# Rješenje 3. školske zadaće za grupe 4, 8 i 10

21. prosinca 2010.

## 1 Grupa A

**Bodovi: 4-3-3**

**ZADATAK 1**

- $D(f) = \mathbb{R} \setminus \{0\} = \langle -\infty, 0\rangle \cup \langle 0, \infty\rangle$
- **Ponašanje na rubovima od $D(f)$:**

  **(1)** $\lim\limits_{x\to-\infty} xe^{-\frac{1}{x}} = -\infty \cdot e^0 = -\infty$

  **(2)** $\lim\limits_{x\to+\infty} xe^{-\frac{1}{x}} = +\infty \cdot e^0 = +\infty$

  **(3)** $\lim\limits_{x\to0-} xe^{-\frac{1}{x}} = \lim\limits_{x\to0-} \frac{e^{-\frac{1}{x}}}{\frac{1}{x}} = \frac{\pm\infty}{-\infty} \overset{L'H}{=} \lim\limits_{x\to0-} \frac{e^{-\frac{1}{x}}\frac{1}{x^2}}{-\frac{1}{x^2}} = \lim\limits_{x\to0-} -e^{-\frac{1}{x}} = -e^{+\infty} = -\infty$

  **(4)** $\lim\limits_{x\to0+} xe^{-\frac{1}{x}} = 0 \cdot e^{-\infty} = 0$

  **Iz** $(3)$ **i** $(4)$ **slijedi da je** $x = 0$ **lijeva vertikalna asimptota.**

- **Kose asimptote:**

$$y = kx + l$$

$$k = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{xe^{-\frac{1}{x}}}{x} = e^0 = 1$$

$$l = \lim_{x\to\pm\infty}(f(x) - kx) = \lim_{x\to\pm\infty}(xe^{-\frac{1}{x}} - x) = \lim_{x\to\pm\infty} \frac{e^{-\frac{1}{x}} - 1}{\frac{1}{x}} = \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x\to\pm\infty} \frac{e^{-\frac{1}{x}}\frac{1}{x^2}}{-\frac{1}{x^2}} = \lim_{x\to\pm\infty} -e^{-\frac{1}{x}} = -e^0 = -1$$

  $y = x - 1$ **je obostrana kosa asimptota**

- **Ekstremi:**

$$f'(x) = 1 \cdot e^{-\frac{1}{x}} + x \cdot e^{-\frac{1}{x}}\frac{1}{x^2} = \frac{e^{-\frac{1}{x}}}{x}(x + 1)$$

  **Jedina stacionarna točka je** $x = -1$ **i to je točka lokalnog maksimuma. Točka** $M(-1, -e)$ **je lokalni maksimum.**

**ZADATAK 2**

$$\int\limits_0^1 \frac{x^9}{x^{20}+3}dx = \frac{1}{10}\int\limits_0^1 \frac{10x^9}{(x^{10})^2+(\sqrt{3})^2} = \left\{t = x^{10},\ dt = 10x^9 dt\right\} =$$
$$= \frac{1}{10}\cdot\frac{1}{\sqrt{3}}\arctan(\frac{t}{\sqrt{3}})\ |_0^1 =$$
$$= \frac{1}{10\sqrt{3}}\cdot\frac{\pi}{6} = \frac{\pi}{60\sqrt{3}}$$

**ZADATAK 3**

$$\int x^2\sin(2x)dx = \{u = x^2, dv = \sin(2x)dx\} = x^2\frac{-\cos(2x)}{2} - \int -\frac{\cos(2x)}{2}\cdot 2xdx =$$
$$= -\frac{1}{2}x^2\cos(2x) + \int x\cos(2x)dx = \{u = x, dv = \cos(2x)dx\} =$$
$$= -\frac{1}{2}x^2\cos(2x) + \left(x\frac{\sin(2x)}{2} - \int\frac{\sin(2x)}{2}dx\right) =$$
$$= -\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C$$

## 2 Grupa B

**Bodovi: 4-3-3**

**ZADATAK 1**

- $D(f) = \mathbb{R} \setminus \{0\} = \langle -\infty, 0\rangle \cup \langle 0, \infty\rangle$
- **Ponašanje na rubovima od $D(f)$:**

  **(1)** $\lim\limits_{x\to-\infty} \frac{e^{-\frac{1}{x}}}{x} = \frac{e^0}{-\infty} = \frac{1}{-\infty} = 0-$

  **(2)** $\lim\limits_{x\to+\infty} \frac{e^{-\frac{1}{x}}}{x} = \frac{e^0}{+\infty} = \frac{1}{+\infty} = 0+$

  **(3)** $\lim\limits_{x\to0-} \frac{e^{-\frac{1}{x}}}{x} = \frac{e^{+\infty}}{-0} = \frac{+\infty}{-0} = -\infty$

  **(4)** $\lim\limits_{x\to0+} \frac{e^{-\frac{1}{x}}}{x} = \{x = \frac{1}{t}, x \to 0+ \Rightarrow t \to +\infty\} = \lim\limits_{t\to+\infty} \frac{e^{-t}}{\frac{1}{t}} = \lim\limits_{t\to+\infty} \frac{t}{e^t} = \frac{+\infty}{+\infty} \overset{L'H}{=} \lim\limits_{t\to+\infty} \frac{1}{e^t} = 0+$

  **Iz** (3) **i** (4) **slijedi da je** $x = 0$ **lijeva vertikalna asimptota.**
  **Iz** (1) **i** (2) **slijedi da je** $y = 0$ **obostrana horizontalna asimptota. Budući da postoje horizontalne asimptote, kose nije ni potrebno tražiti.**

- **Ekstremi:**

$$f'(x) = -\frac{1}{x^2} \cdot e^{-\frac{1}{x}} + \frac{1}{x} \cdot e^{-\frac{1}{x}} \frac{1}{x^2} = \frac{e^{-\frac{1}{x}}}{x^3}(1-x)$$

  **Jedina stacionarna točka je** $x = 1$ **i to je točka lokalnog maksimuma. Točka** $M(1, \frac{1}{e})$ **je lokalni maksimum.**

**ZADATAK 2**

$$\int\limits_0^1 \frac{x^5}{x^{12}+3}dx = \frac{1}{6}\int\limits_0^1 \frac{6x^5}{(x^6)^2 + \sqrt{(3)}^2} = \{t = x^6,\, dt = 6x^5dt\} =$$
$$= \frac{1}{6} \cdot \frac{1}{\sqrt{3}} \arctan(\frac{t}{\sqrt{3}}) \,|_0^1 =$$
$$= \frac{1}{6\sqrt{3}} \cdot \frac{\pi}{6} = \frac{\pi}{36\sqrt{3}}$$

**ZADATAK 3**

$$\int x^2 \cos(3x)dx = \{u = x^2, dv = \cos(3x)dx\} = x^2\frac{\sin(3x)}{3} - \int \frac{\sin(3x)}{3} \cdot 2xdx =$$

$$= \frac{1}{3}x^2 \sin(3x) - \frac{2}{3}\int x\sin(3x)dx = \{u = x, dv = \sin(3x)dx\} =$$

$$= \frac{1}{3}x^2 \sin(3x) - \frac{2}{3}\left(x\frac{-\cos(3x)}{3} - \int \frac{-\cos(3x)}{3}dx\right) =$$

$$= \frac{1}{3}x^2 \sin(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + C$$