Rješenja 1. školske zadaće, grupe 2, 6; GRUPA A

1.
$$\overline{z}^3 = z(-1-i) \qquad (*)$$

Uočimo, jedno rješenje dane jednadžbe je trivijalno, tj. $z = 0$.

Pronađimo ostala netrivijalna rješenja dane jednadžbe; jednadžbu (*) pomnožimo sa z^3, imamo
$$|z|^6 = z^4(-1-i). \qquad (\diamondsuit)$$

Rješenje jednadžbe $(\diamondsuit)$ $(z \neq 0)$ tražimo u obliku $z = r(\cos\varphi + i\sin\varphi)$, te jer je $-1-i = \sqrt{2}(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4})$, uvrštavanjem u $(\diamondsuit)$ slijedi

$$r^2(\cos 0 + i\sin 0) = \sqrt{2}\Big(\cos\big(4\varphi + \frac{5\pi}{4}\big) + i\sin\big(4\varphi + \frac{5\pi}{4}\big)\Big),$$

odakle, imamo
$$r^2 = \sqrt{2} \qquad \Longrightarrow \qquad r = \sqrt[4]{2},$$

te
$$4\varphi + \frac{5\pi}{4} = 2k\pi \qquad (k \in \mathbb{Z}),$$

tj.
$$\varphi_k = \frac{k\pi}{2} - \frac{5\pi}{16} \qquad (k \in \mathbb{Z}).$$

No, jer je $\varphi_k \in [0, 2\pi\rangle$, posljednja relacija ima smisla za $k = 1, 2, 3, 4$, tj. $\varphi_1 = \frac{3\pi}{16}$, $\varphi_2 = \frac{11\pi}{16}$, $\varphi_3 = \frac{19\pi}{16}$, $\varphi_4 = \frac{27\pi}{16}$.

Dakle, sva rješenja jednadžbe (*) jesu:

$$\begin{aligned}
z_0 &= 0,\\
z_1 &= \sqrt[4]{2}\Big(\cos\big(\frac{3\pi}{16}\big) + i\sin\big(\frac{3\pi}{16}\big)\Big),\\
z_2 &= \sqrt[4]{2}\Big(\cos\big(\frac{11\pi}{16}\big) + i\sin\big(\frac{11\pi}{16}\big)\Big),\\
z_3 &= \sqrt[4]{2}\Big(\cos\big(\frac{19\pi}{16}\big) + i\sin\big(\frac{19\pi}{16}\big)\Big),\\
z_4 &= \sqrt[4]{2}\Big(\cos\big(\frac{27\pi}{16}\big) + i\sin\big(\frac{27\pi}{16}\big)\Big).
\end{aligned}$$

2. a) $D(f) = \mathbb{R}\backslash\{\ln\sqrt{2}\}$.
b)
$$x = \frac{2\mathrm{e}^y + \mathrm{e}^{-y}}{\mathrm{e}^y - 2\mathrm{e}^{-y}} \qquad \Longrightarrow \qquad x(\mathrm{e}^y - 2\mathrm{e}^{-y}) = 2\mathrm{e}^y + \mathrm{e}^{-y},$$

odakle, pak, nakon množenja s e^y, te uvođenjem supstitucije $t = \mathrm{e}^y$, dobivamo

$$t^2 = \frac{1+2x}{x-2}.$$

Djelujemo li s ln-om u posljednjoj jednakosti, koristeći činjenicu $\ln t^2 = 2\ln t = 2y$, slijedi

$$y = \frac{1}{2}\ln\frac{1+2x}{x-2},$$

odnosno

$$f^{-1}(x) = \frac{1}{2}\ln\frac{1+2x}{x-2}.$$

c) $D(f^{-1}) = \langle -\infty, -\frac{1}{2}\rangle \cup \langle 2, +\infty\rangle$.

3.

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}.$$

a)

$$AB = \begin{pmatrix} 2 & 5 & 8 \\ 0 & 2 & 4 \\ -2 & 1 & 4 \end{pmatrix}.$$

b) $\det(AB) = 0$.

Rješenja 1. školske zadaće, grupe 2, 6; GRUPA B

1.
$$z^3 = \frac{\overline{z}}{1-i} \qquad (*)$$

Uočimo, jedno rješenje dane jednadžbe je trivijalno, tj. $z = 0$.

Pronađimo ostala netrivijalna rješenja dane jednadžbe; jednadžbu (*) pomnožimo sa z, imamo
$$z^4 = \frac{|z|^2}{1-i}. \qquad (\diamondsuit)$$

Rješenje jednadžbe $(\diamondsuit)$ $(z \neq 0)$ tražimo u obliku $z = r(\cos\varphi + i\sin\varphi)$, te jer je $1 - i = \sqrt{2}(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$, uvrštavanjem u $(\diamondsuit)$ slijedi

$$r^2\Big(\cos\big(4\varphi + \frac{7\pi}{4}\big) + i\sin\big(4\varphi + \frac{7\pi}{4}\big)\Big) = \frac{1}{\sqrt{2}}(\cos 0 + i\sin 0),$$

odakle, imamo
$$r^2 = \frac{1}{\sqrt{2}} \qquad \Longrightarrow \qquad r = \frac{1}{\sqrt[4]{2}},$$

te
$$4\varphi + \frac{7\pi}{4} = 2k\pi \qquad (k \in \mathbb{Z}),$$

tj.
$$\varphi_k = \frac{k\pi}{2} - \frac{7\pi}{16} \qquad (k \in \mathbb{Z}).$$

No, jer je $\varphi_k \in [0, 2\pi\rangle$, posljednja relacija ima smisla za $k = 1, 2, 3, 4$, tj. $\varphi_1 = \frac{\pi}{16}$, $\varphi_2 = \frac{9\pi}{16}$, $\varphi_3 = \frac{17\pi}{16}$, $\varphi_4 = \frac{25\pi}{16}$.

Dakle, sva rješenja jednadžbe (*) jesu:

$$\begin{aligned}
z_0 &= 0,\\
z_1 &= \frac{1}{\sqrt[4]{2}}\Big(\cos\big(\frac{\pi}{16}\big) + i\sin\big(\frac{\pi}{16}\big)\Big),\\
z_2 &= \frac{1}{\sqrt[4]{2}}\Big(\cos\big(\frac{9\pi}{16}\big) + i\sin\big(\frac{9\pi}{16}\big)\Big),\\
z_3 &= \frac{1}{\sqrt[4]{2}}\Big(\cos\big(\frac{17\pi}{16}\big) + i\sin\big(\frac{17\pi}{16}\big)\Big),\\
z_4 &= \frac{1}{\sqrt[4]{2}}\Big(\cos\big(\frac{25\pi}{16}\big) + i\sin\big(\frac{25\pi}{16}\big)\Big).
\end{aligned}$$

2. a) $D(f) = \langle -\infty, -\frac{1}{3}\rangle \cup \langle 3, +\infty\rangle$.

b)

$$x = \ln\frac{3y+1}{y-3} \qquad \Longrightarrow \qquad \mathrm{e}^x = \frac{3y+1}{y-3} \qquad \Longrightarrow \qquad y = \frac{1+3\mathrm{e}^x}{\mathrm{e}^x - 3},$$

odnosno

$$f^{-1}(x) = \frac{1+3\mathrm{e}^x}{\mathrm{e}^x - 3}.$$

c) $D(f^{-1}) = \mathbb{R}\backslash\{\ln 3\}$.

3.

$$A = \begin{pmatrix} -2 & -1 & 0 \\ -1 & 1 & 3 \\ 0 & 3 & 6 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}.$$

a)

$$AB = \begin{pmatrix} -1 & 2 & 5 \\ 7 & 4 & 1 \\ 15 & 6 & -3 \end{pmatrix}.$$

b) $\det(AB) = 0$.