Rješenja ponovljenog drugog međuispita iz Matematike 1

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1. Niz je padajući:

   $a_1 = 2 > 3/2 = a_2$. Pretpostavimo da je $a_n < a_{n-1}$ što povlači $\dfrac{1+a_n}{2} < \dfrac{1+a_{n-1}}{2}$, tj. $a_{n+1} < a_n$.

   Niz je omeđen odozdo s 1:

   $a_1 = 2 > 1$ i pretpostavimo da je $a_n > 1$ što povlači $a_{n+1} = \dfrac{1+a_n}{2} > \dfrac{1+1}{2} = 1$.

   Dakle, niz je konvergentan i vrijedi $L = \lim\limits_n a_n = \lim\limits_n a_{n+1} = \lim\limits_n \dfrac{1+a_n}{2} = \dfrac{1+L}{2}$.

   $2L = L + 1 \Rightarrow L = 1$.

2. (a) Predavanja.

   (b) $\lim\limits_{x\to x_0} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to x_0} \left[\dfrac{f(x)}{m(x)} \cdot \dfrac{n(x)}{g(x)} \cdot \dfrac{m(x)}{n(x)}\right] = \lim\limits_{x\to x_0} \dfrac{f(x)}{m(x)} \lim\limits_{x\to x_0} \dfrac{n(x)}{g(x)} \lim\limits_{x\to x_0} \dfrac{m(x)}{n(x)} = \lim\limits_{x\to x_0} \dfrac{m(x)}{n(x)}$.

   (c) $\frac{1}{9}$.

3. $\ln' x = \lim\limits_{h\to 0} \dfrac{\ln(x+h) - \ln h}{h} = \lim\limits_{h\to 0} \dfrac{\ln \frac{x+h}{x}}{h} = \lim\limits_{h\to 0} \dfrac{\ln\left(1+\frac{h}{x}\right)}{h} = \left[\ln\left(1+\dfrac{h}{x}\right) \sim \dfrac{h}{x}\right] = \dfrac{1}{x}$.

4. Neprekidnost: $a - 1 = b$. Diferencijabilnost: $-3 = 2b$. Dakle, $a = -\dfrac{1}{2},\ b = -\dfrac{3}{2}$.

5. $y = shx$

   $(arshy)' = \dfrac{1}{(shx)'|_{x=arshy}} = \dfrac{1}{chx|_{x=arshy}} = \dfrac{1}{\sqrt{1+sh^2x}|_{x=arshy}} = \dfrac{1}{\sqrt{1+y^2}}$.

6. $y' = \dfrac{2t - e^t}{1+2e^{2t}} \Rightarrow y'|_{T(t=0)} = -\dfrac{1}{3}$.

   $x(0) = 1,\ y(0) = -1 \Rightarrow \text{t} \ldots y + 1 = -\dfrac{1}{3}(x-1)$.

   $\text{t} \ldots y = -\dfrac{1}{3}x - \dfrac{2}{3}$.

7. a) $chx = 1 + \dfrac{x^2}{2} + \dfrac{x^4}{24} + \dfrac{ch(x_1)}{720}x^6$ za neki $x_1 \in \langle 0, x\rangle$ ili $x_1 \in \langle x, 0\rangle$.

   $b)|R_5(1)| < \frac{1}{720} < 10^{-2}$

8. $\lim\limits_{x\to\infty} x \cdot arcctg(2x) = (\infty \cdot 0) = \lim\limits_{x\to\infty} \dfrac{arcctg(2x)}{\frac{1}{x}} = [L'H] \ldots = \dfrac{1}{2}$.