Rješenja 3. školske zadaće

Grupa A:

1. Neka su preostala dva vrha trapeza točke (x_0, y_0) i $(-x_0, y_0)$. Tada je površina

$$P = \frac{(4+2x_0)\cdot y_0}{2} = (2+x_0)(4-x_0^2) = -x_0^3 - 2x_0^2 + 4x_0 + 8.$$

Ekstrem funkcije $P(x) = -x^3 - 2x^2 + 4x + 8$ je u točki za koju je $0 = P'(x) = -3x^2 - 4x + 4$, a budući da mora vrijediti $x > 0$ to je točka $x_0 = \frac{2}{3}$, tj. $(x_0, y_0) = (\frac{2}{3}, \frac{32}{9})$.

2. $f(x) = e^{-\frac{1}{x}} \Rightarrow D_f = \mathbb{R} \setminus \{0\}$.

$$\lim_{x\to 0^-} f(x) = +\infty, \lim_{x\to 0^+} f(x) = 0, \lim_{x\to +\infty} f(x) = \lim_{x\to -\infty} f(x) = e^0 = 1.$$

$f'(x) = \frac{1}{x^2} \cdot e^{-\frac{1}{x}} \Rightarrow f(x) > 0$ za sve x, tj. f je svugdje strogo rastuća. (Graf je na 3. stranici).

3.

$$\int \frac{\arcsin\sqrt{x}}{\sqrt{x}}dx = \left\{ \begin{array}{l} u = \sqrt{x} \\ du = \frac{1}{2\sqrt{x}}dx \end{array} \right\} = 2\int \arcsin u du = (*) = 2u\arcsin u + 2\sqrt{1-u^2} + C =$$

$$= 2\sqrt{x}\arcsin\sqrt{x} + 2\sqrt{1-x} + C.$$

$(*)$=za detalje vidjeti knjižicu 11.

4.

$$\int \frac{dx}{x(\ln^2 x - \ln^3 x)} = \left\{ \begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \end{array} \right\} = \int \frac{du}{u^2 - u^3} = \int \frac{du}{u^2(1-u)} = \text{(rastav na parc.razlomke)} =$$

$$\int (\frac{1}{u} + \frac{1}{u^2} + \frac{1}{1-u})du = \ln u - \frac{1}{u} - \ln(1-u) + C = \ln\ln x - \frac{1}{\ln x} - \ln(1-\ln x) + C.$$

Grupa B:

1. Neka su preostala dva vrha trapeza točke (x_0, y_0) i $(-x_0, y_0)$. Tada je površina

$$P = \frac{(2+2x_0)\cdot y_0}{2} = (1+x_0)(2-2x_0^2) = -2x_0^3 - 2x_0^2 + 2x_0 + 2.$$

Ekstrem funkcije $P(x) = -2x^3 - 2x^2 + 2x + 2$ je u točki za koju je $0 = P'(x) = -6x^2 - 4x + 2$, a budući da mora vrijediti $x > 0$ to je točka $x_0 = \frac{1}{3}$, tj. $(x_0, y_0) = (\frac{1}{3}, \frac{16}{9})$.

2. $f(x) = e^{\frac{1}{x-1}} \Rightarrow D_f = \mathbb{R} \setminus \{1\}$.

$$\lim_{x\to 1^-} f(x) = 0, \lim_{x\to 1^+} f(x) = +\infty, \lim_{x\to +\infty} f(x) = \lim_{x\to -\infty} f(x) = e^0 = 1.$$

$f'(x) = \frac{-1}{(x-1)^2}\cdot e^{\frac{1}{x-1}} \Rightarrow f(x) < 0$ za sve x, tj. f je svugdje strogo padajuća. (Graf je na 3.stranici).

3.

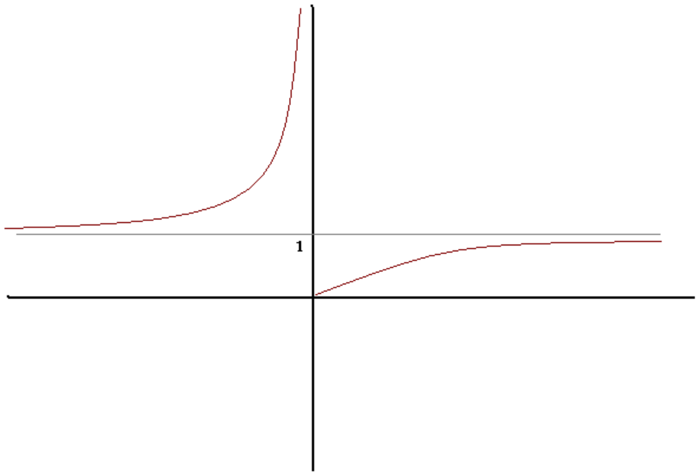
$$\int x \operatorname{arcctg} x dx = \left\{ \begin{array}{l} u = \operatorname{arcctg} x \\ du = \frac{-1}{x^2+1}dx \\ dv = xdx \\ v = \frac{x^2}{2} \end{array} \right\} = \frac{x^2}{2}\operatorname{arcctg} x + \int \frac{1}{2}x^2\frac{1}{x^2+1}dx =$$

$$= \frac{x^2}{2}\operatorname{arcctg} x + \frac{1}{2}\int (1 - \frac{1}{x^2+1})dx = \frac{x^2}{2}\operatorname{arcctg} x + \frac{x}{2} + \frac{1}{2}\operatorname{arcctg} x + C.$$

4.

$$\int \frac{dx}{(1+x^2)(\operatorname{arctg}^2 x + \operatorname{arctg}^3 x)} = \left\{ \begin{array}{l} u = \operatorname{arctg} x \\ du = \frac{1}{(1+x^2)}dx \end{array} \right\} = \int \frac{du}{u^2(1+u)} = (\text{rastav na parc.razlomke}) =$$

$$= \int (\frac{-1}{u} + \frac{1}{u^2} + \frac{1}{1+u})du = -\ln u - \frac{1}{u} + \ln(1+u) + C = -\ln(\operatorname{arctg} x) - \frac{1}{\operatorname{arctg} x} + \ln(1+\operatorname{arctg} x) + C.$$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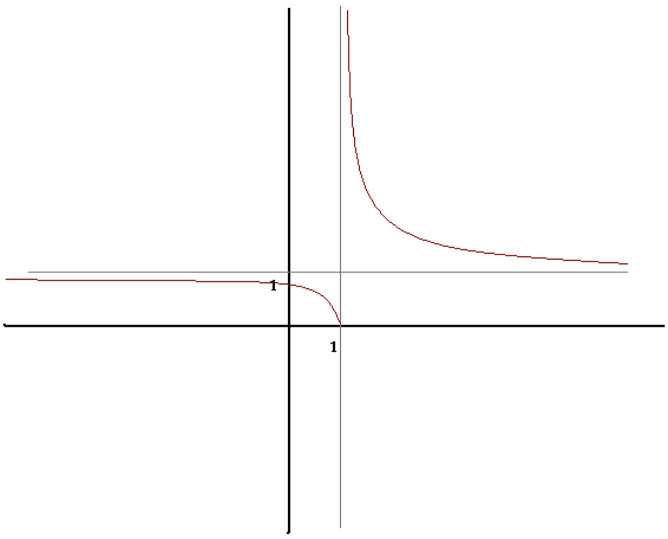
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