# Rješenja druge školske zadaće za grupe 4., 8. i 10.

## Grupa A

**Zadatak 1.**

a) $\lim_{n\to+\infty} \frac{2n\cos n}{(3n+1)(1-3n)} = \lim_n \frac{2\frac{\cos n}{n}}{(3+1/n)(1/n-3)} = \frac{2\lim_n \frac{\cos n}{n}}{\lim_n(3+1/n)\lim_n(1/n-3)} = \frac{2\cdot 0}{3(-3)} =$ 0

b) $\lim_{n\to+\infty} (\sqrt{n^2-5n+6} - n)\frac{\sqrt{n^2-5n+6}+n}{\sqrt{n^2-5n+6}+n} = \lim_n \frac{-5n+6}{\sqrt{n^2-5n+6}+n} = \lim_n \frac{-5+6/n}{\sqrt{1-5/n+6/n^2}+1} =$ $\frac{-5}{\sqrt{1}+1} = -\frac{5}{2}$

**Zadatak 2.**

a) $\lim_{n\to+\infty} \left(\frac{n^2+1}{2n^2+1}\right)^{n^2} = \left(\lim_n \frac{n^2+1}{2n^2+1}\right)^{\lim_n n^2} = \left(\frac{1}{2}\right)^{\infty} = 0$

b) $\lim_{n\to+\infty} \left(1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots + \frac{(-1)^{n-1}}{3^{n-1}}\right) = \lim_n \frac{1-\left(-\frac{1}{3}\right)^n}{1+1/3} = \frac{1-\lim_n\left(-\frac{1}{3}\right)^n}{4/3} = \frac{1}{4/3} = \frac{3}{4}$

**Zadatak 3.**

a) $\lim_{n\to+0} \frac{\sin(2x)}{\sin 3x} = \lim_{n\to+0} \frac{2x}{3x} = \frac{2}{3}$

b) $\lim_{n\to+0} \frac{x^2}{\sin 3x\cos(2x)} = \lim_{n\to+0} \frac{x^2}{3x\cos(2x)} = \lim_{n\to+0} \frac{x}{3\cos(2x)} = 0$

c) $\lim_{n\to+0} \frac{1-\cos(x)}{x\sin x} = \lim_{n\to+0} \frac{2\sin^2(x/2)}{x\sin(x)} = \lim_{n\to+0} \frac{x^2/2)}{x^2} = \frac{1}{2}$

**Zadatak 4.**
Funkcija je očito neprekidna u svakom $x \in \mathbb{R}$
0. U $x = 0$ vrijedi

$$\lim_{x\to 0-} f(x) = \lim_{x\to 0-} x + a = a,$$

$$\lim_{x\to 0+} f(x) = \lim_{x\to 0+} e^{-2x} + 1 = e^0 + 1 = 2.$$

U 0 dakle postoje lijevi i desni limes. Obostrani limes postoji ako i samo ako su ta dva limesa jednaka, tj. ako je

$$a = 2.$$

Dakle, Funkcija je neprekidna na $\mathbb{R}$ ako i samo ako je $a = 2$.

## Grupa B

**Zadatak 1.**

a) $\lim_{n\to+\infty} \frac{n^2 \sin n}{(4n^2-3)(2-n)} = \lim_n \frac{\frac{\sin n}{n}}{(4-\frac{3}{n^2})(\frac{2}{n}-1)} = \frac{\lim_n \frac{\sin n}{n}}{\lim_n(4-\frac{3}{n^2})\lim_n(\frac{2}{n}-1)} = \frac{0}{4(-1)} = 0$

b) $\lim_{n\to+\infty}(\sqrt{n(n+7)}-n)\frac{\sqrt{n(n+7)}+n}{\sqrt{n(n+7)}+n} = \lim_n \frac{n(n+7)-n^2}{\sqrt{n(n+7)}+n} = \lim_n \frac{7n}{\sqrt{n(n+7)}+1} =$
$\frac{7}{\sqrt{1}+1} = \frac{7}{2}$

**Zadatak 2.**

a) $\lim_{n\to+\infty}\left(\frac{3n+3}{3n+2}\right)^{n+1} = \lim_n\left[\left(1+\frac{1}{3n+2}\right)^{3n+2}\right]^{\frac{n+1}{3n+2}} = \left[\lim_n\left(1+\frac{1}{3n+2}\right)^{3n+2}\right]^{\lim_n \frac{n+1}{3n+2}} =$
$e^{1/3}$

b) $\lim_{n\to+\infty}\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^{n-1}}\right) = \lim_n \frac{1-\left(\frac{1}{2}\right)^n}{1-1/2} = \frac{1-\lim_n\left(\frac{1}{2}\right)^n}{1/2} = \frac{1}{1/2} = 2$

**Zadatak 3.**

a) $\lim_{n\to+0}\frac{\sin^2 x}{x\cos(3x)} = \lim_{n\to+0}\frac{x^2}{x\cos(3x)} = \lim_{n\to+0}\frac{x}{\cos(3x)} = 0$

b) $\lim_{n\to+0}\frac{\tan(5x)}{\tan 2x} = \lim_{n\to+0}\frac{5x}{2x} = \frac{5}{2}$

c) $\lim_{n\to+0}\frac{1-\cos(4x)}{x^2} = \lim_{n\to+0}\frac{2\sin^2(2x)}{x^2} = \lim_{n\to+0}\frac{8x^2}{x^2} = 8$

**Zadatak 4.**
Funkcija je očito neprekidna u svakom $x \in \mathbb{R}$
0. U $x = 0$ vrijedi

$$\lim_{x\to 0-} f(x) = \lim_{x\to 0-}\left(a-e^{-2x^2}\right) = a - e^0 = a-1,$$

$$\lim_{x\to 0+} f(x) = \lim_{x\to 0+} 4 + x^2/2 = 4$$

U 0 dakle postoje lijevi i desni limes. Obostrani limes postoji ako i samo ako su ta dva limesa jednaka, tj. ako je

$$a-1=4.$$

Dakle, Funkcija je neprekidna na $\mathbb{R}$ ako i samo ako je $a = 5$.