

# 1. školska zadaća - Rješenja za grupe 4, 8, 10

## GRUPA A

### ZADATAK 1: (a) (2 BODA)

$$|z| + |z + 2i| = 3$$

$$\Rightarrow \sqrt{x^2 + y^2} + \sqrt{x^2 + (y + 2)^2} = 3$$

nakon kvadriranja i sredivanja izraza dobijemo:

$$36x^2 + 20y^2 + 40y = 25$$

$$\Rightarrow 36x^2 + 20(y + 1)^2 = 45$$

odnosno tražena jednačba elipse je:

$$\frac{x^2}{\frac{5}{4}} + \frac{(y + 1)^2}{\frac{9}{4}} = 1.$$

### (b) (2 BODA)

$$\arg(z^7) = \arg(z^2)$$

$$\Rightarrow 7\arg(z) = 2\arg(z) + 2k\pi$$

$$\Rightarrow 5\arg(z) = 2k\pi, \text{ tj. } \arg(z) = \frac{2k\pi}{5}, k = 0, 1, 2, 3, 4.$$

### ZADATAK 2: (a) (2 BODA)

$$D_f = \mathbb{R}$$

$$Im_f = \langle -\infty, 1 \rangle$$

### (b) (2 BODA)

$$y = 1 - e^x$$

- Zrcaljenje obzirom na pravac  $y = x$  ( $y \leftrightarrow x, x \leftrightarrow y$ ):

$$x = 1 - e^y \Rightarrow e^y = 1 - x \Rightarrow y = \ln(1 - x)$$

- Zrcaljenje obzirom na os  $y$  ( $x \leftrightarrow -x$ )

$$y = \ln(1 - x) \Rightarrow y = \ln(1 + x)$$

**ZADATAK 3: (2 BODA)**

$$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 10 \end{pmatrix}$$

$$\begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 10 \end{pmatrix}$$

$$\begin{cases} a^2 + bc = 5 \\ b(a + d) = 0 \\ c(a + d) = 0 \\ bc + d^2 = 10 \end{cases} \Rightarrow \begin{cases} b = 0 \\ c = 0 \\ a = \pm\sqrt{5} \\ d = \pm\sqrt{10} \end{cases}$$

$$A_1 = \begin{pmatrix} \sqrt{5} & 0 \\ 0 & \sqrt{10} \end{pmatrix}, A_2 = \begin{pmatrix} -\sqrt{5} & 0 \\ 0 & \sqrt{10} \end{pmatrix},$$

$$A_3 = \begin{pmatrix} \sqrt{5} & 0 \\ 0 & -\sqrt{10} \end{pmatrix}, A_4 = \begin{pmatrix} -\sqrt{5} & 0 \\ 0 & -\sqrt{10} \end{pmatrix}.$$

## GRUPA B

### ZADATAK 1: (a) (2 BODA)

$$|z| + |z - 2| = 3$$

$$\Rightarrow \sqrt{x^2 + y^2} + \sqrt{(x - 2)^2 + y^2} = 3$$

nakon kvadriranja i sredivanja izraza dobijemo:

$$20x^2 - 40x + 36y^2 = 25$$

$$\Rightarrow 20(x - 1)^2 + 36y^2 = 45$$

odnosno tražena jednačba elipse je:

$$\frac{(x - 1)^2}{\frac{9}{4}} + \frac{y^2}{\frac{5}{4}} = 1.$$

### (b) (2 BODA)

$$\operatorname{Re}(z^2) = \operatorname{Im}(z^2)$$

$$\Rightarrow r^2 \cos(2\varphi) = r^2 \sin(2\varphi)$$

- 1. slučaj:  $r = 0$
- 2. slučaj:

$$\operatorname{tg}(2\varphi) = 1$$

$$\Rightarrow 2\varphi = \frac{\pi}{4} + k\pi, \text{ tj. } \varphi = \frac{\pi}{8} + \frac{k\pi}{2}, k = 0, 1, 2, 3.$$

### ZADATAK 2: (a) (2 BODA)

$$D_f = \langle -1, +\infty \rangle$$

$$I_{m_f} = \mathbb{R}$$

### (b) (2 BODA)

$$y = -\ln(x + 1)$$

- Zrcaljenje obzirom na os  $y$  ( $x \leftrightarrow -x$ )

$$y = -\ln(x + 1) \Rightarrow y = -\ln(1 - x)$$

- Zrcaljenje obzirom na pravac  $y = x$  ( $y \leftrightarrow x, x \leftrightarrow y$ ):

$$x = -\ln(1 - y) \Rightarrow e^{-x} = 1 - y \Rightarrow y = 1 - e^{-x}$$

**ZADATAK 3: (2 BODA)**

$$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$$

$$\begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$$

$$\begin{cases} a^2 + bc = 2 \\ b(a + d) = 0 \\ c(a + d) = 0 \\ bc + d^2 = 4 \end{cases} \Rightarrow \begin{cases} b = 0 \\ c = 0 \\ a = \pm\sqrt{2} \\ d = \pm\sqrt{4} = \pm 2 \end{cases}$$

$$A_1 = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 2 \end{pmatrix}, A_2 = \begin{pmatrix} -\sqrt{2} & 0 \\ 0 & 2 \end{pmatrix},$$

$$A_3 = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & -2 \end{pmatrix}, A_4 = \begin{pmatrix} -\sqrt{2} & 0 \\ 0 & -2 \end{pmatrix}.$$