

A grupa:

1. $\arg(z^2) = \frac{3\pi}{2}$, $|z| = \sqrt{2}$, $z = ?$

$$\arg z = \frac{1}{2} \left(\frac{3\pi}{2} + 2k\pi \right), \quad k = 0, 1 \Rightarrow \arg z_1 = \frac{3\pi}{4}, \quad \arg z_2 = \frac{7\pi}{4}$$

$$\operatorname{tg} \frac{3\pi}{4} = \operatorname{tg} \frac{7\pi}{4} = -1 \Rightarrow \frac{y}{x} = -1 \Rightarrow y = -x$$

$$|z| = \sqrt{2} \Rightarrow x^2 + y^2 = 2 \Rightarrow 2x^2 = 2 \Rightarrow x = \pm 1, \quad y = \mp 1$$

$$\Rightarrow z_1 = -1 + i, \quad z_2 = 1 - i$$

$$\frac{1}{z_1 + 1 + 3i} = \frac{1}{-1 + i + 1 + 3i} = \frac{1}{4i} = \frac{-i}{4} = \frac{1}{4} \operatorname{cis} \frac{3\pi}{2}$$

$$\frac{1}{z_2 + 1 + 3i} = \frac{1}{1 - i + 1 + 3i} = \frac{1}{2 + 2i} = \frac{1-i}{4} = \frac{\sqrt{2}}{4} \operatorname{cis} \frac{7\pi}{4}$$

2. $f(x) = \sqrt{\sin(x^2)}$; $g(x) = \sqrt{2 - \sin(x^2)}$

a) $D_g = ?$

$$2 - \sin(x^2) \geq 0 \Rightarrow \sin(x^2) \leq 2 \Rightarrow D_g = \mathbb{R} \quad \text{jer je } \sin x \in [-1, 1] \text{ za } \forall x \in \mathbb{R}$$

$$D_f = ?$$

$$\sin(x^2) \geq 0 \Rightarrow 2k\pi \leq x^2 \leq (2k+1)\pi, \quad k \in \mathbb{N}_0 \quad (\text{za } k < 0 \text{ nema smisla gledati jer je } x^2 \geq 0)$$

$$\Rightarrow D_f = \bigcup_{k \in \mathbb{N}_0} \left(\left[-\sqrt{(2k+1)\pi}, -\sqrt{2k\pi} \right] \cup \left[\sqrt{2k\pi}, \sqrt{(2k+1)\pi} \right] \right)$$

b) $f(-x) = \sqrt{\sin((-x)^2)} = \sqrt{\sin(x^2)} = f(x) \Rightarrow f$ je parna funkcija.

3. $ABAB = \begin{bmatrix} 46 & 50 \\ 60 & 66 \end{bmatrix}$

B grupa

$$1. z^6 + 5z^3 + 6 = 0 \Rightarrow (z^3 + 2)(z^3 + 3) = 0 \Rightarrow z^3 + 2 = 0 \text{ ili } z^3 + 3 = 0$$

$$z^3 = -2 = 2 \operatorname{cis} \pi \Rightarrow z_{0,1,2} = \sqrt[3]{2} \operatorname{cis} \left(\frac{\pi + 2k\pi}{3} \right), k = 0, 1, 2$$

$$z^3 = -3 = 3 \operatorname{cis} \pi \Rightarrow z_{3,4,5} = \sqrt[3]{3} \operatorname{cis} \left(\frac{\pi + 2k\pi}{3} \right), k = 0, 1, 2$$

Rješenja za koja je $\operatorname{Re} z > 0$ su: z_0, z_2, z_3, z_5 .

$$2. f(x) = \ln(\cos(x^2)), g(x) = \ln(\sqrt{3} - \cos(x^2))$$

$$a) D_g = ?$$

$$\sqrt{3} - \cos(x^2) > 0 \Rightarrow \cos(x^2) < \sqrt{3} \Rightarrow D_g = \mathbb{R} \text{ jer je } \cos y \in [-1, 1] \text{ za } \forall y \in \mathbb{R}$$

$$D_f = ?$$

$$\cos(x^2) > 0 \Rightarrow \frac{-\pi}{2} + 2k\pi < x^2 < \frac{\pi}{2} + 2k\pi$$

Za $k < 0$ nema smisla gledati jer je $x^2 \geq 0$.

$$\text{Za } k = 0 \Rightarrow \frac{-\pi}{2} < x^2 < \frac{\pi}{2} \Rightarrow 0 \leq x^2 < \frac{\pi}{2} \Rightarrow x \in \left\langle -\sqrt{\frac{\pi}{2}}, \sqrt{\frac{\pi}{2}} \right\rangle$$

$$\text{Za } k > 0 \Rightarrow x \in \left\langle -\sqrt{\frac{\pi}{2} + 2k\pi}, -\sqrt{\frac{-\pi}{2} + 2k\pi} \right\rangle \cup \left\langle \sqrt{\frac{-\pi}{2} + 2k\pi}, \sqrt{\frac{\pi}{2} + 2k\pi} \right\rangle, k \in \mathbb{N}$$

$$\Rightarrow D_f = \left\langle -\sqrt{\frac{\pi}{2}}, \sqrt{\frac{\pi}{2}} \right\rangle \cup \left(\bigcup_{k \in \mathbb{N}} \left(\left\langle -\sqrt{\frac{\pi}{2} + 2k\pi}, -\sqrt{\frac{-\pi}{2} + 2k\pi} \right\rangle \cup \left\langle \sqrt{\frac{-\pi}{2} + 2k\pi}, \sqrt{\frac{\pi}{2} + 2k\pi} \right\rangle \right) \right)$$

$$f(-x) = \ln(\sin((-x)^2)) = \ln(\sin(x^2)) = f(x) \Rightarrow f$$

je parna funkcija

$$3. AB = \begin{bmatrix} 3 & 0 & -6 \\ -2 & -1 & -3 \\ 6 & 2 & 2 \end{bmatrix}, \det(AB) = 0$$