Rješenja drugog međuispita iz Matematike 1

26. studenog 2010.

1. (a) $\lim\limits_{x\to\infty} \dfrac{x^2+x-(x^2+4x+4)}{x+1} = \lim\limits_{x\to\infty} \dfrac{-3x+4}{x+1} = -3.$

   (b) $\lim\limits_{x\to\infty} \dfrac{x+\frac{1}{x}}{1+\sqrt{1+\frac{1}{x}}} = \infty.$

   (c) $\lim\limits_{x\to\ 0} \dfrac{e^x \sin(2x)}{x\cos(3x)} = \lim\limits_{x\to 0} \dfrac{2xe^x}{x\cos(2x)} = 2.$

2. (a) i (b) ... predavanja.

   (c) $\sin\dfrac{1}{x^2} \in [-1,1] \Rightarrow \lim\limits_{x\to 0} x^2 \sin\dfrac{1}{x^2} = 0 \Rightarrow a = 0.$

   $f'(0) = \lim\limits_{h\to 0} \dfrac{h^2 \sin\frac{1}{h^2}}{h} = \lim\limits_{h\to 0} h\sin\dfrac{1}{h^2} = 0.$

3. (a) Knjižica 8, str. 7.

   (b) Knjižica 8, str. 9.

4. $y' = \operatorname{arctg}(e^{-x^2}) - \dfrac{2x^2 e^{-x^2}}{1+e^{-2x^2}}$

5. (a) $y' = \dfrac{2x(y+x)}{2y^2-x^2}$

   (b) $y' = 0 \Leftrightarrow x = 0$ ili $x+y=0.$

   $x = 0 \Rightarrow y = -1.$

   $x + y = 0 \Rightarrow x = \sqrt[3]{2},\ y = -\sqrt[3]{2}.$

6. $f(x) = \sin x + \cos x \Rightarrow f(0) = 1,$

   $f'(x) = \cos x - \sin x \Rightarrow f'(0) = 1,$

   $f''(x) = -\sin x - \cos x \Rightarrow f''(0) = -1,$

   $f'''(x) = -\cos x + \sin x \Rightarrow f'''(0) = -1,$

   $f^{(4)}(x) = \sin x + \cos x \Rightarrow f^{(4)}(0) = 1,$

   $f^{(5)}(x) = \cos x - \sin x.$

   $f(x) = 1 + x - \dfrac{x^2}{2} - \dfrac{x^3}{6} + \dfrac{x^4}{24} + \dfrac{\cos x_1 - \sin x_1}{120} x^5.$

   Za neki $x_1 \in \langle 0, x\rangle$ ili $x_1 \in \langle x, 0\rangle.$

7. $\lim\limits_{x\to 0+} \dfrac{(x\cos x + a\sin x)'}{(x^3)'} = \lim\limits_{x\to 0+} \dfrac{\cos x - x\sin x + a\cos x}{3x^2} =$

$= \lim\limits_{x\to 0+} \dfrac{(1+a)\cos x - x\sin x}{3x^2} = \lim\limits_{x\to 0+} \left(-\dfrac{1}{3}\dfrac{\sin x}{x} + \dfrac{(1+a)\cos x}{3}\dfrac{1}{x^2} \right) =$

$= \begin{cases} -\frac{1}{3}, & a = -1 \\ +\infty, & a > -1 \\ -\infty, & a < -1. \end{cases}$