F \theta
$$

subject to $C U \leq c + S \theta$.

This is a special form of a mathematical program, where $U \in \mathbb{R}^{n_u}$ is the optimization vector, $\theta \in \mathbb{R}^p$ is the vector of parameters, and $C \in \mathbb{R}^{p \times n_u}$, $c$ is in $\mathbb{R}^p$, $S \in \mathbb{R}^{q \times n_u}$, $G \in \mathbb{R}^{q \times n_u}$, $H \in \mathbb{R}^{n_u \times n_u}$, $F \in \mathbb{R}^{n_u \times q}$ are constant matrices.

The problem of computing the optimizer $U^*(\theta)$ and the value function $J^*(\theta)$ in (19) is referred to as the (right-hand-side) multi-parametric quadratic program (mp-QP). Efficient mp-QP solvers are developed and readily available, e.g. [14].

Once the mp-QP is solved off-line, i.e. the solution $U^*(\theta)$ of the CFTOC is found, the state feedback piece-wise affine receding-horizon control law is simply:

$$
u(t) = [I_{n_u} \ 0 \ldots 0] U^*(\theta(t)). \quad (20)$$

The following theorem describes the properties of the solution of the mp-QP that are used in this control design.

**Theorem IV.1** ([13]). Consider the mp-QP (19). Assume $H \succ 0$ and $[Y \ F] H [Y \ F]' \succeq 0$. The set $K^*$ is a polyhedral set, the value function $J^* : K^* \rightarrow \mathbb{R}$ is a piecewise affine function on polyhedra (PPWA), convex and continuous and the optimizer $U^* : K^* \rightarrow \mathbb{R}^{n_u}$ is a piecewise affine function on polyhedra (PPWA) and continuous.

Here, $K^*$ denotes the set of feasible parameters. For definitions of feasibility, polyhedra, convexity, PPWA see [15], [16].

### B. Reconfiguration algorithm

Once the local optimization program for each wind turbine is solved off-line, the on-line reconfiguration algorithm distributes the power references in such a manner that the delivered power follows the demand set by TSO. The distribution is based on the collected measurements and the solutions of the local problem. The principle of reconfiguration algorithm is explained in the following.

Let $U^j(t)$ and $J^j(t)$ be the optimizer and the value function expressions for the $j$-th wind turbine, and let $\theta^j$ be the space of parameters ($\theta^j = [x_0^j \ u^j \ x_{ref}^j \ y_{ref}^j \ u_{max}^j]$) for which those expressions are valid. Note that the superscript $j$ is used to denote variables related to the $j$-th wind turbine.

At the time $t$, the state vector $x_0^j(t)$, the wind speed (disturbance) $v(t)$, the references $x_{ref}^j(t)$ and $y_{ref}^j(t)$ (provided by the supervisory controller), and $u_{max}(t)$, are known and fixed. Then, from a pre-solved local control problem (17), we obtain the optimizer

$$
z^j(U_{ref}^j) := U^j(x_0^j(t), v(t), U_{ref}^j, x_{ref}^j(t), y_{ref}^j(t), u_{max}(t)), \quad (21)$$

and the value function

$$
V^j(U_{ref}^j) := J^j(x_0^j(t), v(t), U_{ref}^j, x_{ref}^j(t), y_{ref}^j(t), u_{max}(t)), \quad (22)
$$

as functions of only input reference of $j$-th turbine, $U_{ref}^j$. Note that wind turbines operate in different operating conditions and therefore have different models and constraints. Consequently, in general, for $i \neq j$ we have $z^i(\cdot) \neq z^j(\cdot), V^i(\cdot) \neq V^j(\cdot)$.
The global control objective for all turbines, at any time instant \( t \), is to produce the power demanded by the TSO:

\[
N_{\text{on\ wt}} \sum_{j=1}^{N_{\text{on\ wt}}} P_j(U_j) = P_{\text{TSO}}^{\text{wf}}(t), \quad \forall t,
\]

(23)

where \( P_j \) denotes the power of the \( j \)-th wind turbine (the power of the wind turbine is considered a part of the wind turbine’s output vector \( y_j \)), \( N_{\text{on\ wt}} \) denotes the number of wind turbines in operation, and \( P_{\text{TSO}}^{\text{wf}} \) is the power demand provided by the TSO.

The reconfiguration algorithm should, by redistributing the references \( U_j \), find the smallest total cost for which the condition (23) is satisfied, i.e., one has to solve the following problem:

\[
\min_{U_1, \ldots, U_{N_{\text{on\ wt}}}} \sum_{j=1}^{N_{\text{on\ wt}}} V_j(U_j)
\]

subj. to

\[
\left\{ \begin{array}{l}
\sum_{j=1}^{N_{\text{on\ wt}}} P_j(U_j) = P_{\text{TSO}}^{\text{wf}} \\
x_j(t) \psi_j(t) U_j \in \Theta_j, \quad j = 1, \ldots, N_{\text{on\ wt}}
\end{array} \right.
\]

(24)

Here, \( \Theta_j \) denotes the set of feasible parameters of the \( j \)-th wind turbine. After solving (24) the optimizers \( z_j \) from (21) can also be computed and applied to the wind farm.

The expression (24) is a convex optimization problem with strictly convex, piecewise quadratic cost over polyhedral constraints. Such problems can be, in principle, solved with some type of a gradient or Newton based approach. At the points where gradient is not a continuous function one may have to compute subgradients, cf. [17]. Note that (24) can be readily formulated as an Mixed-Integer Quadratic Program (MIQP) [16] and then solved either by enumeration (as it is done for now) or some existing (branch & bound) MIQP solvers [18].

The schematic overview of reconfigurable control algorithm is given in Fig. 3.

V. SIMULATION RESULTS

Since the focus of this paper is the reconfigurable control the results only for this part of the wind farm controller are presented. The typical scenarios in which reconfigurable controller should improve the behavior of the wind farm –
the sudden shut-down of a wind turbine and a large wind gust – are tested in simulation. The simplest wind model was used for predicting wind speed – the persistence model \( v(t + 1) = v(t) \). The references which are supposed to be delivered by nominal controller were considered constant in the entire simulation. The wind farm on which the simulation was performed consists of 4 wind turbines. Increase in number of wind turbine does not present a problem from computational point of view, however the results are very hard to demonstrate.

The first scenario examines a sudden shut down of one of the turbines (WT2) in a time instant between two sampling instants of the supervisory controller. As it can be seen from Fig. 5 in the case of a wind turbine sudden shut down reconfigurable control extension enables wind farm to keep tracking the overall wind farm power reference. This is achieved by increase of the power reference of the operating turbines. The redistribution of wind farm power among operating turbines is done considering the local wind turbine operating conditions and constraints. Without reconfigurable control extension shut down of a wind turbine causes a consequent drop in the wind farm production since supervisory controller is not aware of the missing turbine until the next sampling instant.

The second operation scenario in which reconfigurable control improves wind farm operation is the sudden and large wind gust affecting one or some of the turbines. Wind farm behavior with a 4 m/s wind gust affecting only WT2 is shown in Fig. 4.

Unlike the previous scenario, in the event of large wind gust wind farm overall power is maintained even without reconfigurable control extension. This is possible because local wind turbine controller reacts upon wind gust and regulates wind turbine speed and power back to their reference values (demanded by the wind farm supervisory controller). The introduction of the reconfigurable control extension improves the transient behavior of wind turbines in the farm. This improvement is manifested through less oscillatory transients of wind turbine variables such as thrust force, rotor speed, pitch angle and shaft moment. As it can be seen in Fig. 4, the maximum and steady state values of the shaft moment and thrust force on some of the turbines are increased with introduction of reconfigurable control extension what might look like worsening of the wind farm behavior. However, it

\[ v \text{ [m/s]} \]

\[ P_{WF} \text{ [MW]} \]

\[ \omega_r \text{ [rpm]} \]

\[ \beta \text{ [°]} \]

\[ P \text{ [kW]} \]

\[ F_T \text{ [N]} \]

\[ M_{shaft} \text{ [Nm]} \]

Fig. 5. Simulation results – wind gust
should be remembered that reconfigurable control extension was designed with the goal of fatigue reduction on wind turbines. Wind turbine fatigue is a consequence of oscillatory stress and strain resulting from oscillatory loads i.e. forces and moments. Therefore, wind turbine fatigue is not affected by the loads absolute values, but by their amplitude of change and number of periods before the transient oscillations die out. Both of these quantities are reduced with introduction of the reconfigurable control as a consequence of the chosen cost function used for the reconfigurable controller design. The increase of the absolute values of some of the loads poses no problem for wind turbine structure since it has to be designed to withstand steady loads several times larger than nominal loads (see e.g. [19]). The reduction in the loads amplitude and more aperiodic behavior reduces the wind turbine fatigue what is beneficial. Also, the pitch angle activity was decreased and the overshoots of wind turbine speed were reduced.

The control design was also tested on various scenarios with turbulent wind. In all simulations the reduction in oscillations especially of the tower bending moment is observed. With increased control effort the shaft moment becomes (due to direct feed-through between the power reference and the shaft moment (4), (15)), more violent, however, not oscillatory.

VI. CONCLUSION

In this paper the hierarchical wind farm control concept was described. The high level controller computes the optimal distribution of production and loading based on the dynamic wind field model. Due to complex computation this level of control works at slower sampling rate. The low level of control reacts in case of sudden disturbances, e.g. a wind gust or wind turbine shut-down. In such scenarios the response of the wind farm controlled by the supervisory controller is compromised due to slow sampling time of the supervisory controller.

The solution for the low level control problem was proposed. It is based on multi-parametric solution of the Constrained Finite Time Optimal Control problem for each wind turbine – the optimal control action and its associated cost are obtained as explicit expressions in parameters that present measurements, references and non-constant system constraints. Thus, most of computational effort is moved off-line. By using the multi-parametric solutions to local control problems and disregarding the coupling between the wind turbines introduced by the common wind field, the remaining on-line computation becomes much simpler than the overall wind farm optimization. Therefore, it can be run at faster time scale. The simulations of the presented control solutions for typical scenarios show promising results.

Further development of the control approach includes testing the control design on a nonlinear model. One approach for transition to nonlinear models is to use a large amount of models and off-line solution to the local control problems and, correspondingly to the system measurements, use the closest available model. Second approach is modeling the system as a PieceWise-Affine (PWA) system. The methodology described in Sec. IV can be accommodated to PWA systems.

The following actions also include merging of the supervisory and the reconfigurable control. This merging brings a new issue that needs to be considered – the interpolation between supervisory controllers references. Since the nominal controller will also be a model predictive controller, based on prediction of states and outputs reconfigurable control could assist the supervisory controller by interpolation of the references on faster time scale.

REFERENCES


