

Derivacije - dodatni zadatci za vježbu (rješenja zadataka 58.-75.)

58.

$$f'(x) = \frac{-\operatorname{sgn}(x)}{x \cdot \sqrt{x^2 + 1}}$$
$$f''(x) = \operatorname{sgn}(x) \frac{2x^2 + 1}{(x^4 + x^2) \cdot \sqrt{x^2 + 1}}$$

59.

$$f''(-2) = \frac{2 - \frac{7\pi}{3\sqrt{3}}}{12}$$

60.

$$f^{(n)}(x) = \frac{(-1)^{n+1}(n-1)! \cdot a^n}{(ax+b)^n}$$

(dokaz indukcijom)

61.

$$x \cdot y' + y + 2xe^y + x^2 \cdot e^y \cdot y' = 0$$

$$y'|_T = -1$$

62.

$$2x \cdot y^3 + x^2 \cdot 3y^2 \cdot y' + 3x^2 \cdot y^2 + x^3 \cdot 2y \cdot y' = 0$$

$$y'|_T = -1$$

$$t... y = -x + 2$$

63.

$$\ln y + x \cdot \frac{1}{y} \cdot y' + 2xy + x^2 y' = 0$$

$$y' = \frac{-y \ln y - 2xy^2}{x + x^2 y}$$

64.

$$y' = \frac{-4x^3y - y^4}{x^4 + 4xy^3}$$

65.

$$e^{y \ln x} + e^{x \ln y} = 2$$

$$e^{y \ln x} \left(y' \ln x + \frac{y}{x} \right) + e^{x \ln y} (\ln y + x \cdot \frac{y'}{y}) = 0$$

itd.

66.

$$y^3 + x \cdot 3y^2 \cdot y' + 2(3x^2 \cdot y + x^3 \cdot y') = 0$$

u točki $T(1, 1)$:

$$1 + 3y'|_T + 6 + 2y'|_T = 0$$

$$y'|_T = -\frac{7}{5}$$

$$3y^2 \cdot y' + 3(y^2 \cdot y' + x \cdot 2y \cdot (y')^2 + x \cdot y^2 \cdot y'') + 6(2x \cdot y + x^2 \cdot y') + 2(3x^2 \cdot y' + x^3 \cdot y'') = 0$$

u točki $T(1, 1)$:

$$-\frac{21}{5} + 3\left(-\frac{7}{5} + \frac{98}{25} + y''|_T\right) + 6\left(2 - \frac{7}{5}\right) + 2\left(-\frac{21}{5} + y''|_T\right) = 0$$

odakle računamo $y''|_T$.

68.

$$y' = \frac{x - y}{x + y}$$

$$y'' = 2 \frac{y^2 + 2xy - y^2}{(x + y)^3}$$

69.

$$y' = \frac{\sin t}{1 - \cos t}$$

$$y'|_{T(t=\frac{\pi}{4})} = \frac{\sqrt{2}}{2 - \sqrt{2}}$$

$$t... y - 1 + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2 - \sqrt{2}} \left(x - \frac{\pi}{4} + \frac{\sqrt{2}}{2} \right)$$

70.

$$y' = -\operatorname{tgt}$$

$$y'|_{T(t=\frac{\pi}{6})} = \frac{1}{\sqrt{3}}$$

$$t... y = \frac{x}{\sqrt{3}} - \frac{1}{2}$$

71.

$$y = x + 1, y' = 1$$

ili

$$y' = \frac{2\operatorname{cht} \cdot \operatorname{sht}}{2\operatorname{sht} \cdot \operatorname{cht}} = 1$$

(navedeni je zadatak nije baš “sretno zadan”)

72.

$$y' = \frac{2e^{2t} + e^t}{1 + e^t}$$

73.

$$y' = \frac{3t^2 + 1}{1 + \frac{1}{t}}$$

74.

$$y' = -\operatorname{tgt}$$

$$y'' = \frac{-\frac{1}{\cos^2 t}}{3 \cos^2 t \cdot (-\sin t)} = \frac{1}{3 \cos^4 t \cdot \sin t}$$

75. sličan zadatku 18. iz d.z. (točno rješenje) i zad. 29. iz dodatnih zadataka (pazi na ispravljeno rješenje).

(zamijenjeni su x i y u odnosu na navedeni zadatak, ali način rješavanja je isti)