The Coverage Model – Towards high precision image analysis

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Motivation and context

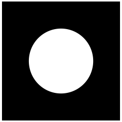
- The task of **Image Analysis** is to **extract relevant information from images**.
- **Numerical descriptors**, such as area, perimeter, and moments of objects are often of interest, for the tasks of shape analysis, classification, etc.

The standard image analysis task (and its solution)

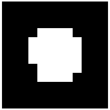
1. Sample preparation and Imaging
2. Pre-processing (optional)
3. Segmentation
   - Usually crisp
4. Feature extraction
   - Discrete representation problematic
5. Classification

Motivation - feature estimation

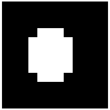
Crisp discrete object representations, especially at low spatial resolutions, put strong limitations to the precision of estimated features:



Area: 28.274
Perim: 18.850



Area: 31.000
Perim: 19.422



Area: 26.000
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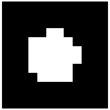

Area: 28.000
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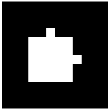

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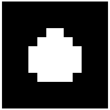

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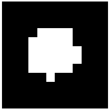


Area: 27.000
Perim: 18.867

Motivation - feature extraction

Unrealistically low resolution?

- Low resolution will always be a challenge; the more powerful imaging devices, the smaller objects are of interest!
- Partial Volume Effect in high resolution 3D images; consistent small displacement of object boundaries leads to significant errors of feature estimates.

Approach based on fuzzy sets

A fuzzy set of a reference set is a set of ordered pairs

$$F = \{\langle x, \mu_F(x)\rangle \mid x \in X\},$$

where the membership function $\mu_F : X \to [0, 1]$ indicates, for each element $x \in X$, to what extent it belongs to the fuzzy set F.

Observations:

- A fuzzy set is defined/identified by its membership function
- A crisp set is a special case of fuzzy set, where membership function takes only two values, 0 and 1.

The Fuzzy Approach

- **Do not throw away information by making crisp decisions.**
- The more nuanced view of a fuzzy approach allows **preservation of more information**.
- A representation based on **fuzzy sets** can provide numerical descriptors with **higher precision** than what can be achieved from a crisp representation.

The Fuzzy Approach
Important observations

- The original grey levels are, in general, **not directly used** as fuzzy representations.
- Meaning of grey levels in a fuzzy segmented image is always a consequence of many criteria, many of them only implicitly present – **fuzzy segmentation methods**.

- Instead of dependence on the properties of imaging devices, we are dependent on criteria used for fuzzy segmentation.
- Interpretation of results can become rather difficult.

The Coverage Model

- Keep good sides of fuzzy; stay close to the digital image, high information content, soft boundaries, robustness.
- Restrict to **one single meaning of memberships**; clear unique interpretation, enabling stronger theoretical results.

The Coverage Model

- **Let the (membership) value of an image element be equal to its relative coverage by the image object.**

- A representation close to the original (discrete) image data.
- Based on very weak assumptions about the imaged objects.
- Utilizing the coverage information, significant improvement in precision of extracted feature values can be reached.

Pixel coverage digitization

Let a square grid in 2D be given. The Voronoi region associated to a grid point $(i,j) \in \mathbb{Z}^2$ is called pixel $p_{(i,j)}$.

Definition (non-quantized case)

For a given continuous object $S \subset \mathbb{R}^2$, inscribed into an integer grid with pixels $p_{(i,j)}$, the *pixel coverage digitization* of S is

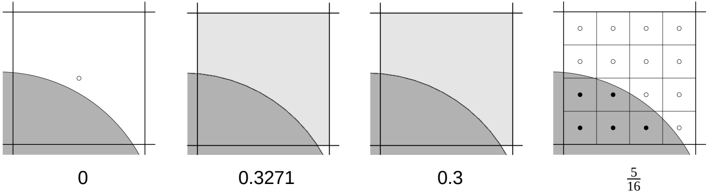
$$\mathcal{D}(S) = \left\{ \left((i,j), \frac{A(p_{(i,j)} \cap S)}{A(p_{(i,j)})} \right) \middle| (i,j) \in \mathbb{Z}^2 \right\} ,$$

where $A(X)$ denotes the area of a set X.

Digital images $\rightarrow$ Quantized grey values

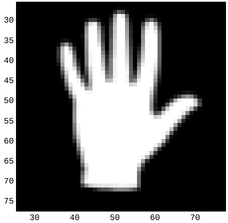
Pixel coverage digitization

The membership value of a pixel is
equal to its relative coverage by an imaged object (here a part of a disk).



- Gauss digitization assigns value 0 to the observed pixel.
- By coverage digitization assigned value is 0.3271.
- If 10-level quantized coverage digitization is used, assigned value is 0.3.
- If coverage is approximated by 4-sampled coverage digitization, assigned value is $\frac{5}{16}$.

Properties of coverage representations



Intuitively, a coverage representation of a crisp object, with a well defined continuous border, is characterized by the presence of **homogeneous connected regions of "pure" pixels**, completely covered by either object or background **separated by thin layers of "mixed" pixels**, i.e., those partially covered by both object and background.

If a crisp continuous set has a reasonably smooth boundary and is represented at a high enough resolution, then **the fuzzy border** of its digital coverage representation **is not more than one pixel thick**.

The Coverage Model

The image analysis task and its coverage solution

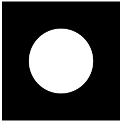
1. Sample preparation and Imaging
2. Pre-processing (optional)
3. **Segmentation**
   - Coverage segmentation (restricted freedom)
4. **Feature extraction**
   - Estimate features from the coverage representation
   - Easy to interpret the results, robustness and precision
5. Classification, statistical evaluation, . . .

New entries in the “standard chain of tasks” required:

- New segmentation methods that result in coverage representations;
- New analysis methods that can be applied to coverage object representations.

However, there is no need for new ways of interpreting and understanding the obtained results.

Feature extraction revisited



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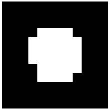

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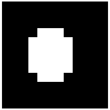

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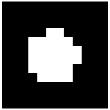

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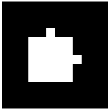

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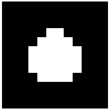

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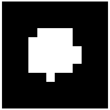

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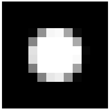
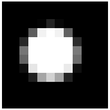


Area: 28.274
Perim: 18.428



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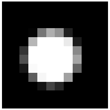
Perim: 18.684



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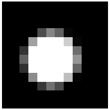

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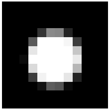

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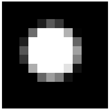


Area: 28.274
Perim: 18.692

Feature estimation - some general observations

- Our aim is to obtain information about continuous real objects, having available their discrete - **coverage** - representation.
- Different numerical descriptors, such as area, perimeter, moments, of the objects are often of interest, for the tasks of shape analysis, classification, etc.
- Estimators should be adjusted/designed so that they utilize in a best way information preserved in a coverage representation.

Some features that benefit from a pixel coverage representation.

Area and other geometric moments

- N. Sladoje and J. Lindblad. Estimation of Moments of Digitized Objects with Fuzzy Borders. ICIAP'05, LNCS-3617, pp. 188-195, Cagliari, Italy, Sept. 2005.

$$m_{p,q}(S) = \frac{1}{r^{p+q+2}}\,\tilde{m}(rS) + \mathcal{O}\left(\frac{1}{r\sqrt{n}}\right)$$

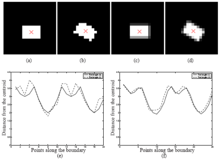
Perimeter and boundary length

- N. Sladoje and J. Lindblad. High Precision Boundary Length Estimation by Utilizing Gray-Level Information. IEEE Trans. on PAMI, Vol. 31, No. 2, pp. 357-363, 2009.

$$\gamma_n^{(0,q)} = \frac{2q}{q + \sqrt{(\sqrt{n^2+q^2} - n)^2 + q^2}}\,, \qquad |\varepsilon_n| = \mathcal{O}(n^{-2})$$

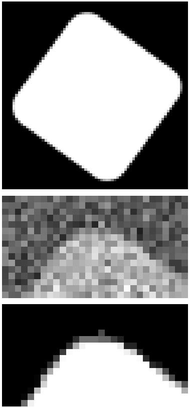
Signature

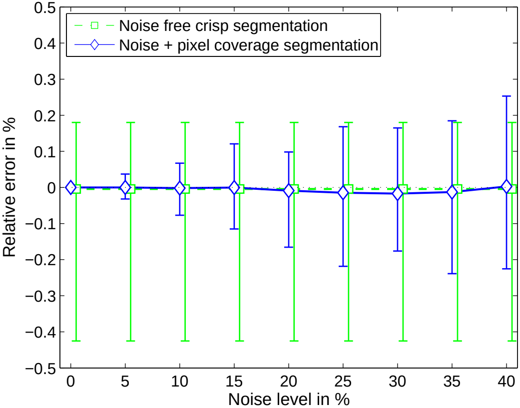
- J. Chanussot, I. Nyström and N. Sladoje, Shape signatures of fuzzy star-shaped sets based on distance from the centroid, Pattern Recognition Letters, vol. 26(6), pp. 735-746, 2005.



Area estimation error in a noisy environment

in combination with coverage segmentation

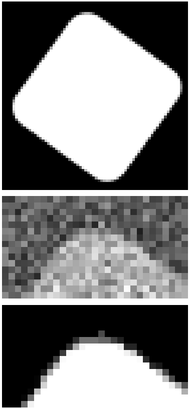


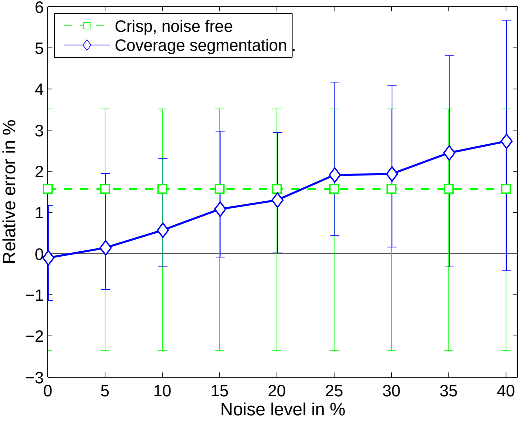


Left: (top) Synthetic test objects. (middle) Part of object with 30% noise added. (bottom) Coverage segmentation result for 30% noise. **Right:** Estimation errors for increasing levels of noise. Green is noise free crisp reference. Lines show averages for 50 observations and bars indicate max and min errors.

Perimeter estimation error in a noisy environment

in combination with coverage segmentation





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Conclusion

- A particular way to represent shapes/objects in images - **Coverage representation** - is presented.
- Main motivation to introduce it is to enable **increased precision** of image processing tasks, compared to the classical - crisp/binary - shape representation.
- Performance of the model, esp. in terms of **feature extraction**, is analysed.

What remains to be shown is:

- Ways how to obtain coverage representation - **coverage segmentation algorithms**.
- Examples of **applications** of the model.