

Skeletonization in 3D via thinning and its applications

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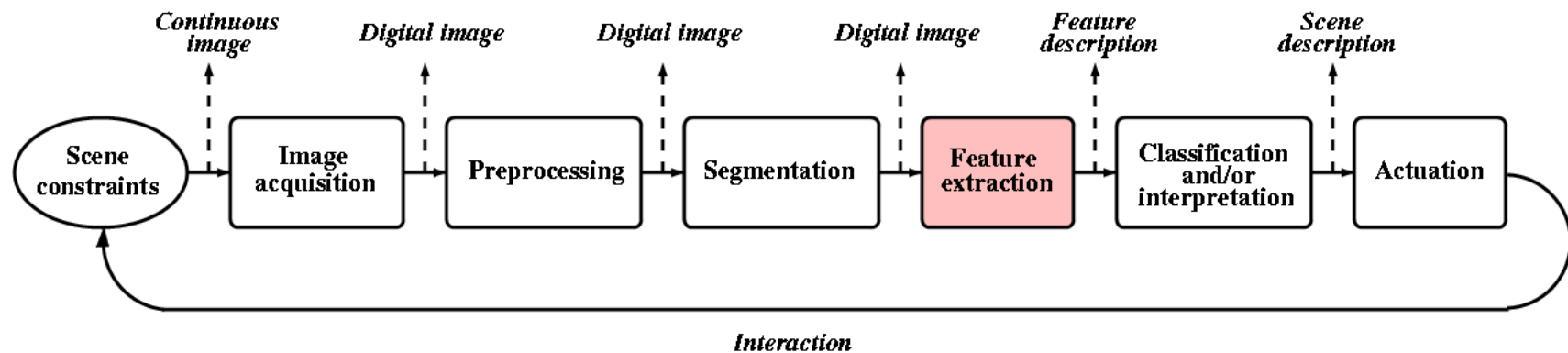




Syllabus

- Shape representation
- Skeleton-like shape features
- Skeletonization via thinning
- Thinning algorithms in 3D
- Applications

The generic model of a modular machine vision system



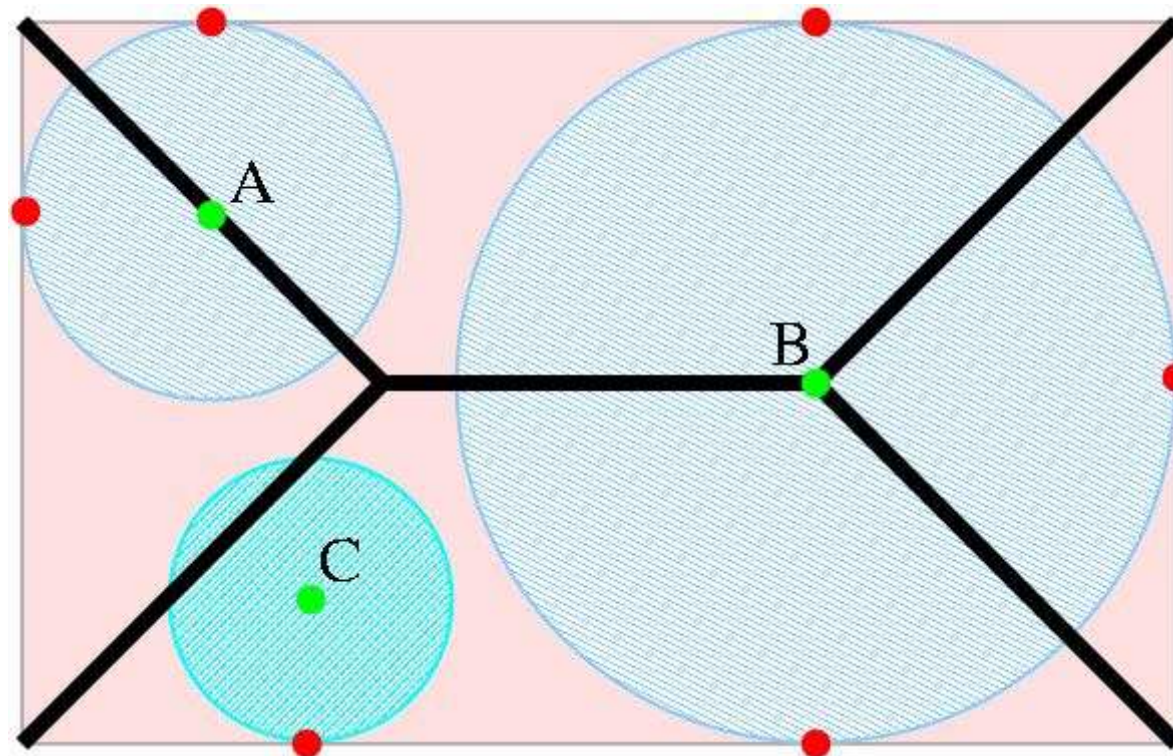
G.W. Awcock, R. Thomas (1996)

Skeleton

- result of the Medial Axis Transform: object points having at least two closest boundary points
- prairie-fire analogy: the boundary is set on fire and skeleton is formed by the loci where the fire fronts meet and quench each other
- the locus of the centers of all the maximal inscribed hyper-spheres

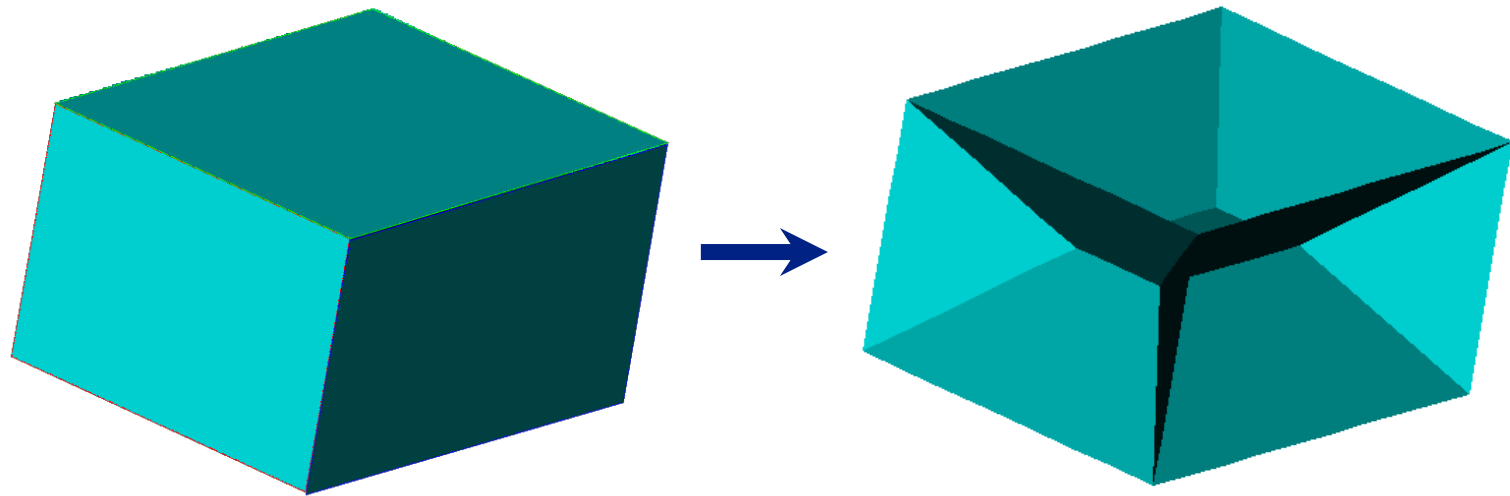


Skeleton in 2D





Skeleton in 3D





Skeleton

- represents
 - the general form of an object,
 - the topological structure of an object, and
 - local object symmetries.
- invariant to
 - translation,
 - rotation, and
 - (uniform) scale change.
- simplified and thin.



Skeleton-like shape features in 2D





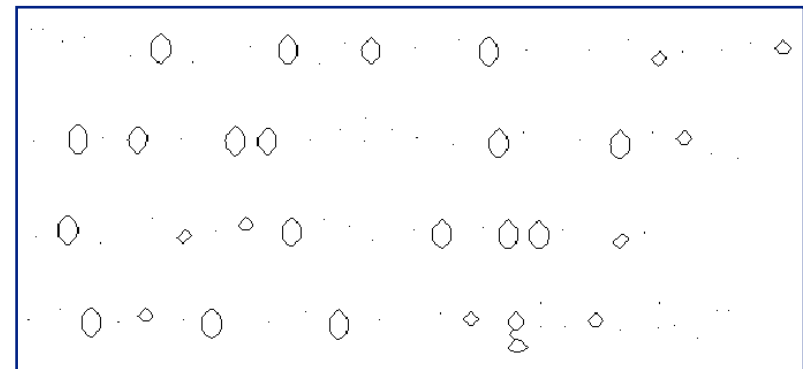
Skeleton-like shape features in 2D

**"If you would know what the
Lord God thinks of money,
you have only to look at
those to whom he gives it."**

original image

"If you would know what the
Lord God thinks of money,
you have only to look at
those to whom he gives it."

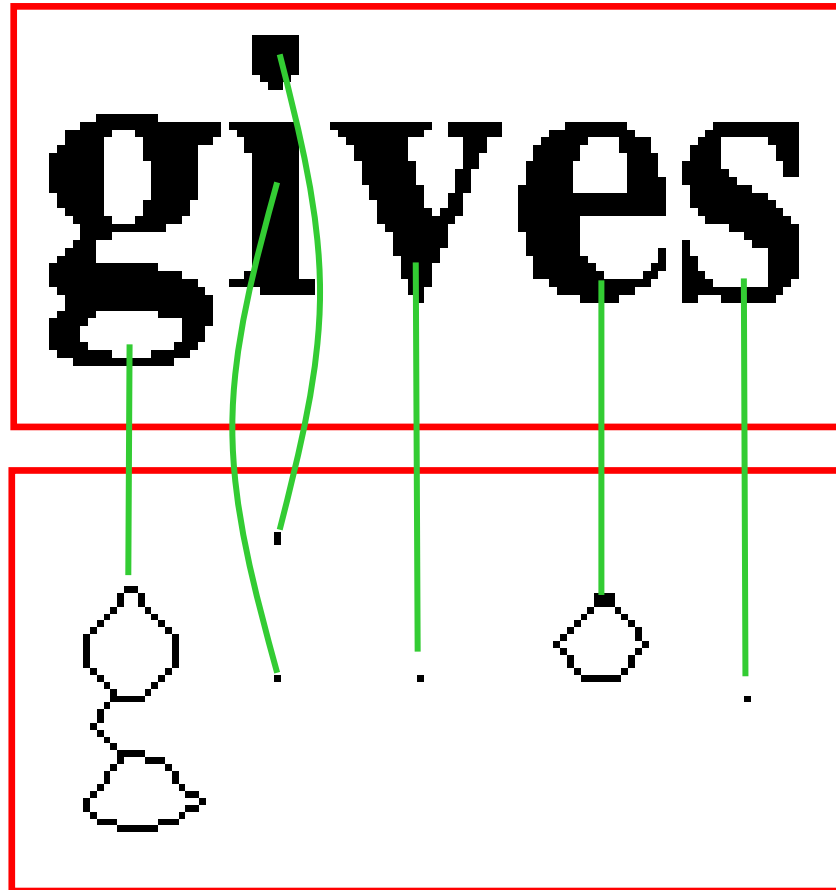
centerline



topological kernel

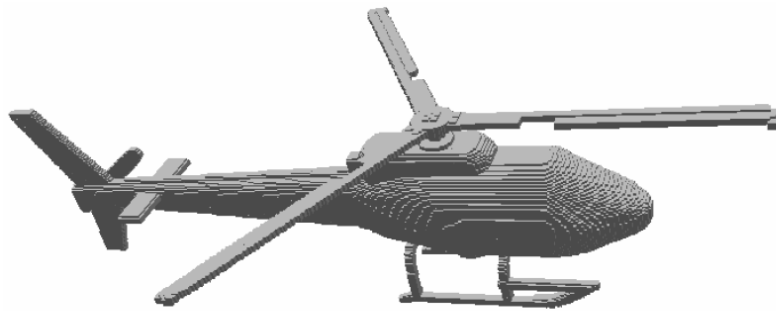


Skeleton-like shape features in 2D

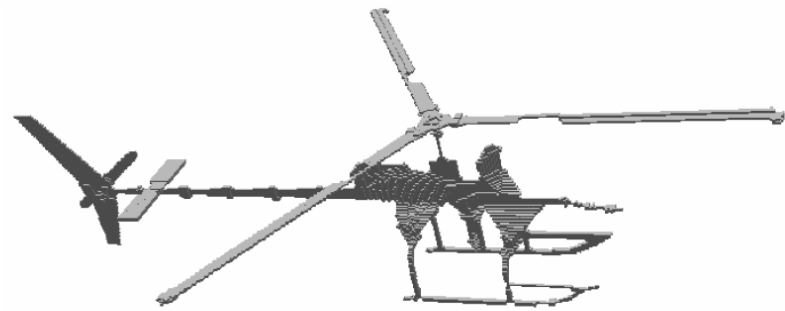


topological kernels
of objects
with/without cavities

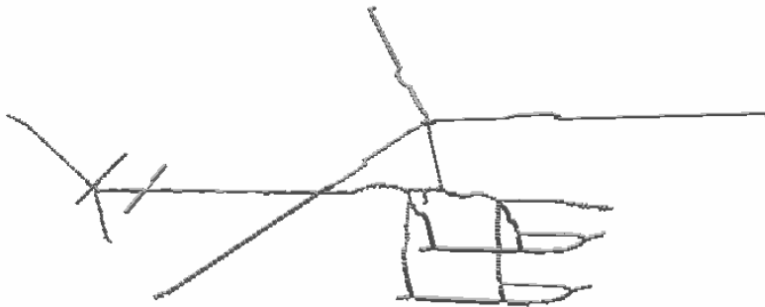
Skeleton-like shape features in 3D



original object



medial surface

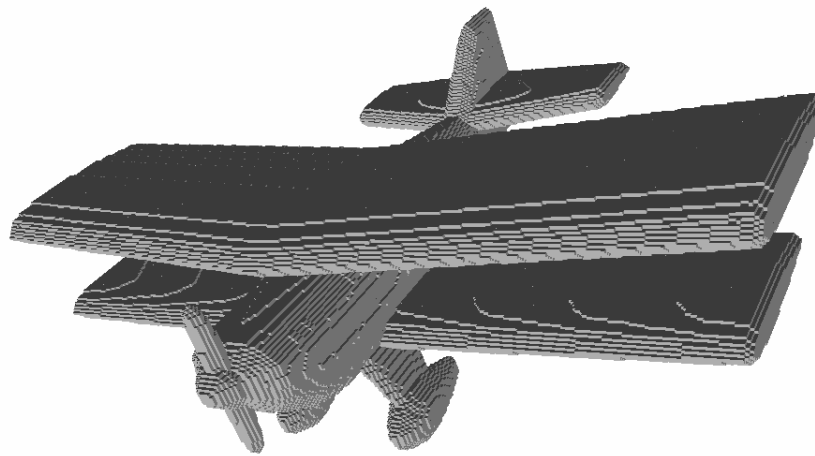


centerline

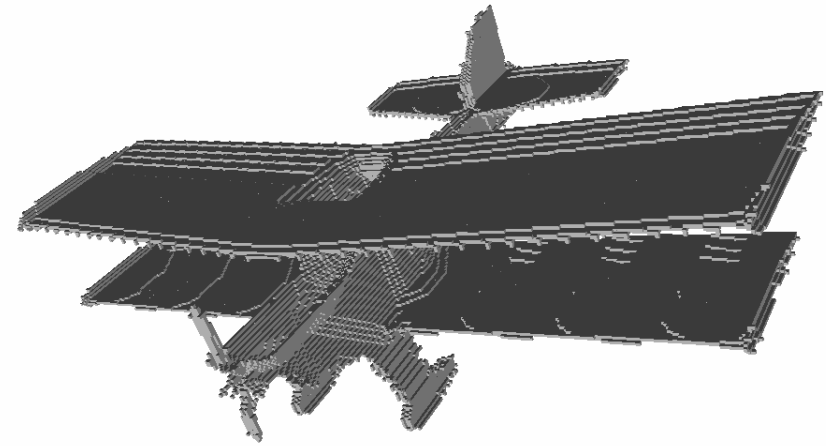


topological kernel

Skeleton-like shape features in 3D



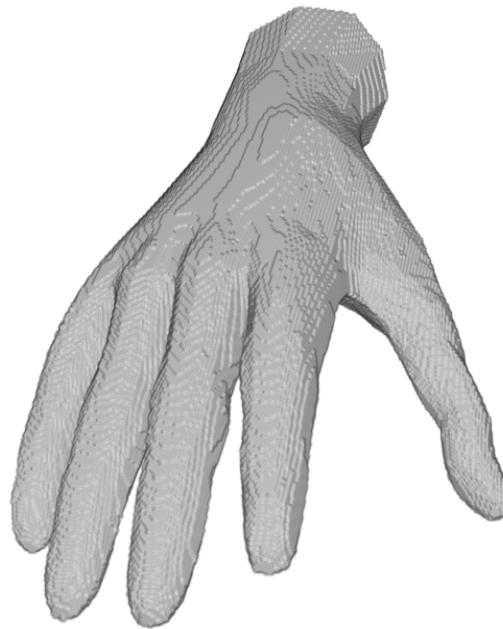
original object



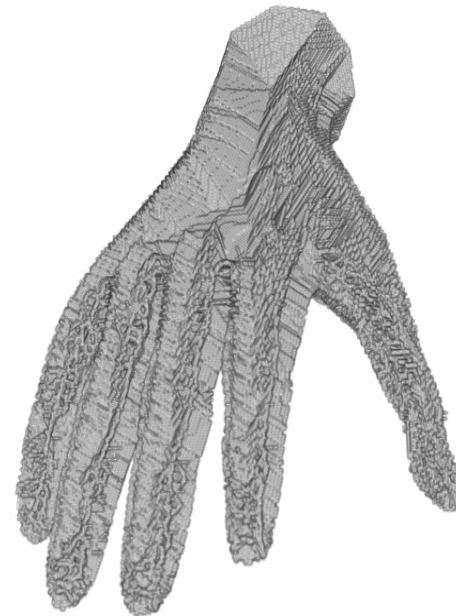
medial surface



Skeleton-like shape features in 3D

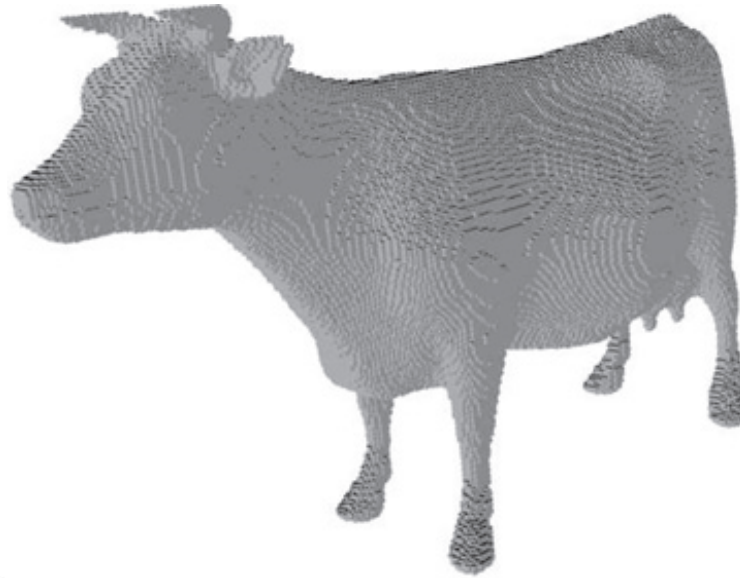


original object



medial surface

Skeleton-like shape features in 3D



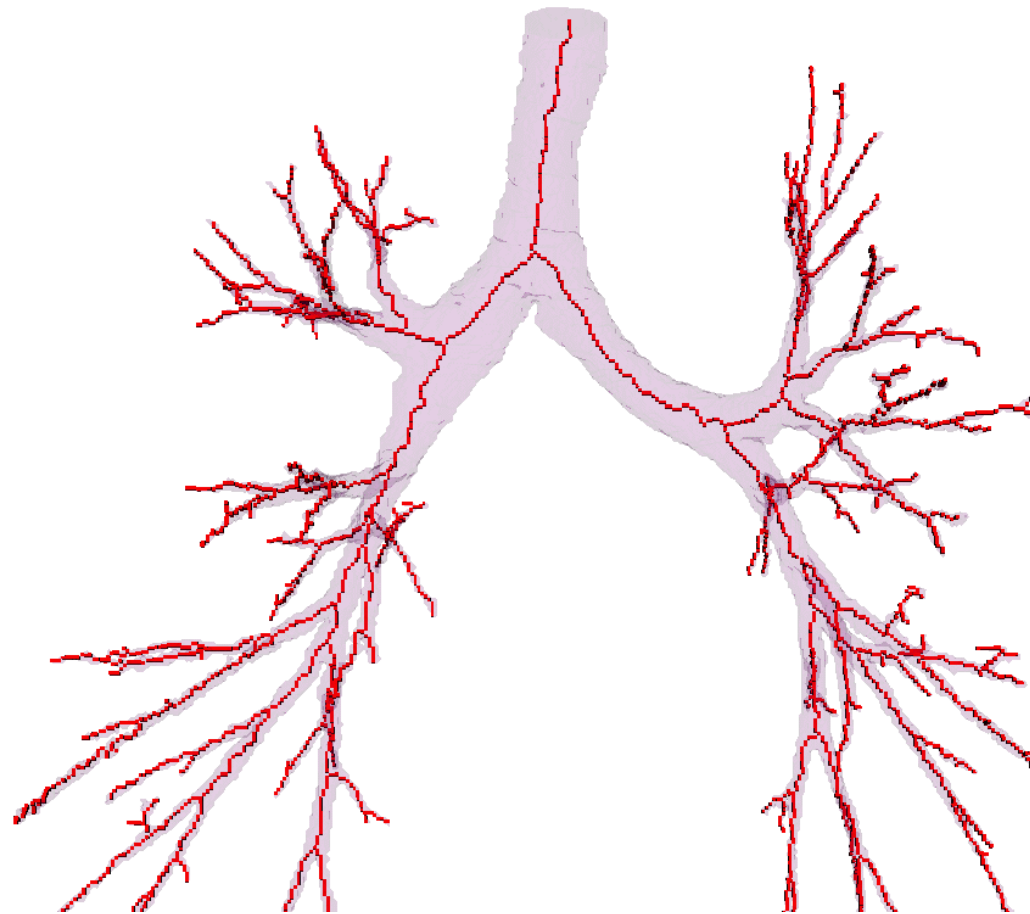
original object



centerline



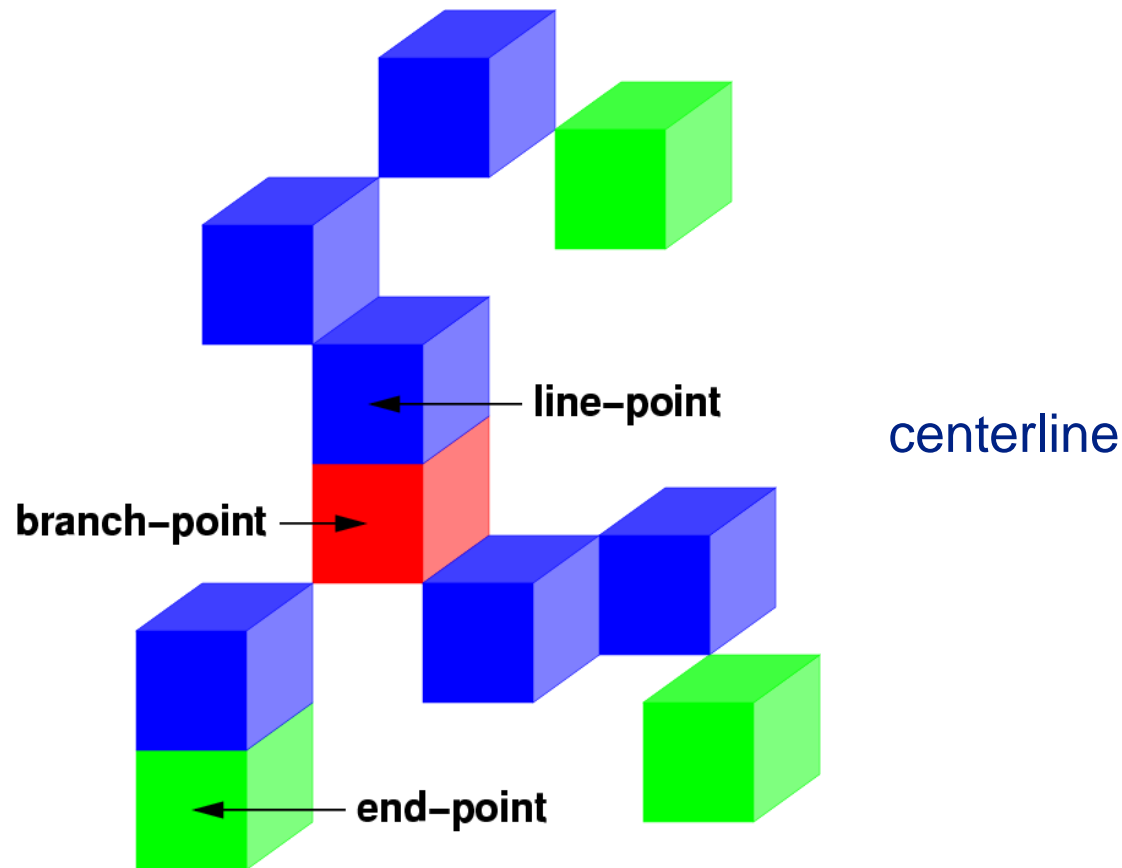
Skeleton-like shape features in 3D



centerline

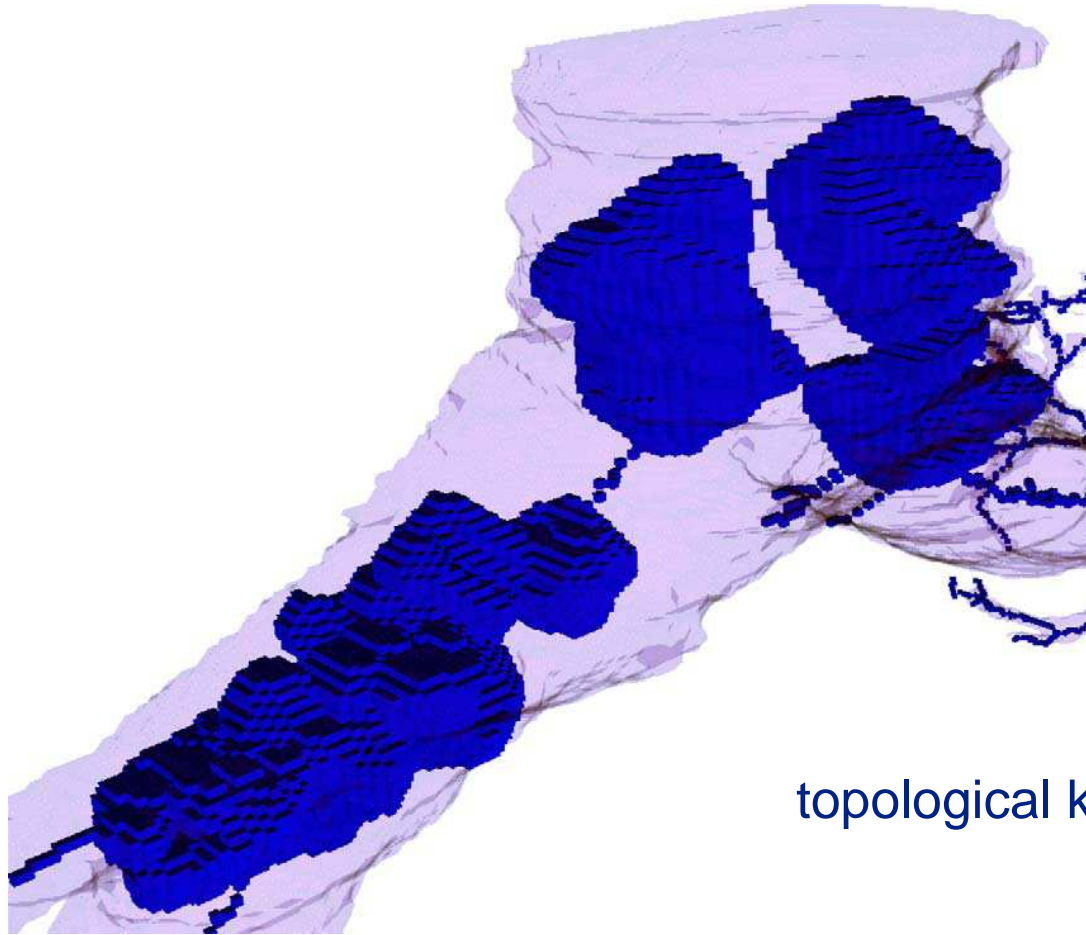


Skeleton-like shape features in 3D





Skeleton-like shape features in 3D



topological kernel



Skeletonization techniques

- Distance-based
- Voronoi-based
- **thinning**

Thinning

*Matryoshka:
Russian nesting wooden doll*



original
object



reduced
structure



Thinning algorithms

repeat

remove „*deletable*” points
from the actual binary image

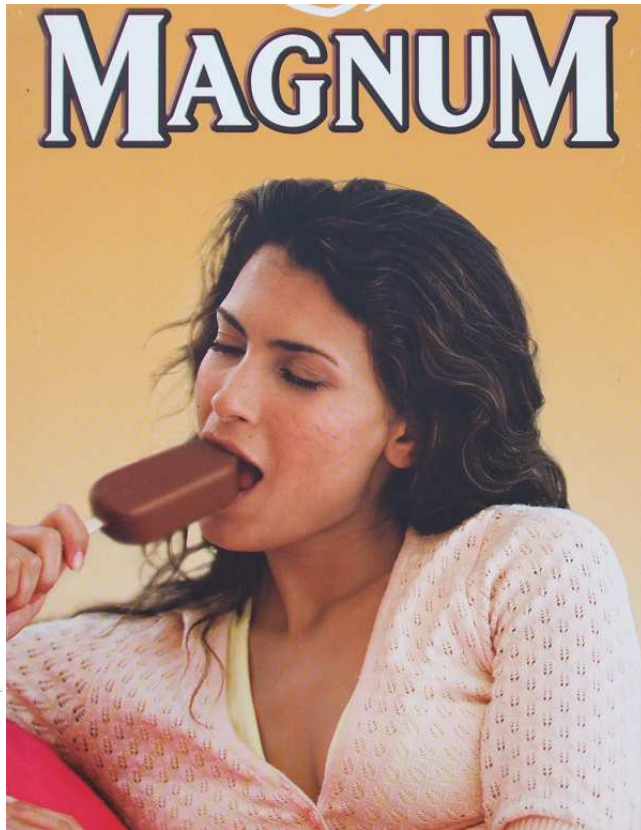
} one iteration step

until no points are deleted

degrees of freedom:

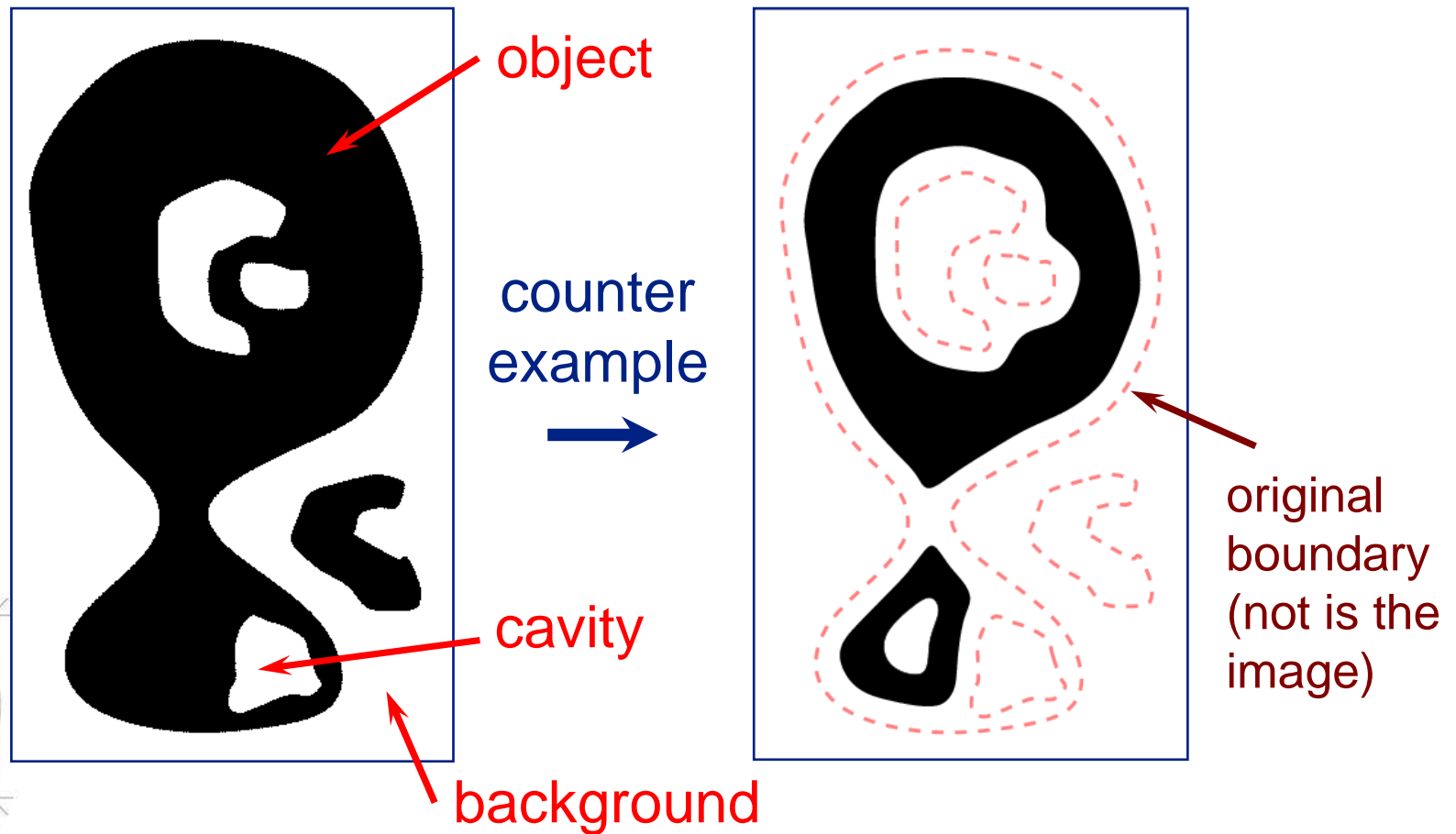
- which points are regarded as „*deletable*”?
- how to organize one iteration step?

Thinning



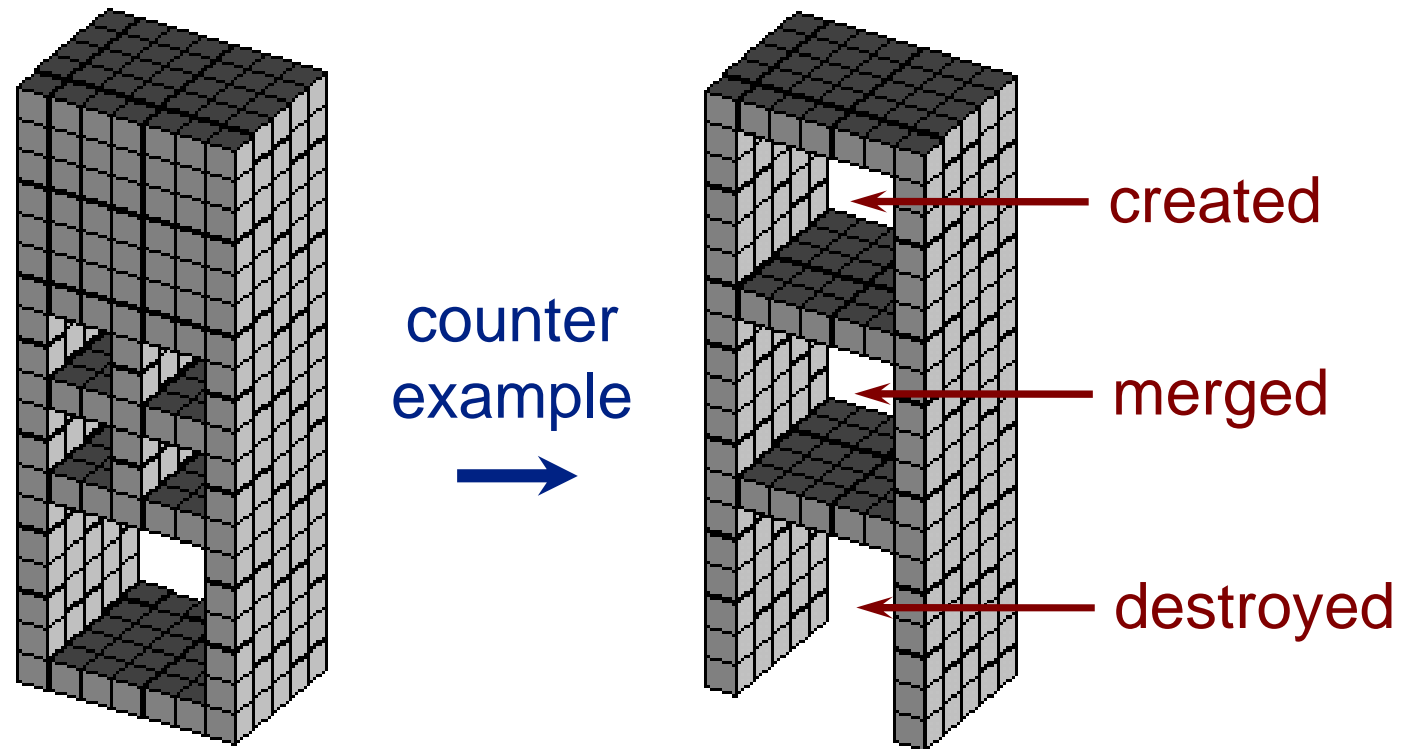
sequential and parallel approaches

Topology preservation in 2D



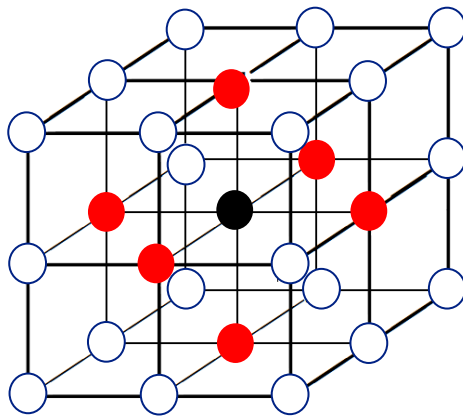


Topology preservation in 3D

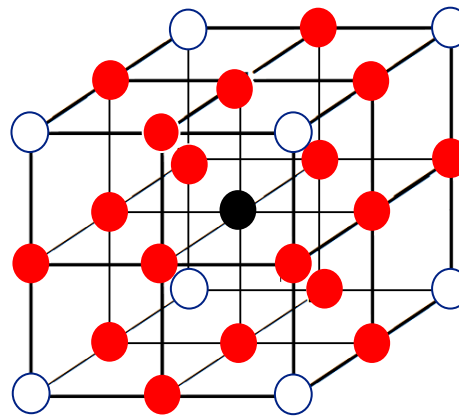


Some concepts

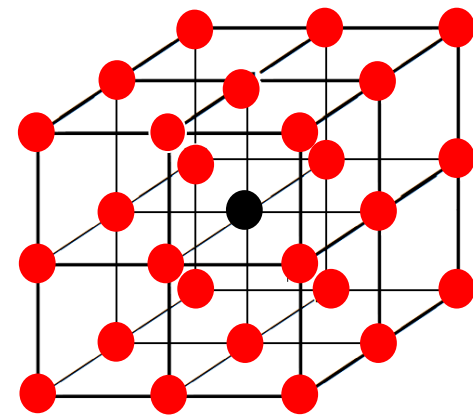
Adjacency relations in \mathbb{Z}^3



6-adjacency



18-adjacency



26-adjacency



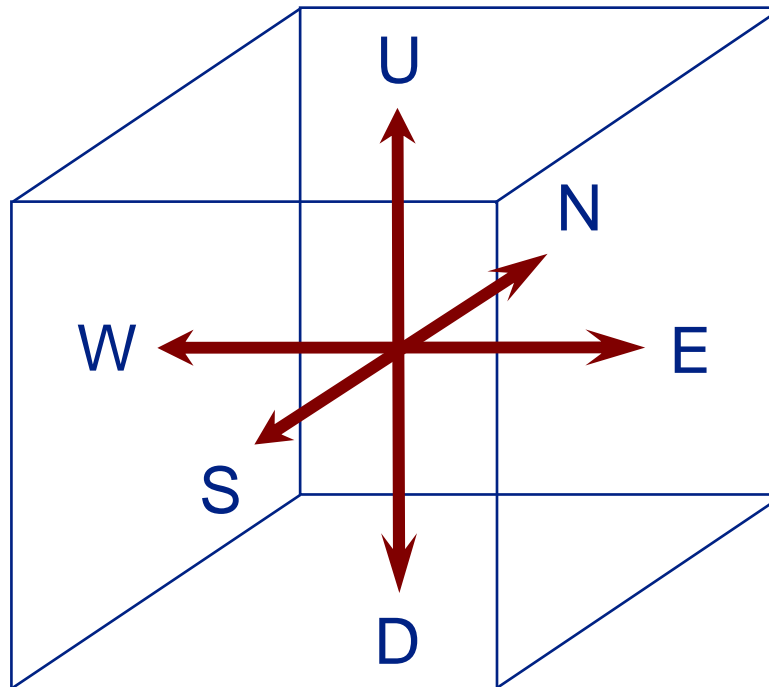
Some concepts

(26,6) binary digital picture: $(Z^3, 26, 6, B)$

- $p \in Z^3$: point/voxel
- B : set of black/object points
 $Z^3 \setminus B$: set of white points
- **object** : maximal 26-connected set of black points
- **background, cavity** : maximal 6-connected set of white points



Some concepts

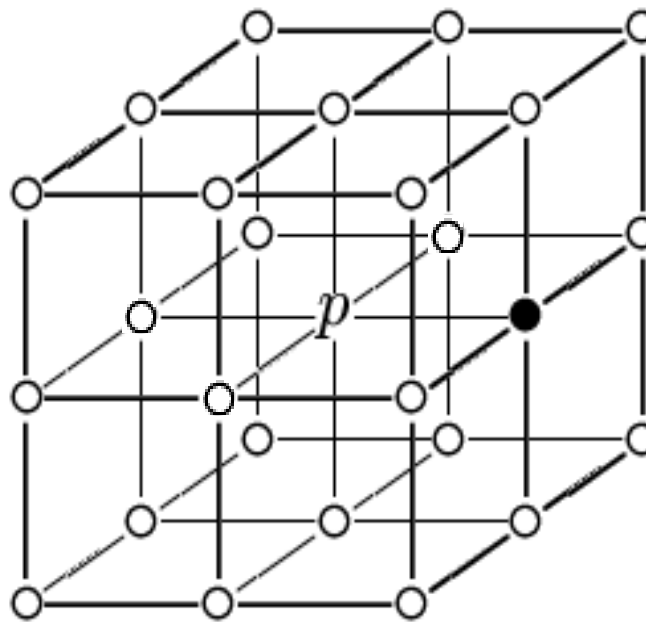


main directions in 3D



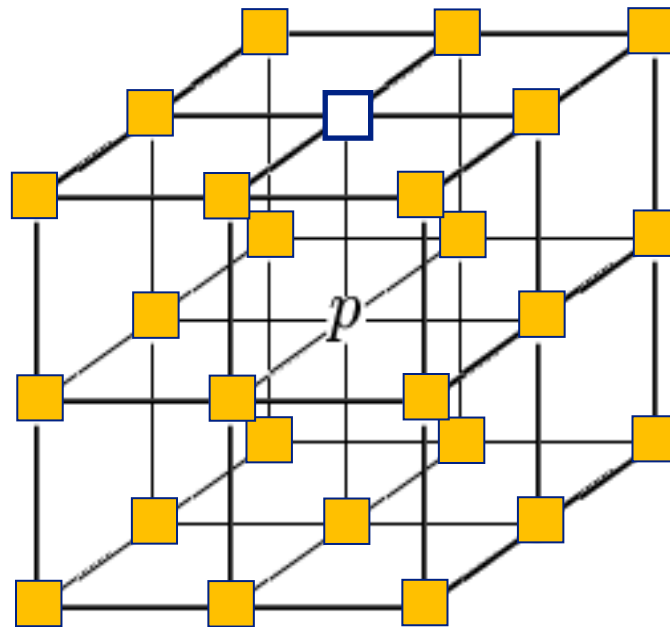
Some concepts

black point p is a line-end point
if it is adjacent to just one black point



Some concepts

black point p is a border point if it is 6-adjacent to at least one white point



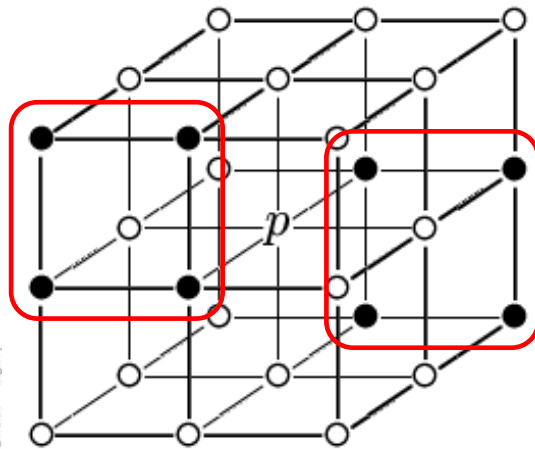
don't care
(either 0 or 1)

border point of type U

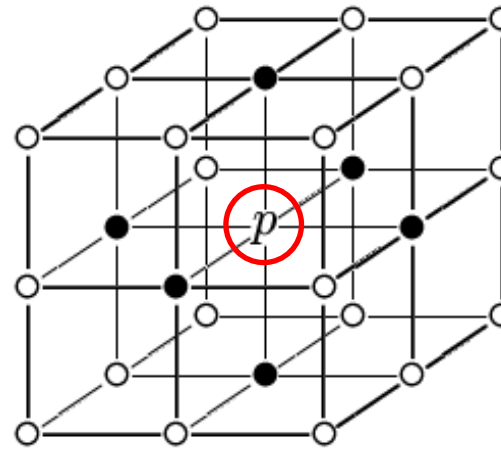
Some concepts

A black point is **simple** if its deletion preserves the topology of the picture.

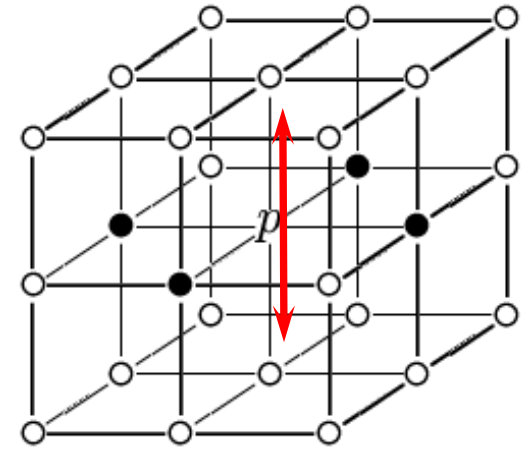
Examples of ***non***-simple points in 3D pictures:



splitting an object



creating a cavity



creating a hole



Some concepts

Simpleness is a local property:
It depends on the 3x3x3 neighborhood
of the point in question.



It can be decided by using a precalculated
LUT (look-up table) of size 8 Mbyte.

A 3D sequential curve-thinning algorithm

Palágyi et al. (2006)

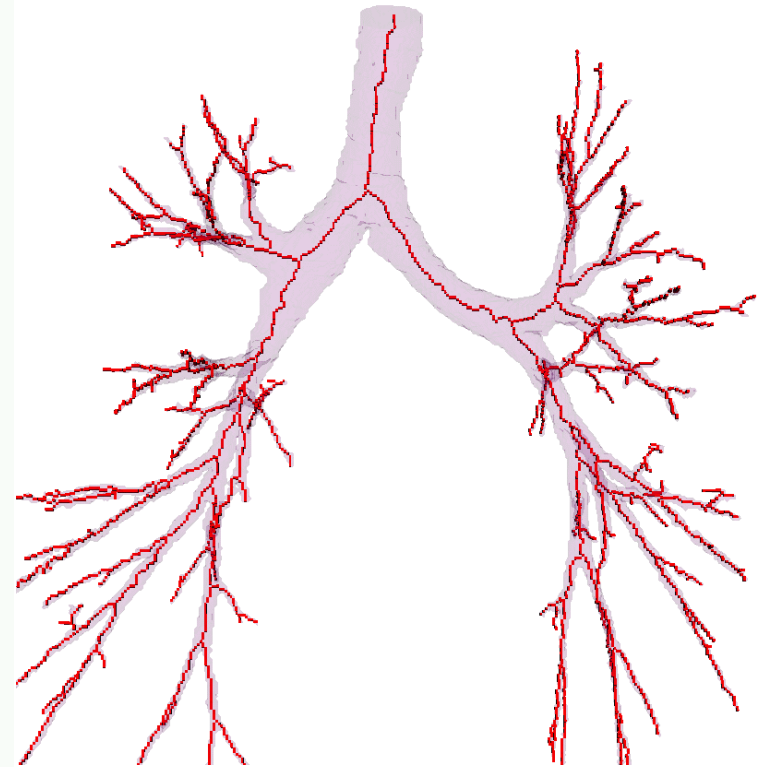
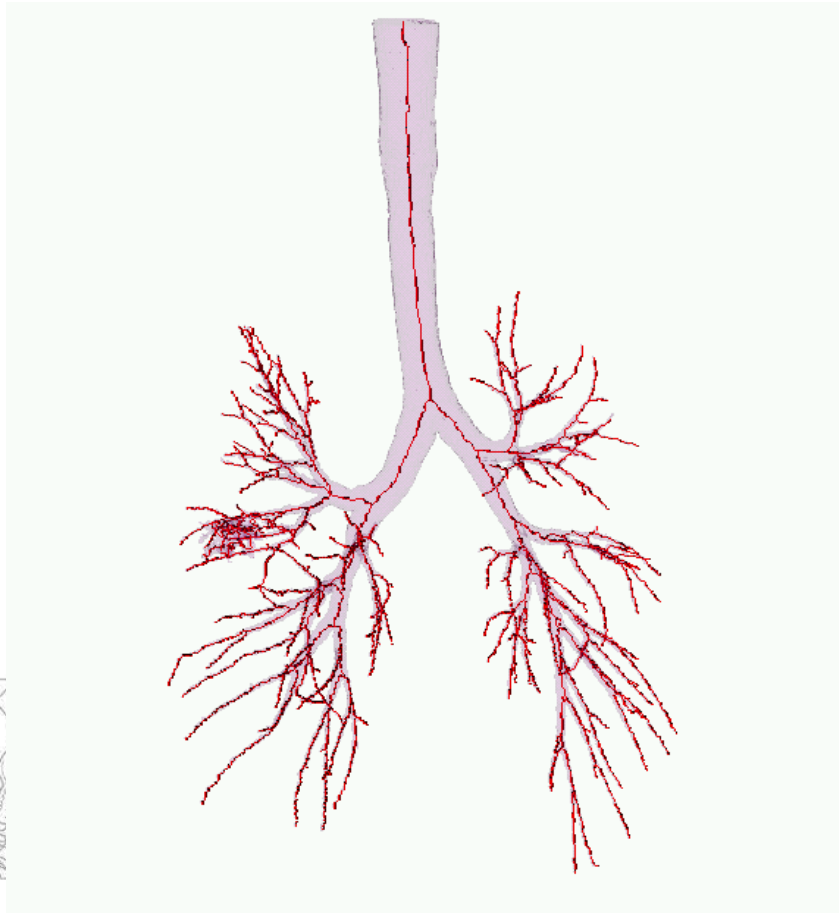
K. Palágyi, J. Tschirren, E.A. Hoffman, M. Sonka:
Quantitative analysis of pulmonary airway tree structures,
Computers in Biology and Medicine 36, 974-996, 2006.

A 3D sequential curve-thinning algorithm

```
repeat
  foreach direction  $d$  in {U,N,E,S,W,D} do
    mark black point  $p$  if it is a
      - border point of type  $d$ ,
      - not line-end point, and
      - simple point
    for each marked point  $q$  do
      delete  $q$  if it is
        - not a line-end point, and
        - simple point in the actual picture
until no points are deleted
```

Palágyi et al. (2006)

A 3D sequential curve-thinning algorithm



Palágyi et al. (2006)



A 3D parallel curve-thinning algorithm

Palágyi & Kuba (1998)

K. Palágyi, A. Kuba:

**A 3D 6-subiteration thinning algorithm for
extracting medial lines,**

Pattern Recognition Letters 19, 613-627, 1998.



A 3D parallel curve-thinning algorithm

repeat

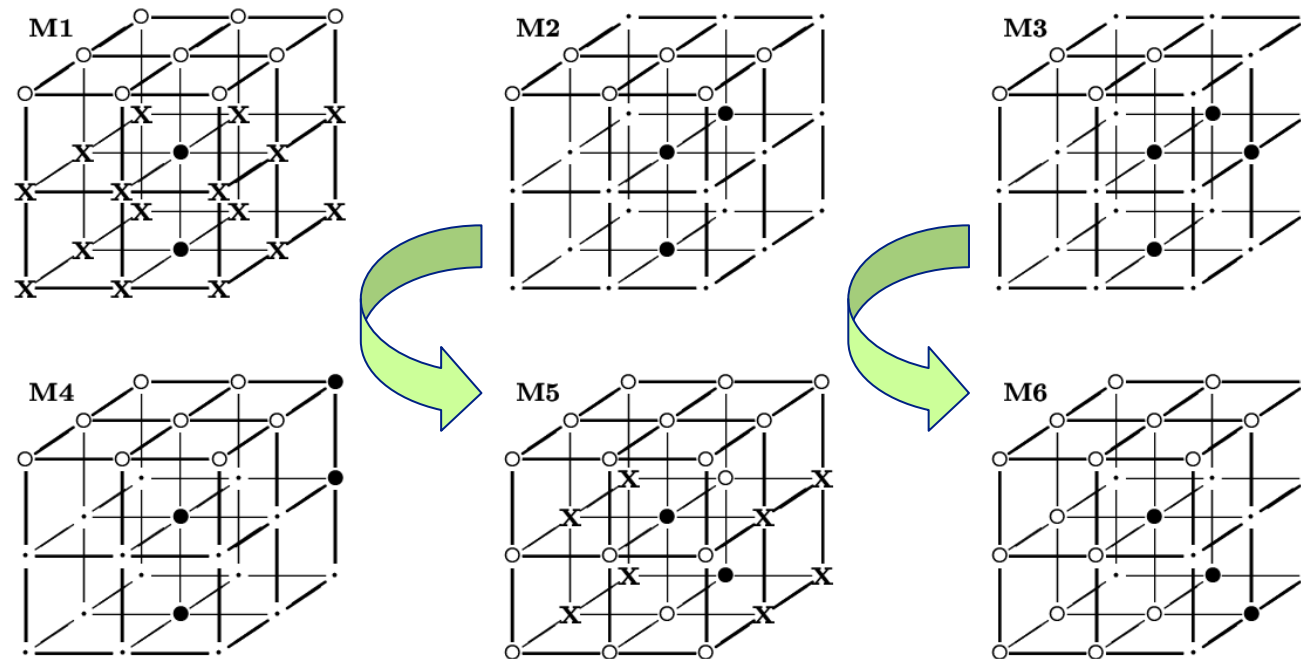
foreach direction d in $\{U, N, E, S, W, D\}$ **do**
simultaneous deletion of black points that
satisfy the deletion conditions assigned to
deletion direction d

until no points are deleted

Palágyi & Kuba (1998)



A 3D parallel curve-thinning algorithm

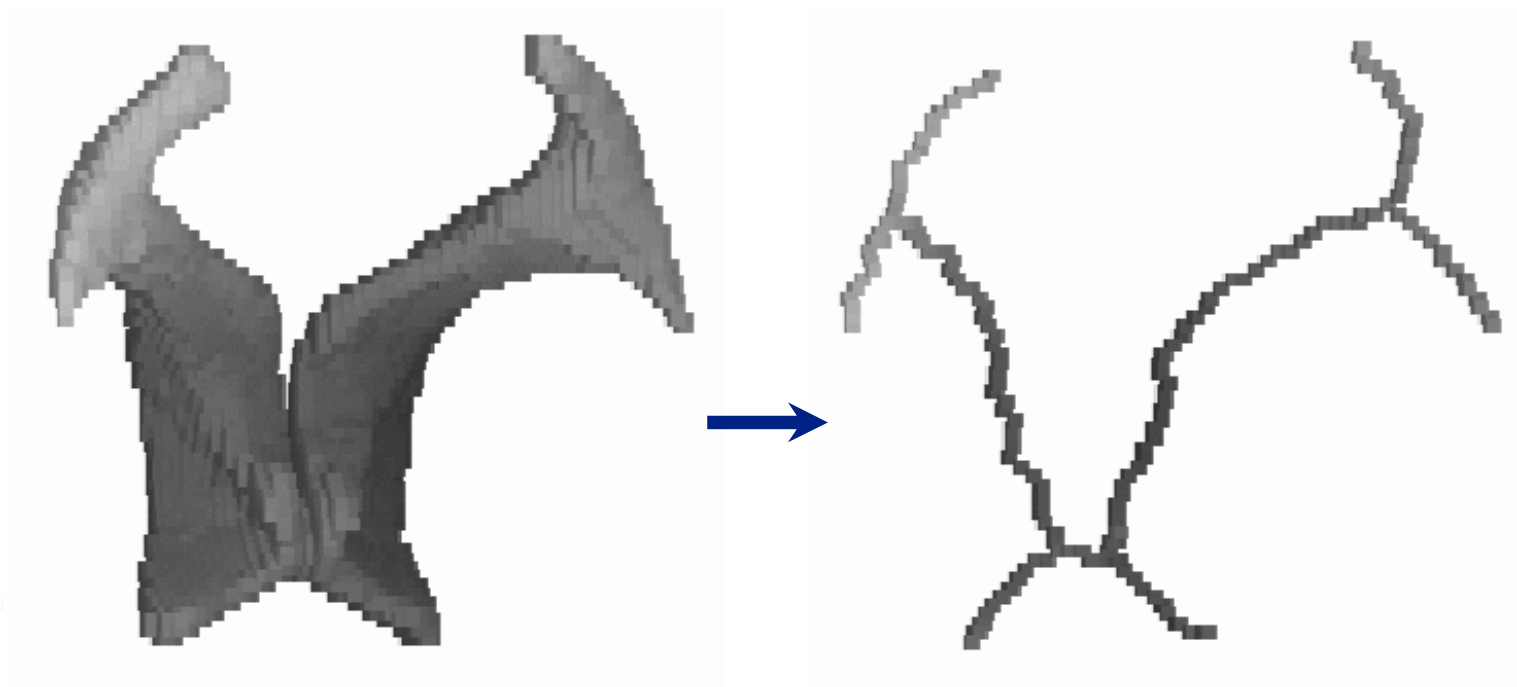


deletion conditions assigned to direction $d=U$
 given by a set of matching templates

Palágyi & Kuba (1998)



A 3D parallel curve-thinning algorithm



Palágyi & Kuba (1998)

Validating conventional parallel thinning algorithms

Theorem (*C.M. Ma, 1994; T.Y. Kong, 1995*)

A parallel reduction operator is topology preserving for (26,6) pictures if all of the following conditions hold:

1. Only simple points are deleted.
2. For any two, three, or four mutually 18-adjacent points are deleted, these points form a simple set.
3. No object containing mutually 26-adjacent points is deleted completely.



Validating conventional parallel thinning algorithms

parallel reduction



sufficient conditions
for topology
preservation



correct
(Y/N)



Constructing a new family of parallel thinning algorithms

Palágyi, Németh, Kardos (2012)

K. Palágyi, G. Németh, P. Kardos:
Topology Preserving Parallel 3D Thinning Algorithms,
in V.E. Brimkov, R.P. Barneva (eds.):
Digital Geometry Algorithms. Theoretical Foundations and
Applications to Computational Imaging,
Springer, 165-188, 2012.

Constructing a new family of parallel thinning algorithms

Theorem

Let T be a parallel reduction operator. Let p be any black point in any picture such that point p is deleted by T . Operator T is topology preserving for (26,6) pictures if all of the following conditions hold:

1. Point p is simple.
2. If point p is an element of any set of simple points Q containing two, three, or four mutually 18-adjacent simple points, then Q is a simple set or point p is not the smallest element of Q .
3. Point p is not the smallest element of any object containing mutually 26-adjacent but not mutually 18-adjacent points.

Palágyi, Németh, Kardos (2012)

Constructing a new family of parallel thinning algorithms

parallel thinning
strategies

geometrical
contrains

new sufficient
conditions
for topology
preservation

topologically
correct
thinning
algorithms

Palágyi, Németh, Kardos (2012)

Constructing a new family of parallel thinning algorithms



original image



3D-FP-TK (194, 114)



3D-6-SI-TK (200, 150)



3D-2-SF-TK (181, 200)



3D-4-SF-TK (171, 324)



3D-8-SF-TK (185, 496)

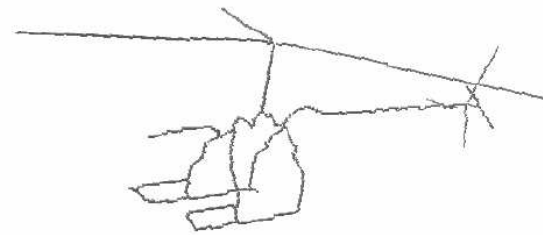
topological kernels
produced by five parallel
3D thinning algorithms

Palágyi, Németh, Kardos (2012)

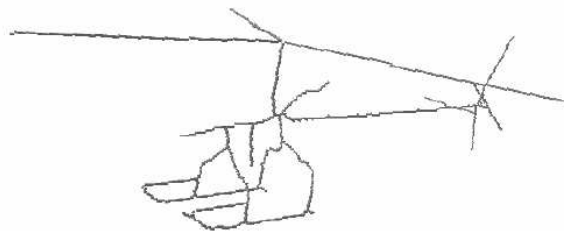
Constructing a new family of parallel thinning algorithms



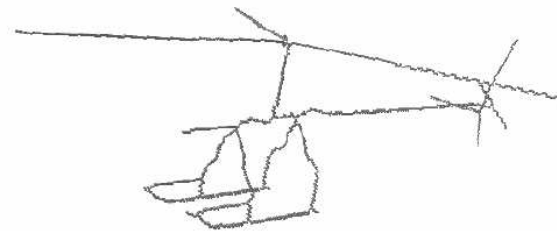
original image



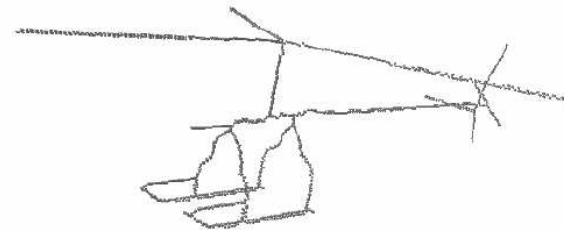
3D-FP-CE (1218, 48)



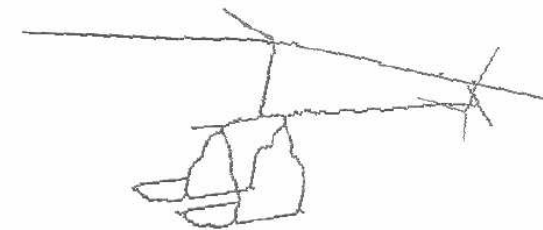
3D-6-SI-CE (1323, 114)



3D-2-SF-CE (1248, 54)



3D-4-SF-CE (1241, 108)

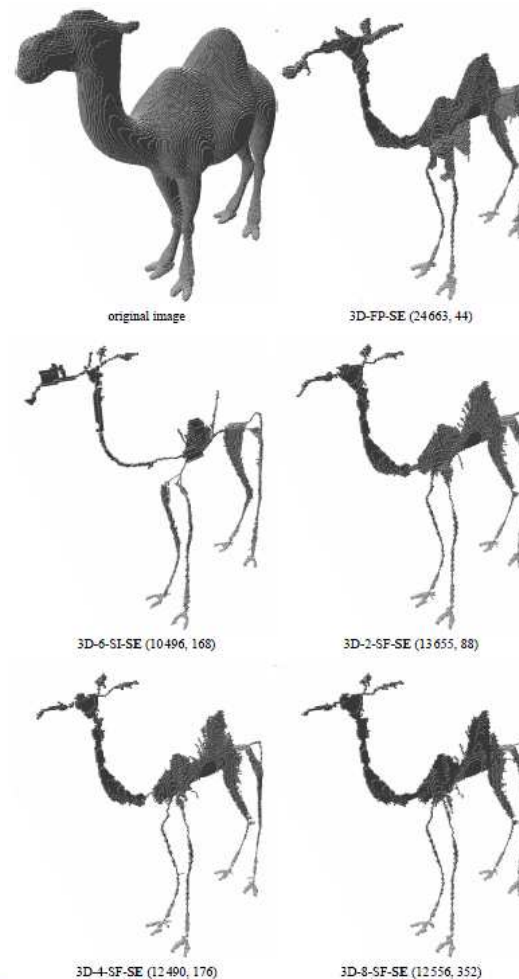


3D-8-SF-CE (1230, 208)

centerlines
 produced by
 five parallel
 3D thinning
 algorithms

Palágyi, Németh, Kardos (2012)

Constructing a new family of parallel thinning algorithms



medial surfaces
 produced by five
 parallel 3D thinning
 algorithms

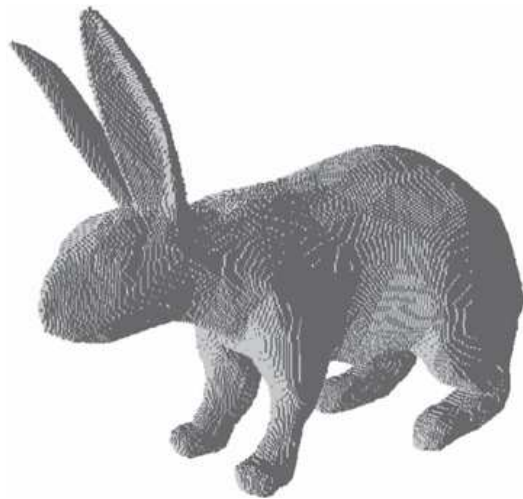
Palágyi, Németh, Kardos (2012)

Thinning combined with iteration-by-iteration smoothing

Németh, Kardos, Palágyi (2011)

G. Németh, P. Kardos, K. Palágyi:
**Thinning combined with iteration-by-iteration
smoothing for 3D binary images,**
Graphical Models 73, 335–345, 2011.

Thinning combined with iteration-by-iteration smoothing



original image



medial surface produced
by Gong & Bertrand (1990)



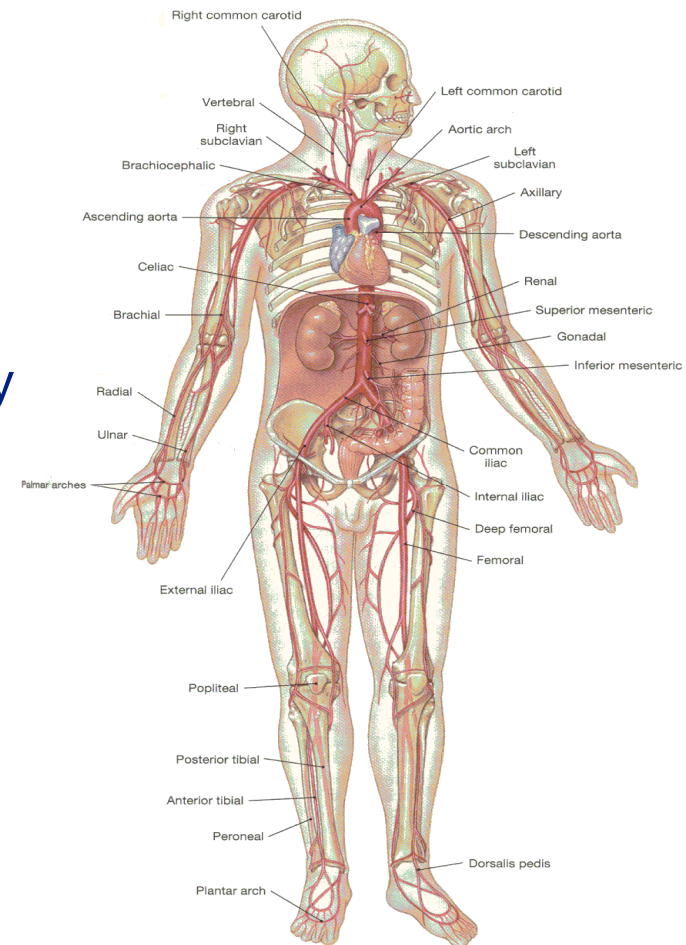
medial surface produced
by Gong & Bertrand (1990)
with iteration-level smoothing

Németh, Kardos, Palágyi (2011)



Medical applications

Tubular structures (e.g., blood vessels, airways) are frequently found in living organs. They can be represented by their centerlines (extracted by 3D curve-thinning algorithms).



Cooperation with Medical University Graz

- assessment of laryngotracheal stenosis
- assessment of infrarenal aortic aneurysm
- unravelling the colon



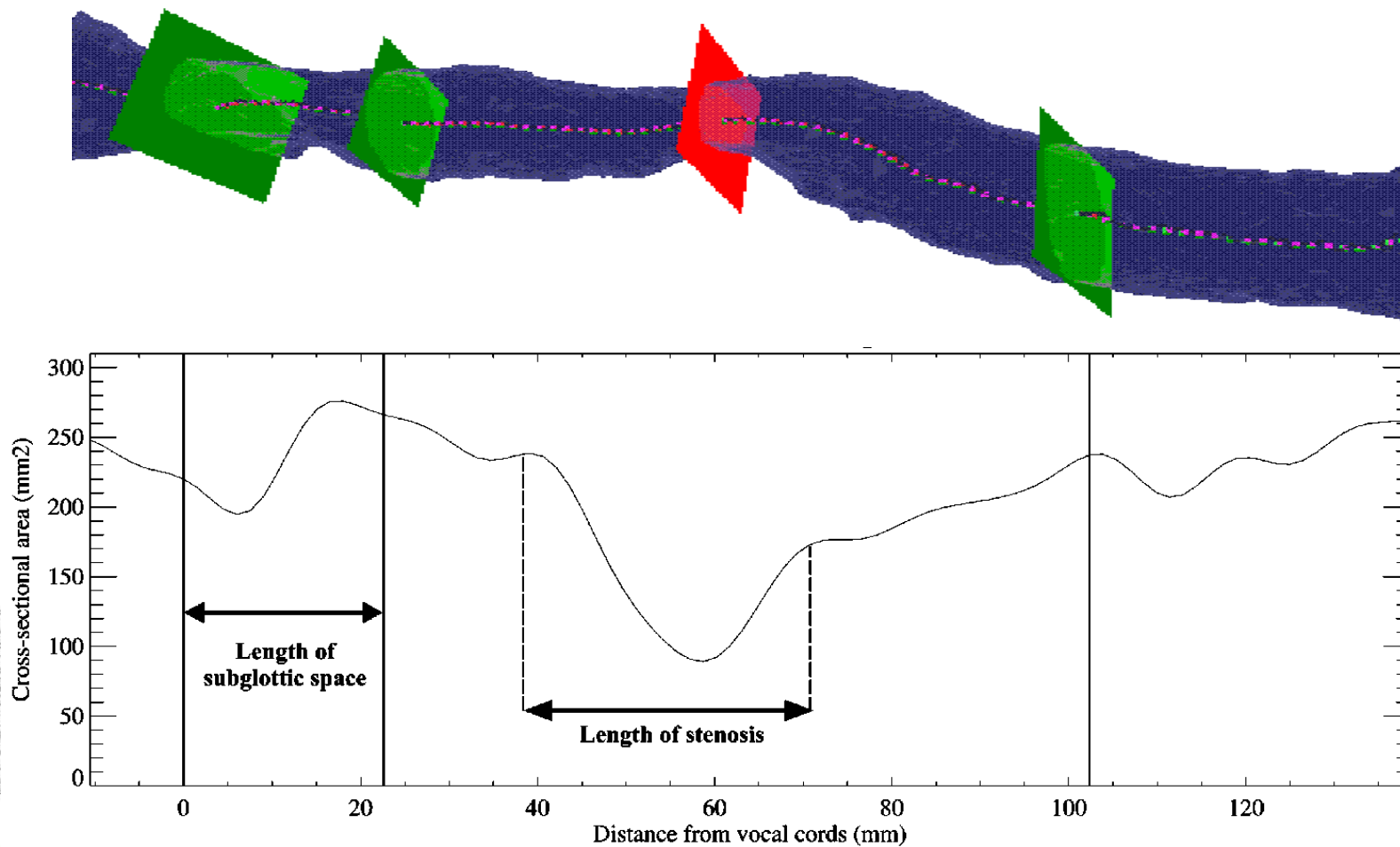
E. Sorantin et al.

Assessment of laryngotracheal stenosis

- Data from multirow detector spiral CT
- Laryngo-Tracheal Tract (LTT) segmentation based on based on fuzzy connectedness
- LTT centerline extraction by 3D curve-thinning
- Diameter estimation based on the LTT cross-sectional profile along the centerline

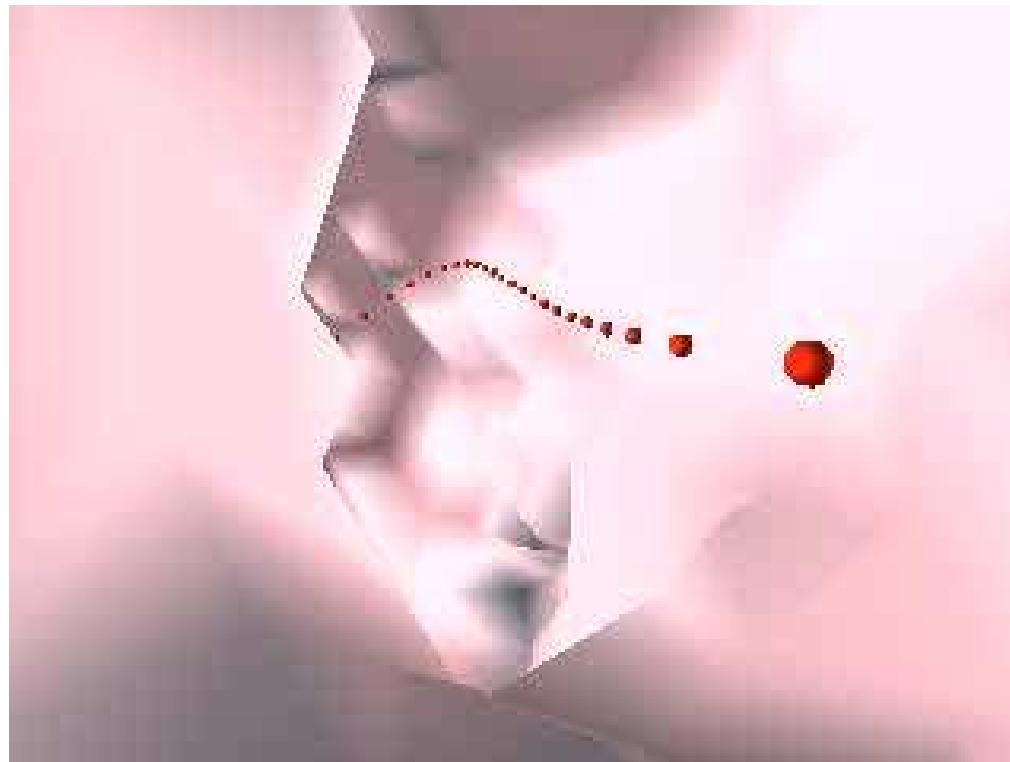


Assessment of laryngotracheal stenosis





Virtual colonoscopy



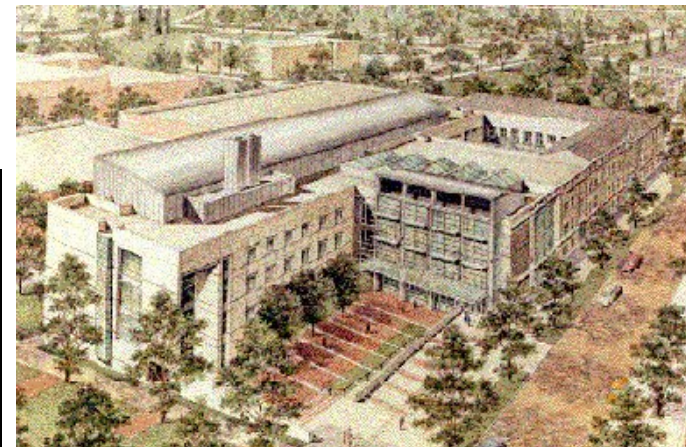
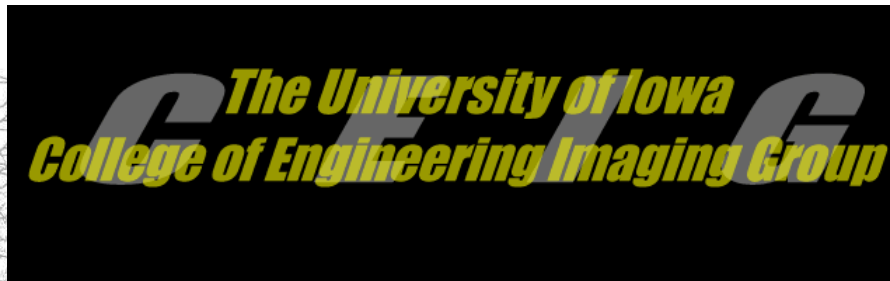


Some publications

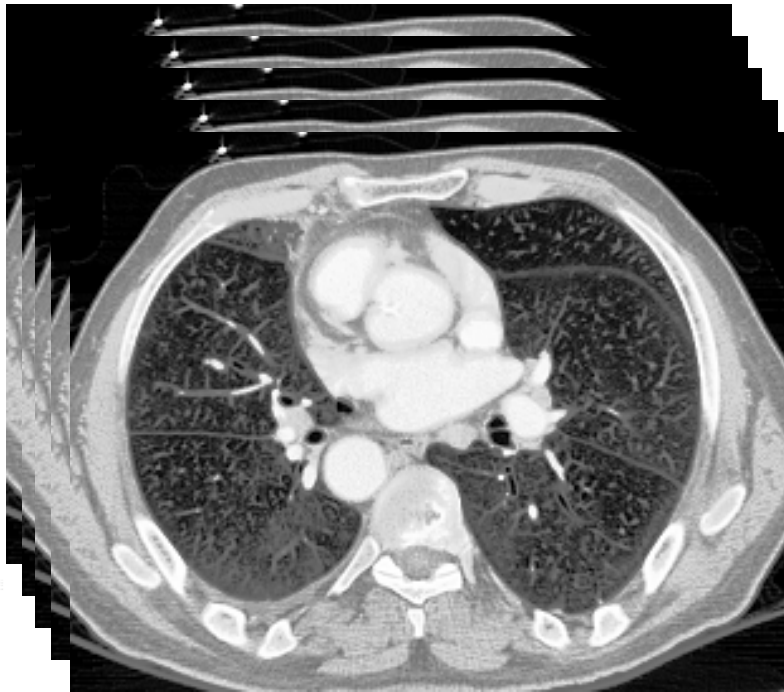
- E. Sorantin, Cs. Halmai, B. Erdôhelyi, K. Palágyi, L.G. Nyúl, L. K. Ollé, B. Geiger, F. Lindbichler, G. Friedrich, K. Kiesler: **Spiral-CT-based assessment of tracheal stenoses using 3-D-skeletonization**, *IEEE Transactions on Medical Imaging* 21, pp. 263-273, 2002.
- E. Sorantin, D. Mohadjer, L.G. Nyúl, K. Palágyi, F. Lindbichler, B. Geiger: **New advances for imaging of laryngotracheal stenosis by post processing of spiral-CT data**, in W. Hruby (ed.) *Digital (R)Evolution in Radiology - Bridging the Future of Health Care*, Springer, pp. 297-308, 2006.
- E. Sorantin, E. Balogh, A. Vilanova i Bartrolí, K. Palágyi, L.G. Nyúl, F. Lindbichler, A. Ruppert: **Virtual dissection of the colon based of spiral CT data**, in E. Neri, D. Caramella, C. Bartolozzi (eds.) *Image Processing in Radiology – Current Applications*, Springer, pp. 257-268, 2008.

Cooperation with The University of Iowa

Quantitative analysis of pulmonary airway trees



Quantitative analysis of pulmonary airway trees



Multi-detector
Row Spiral CT

512 x 512 voxels

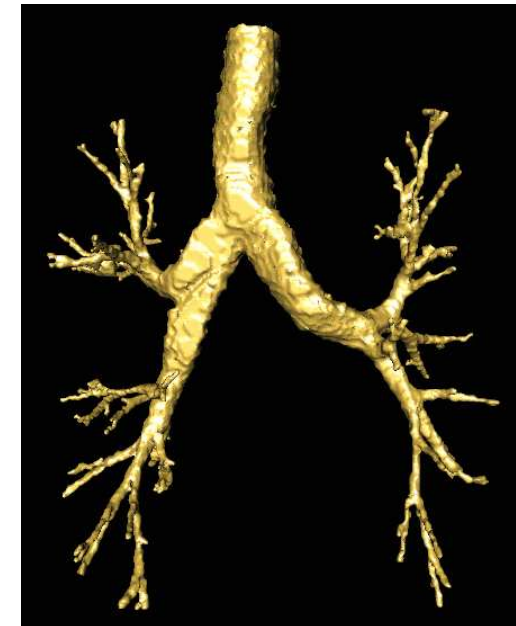
500 – 600 slices

0.65 x 0.65 x 0.6 mm³
(almost isotropic)

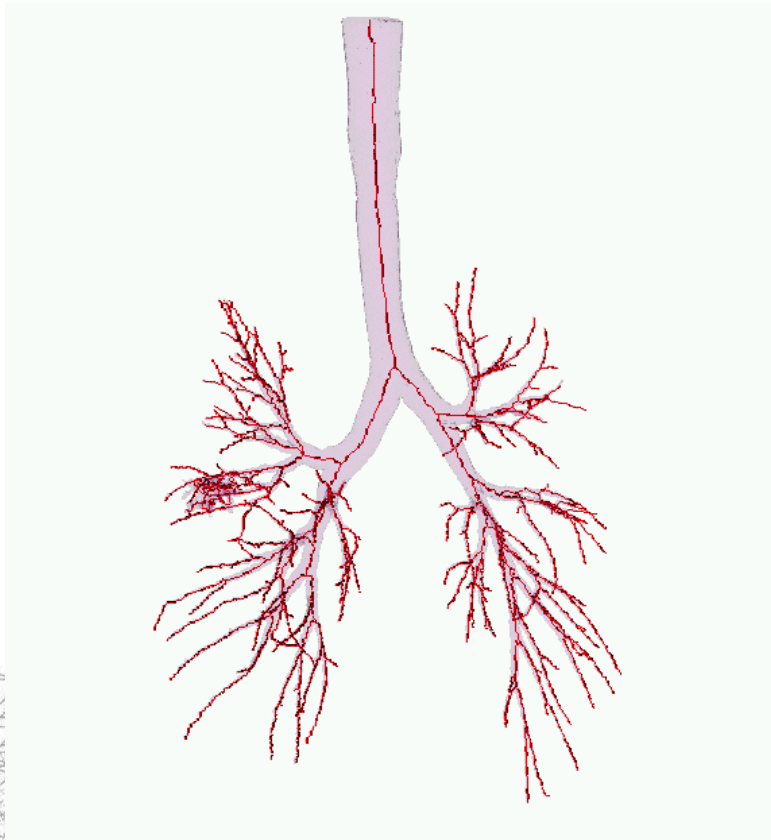
Quantitative analysis of pulmonary airway trees



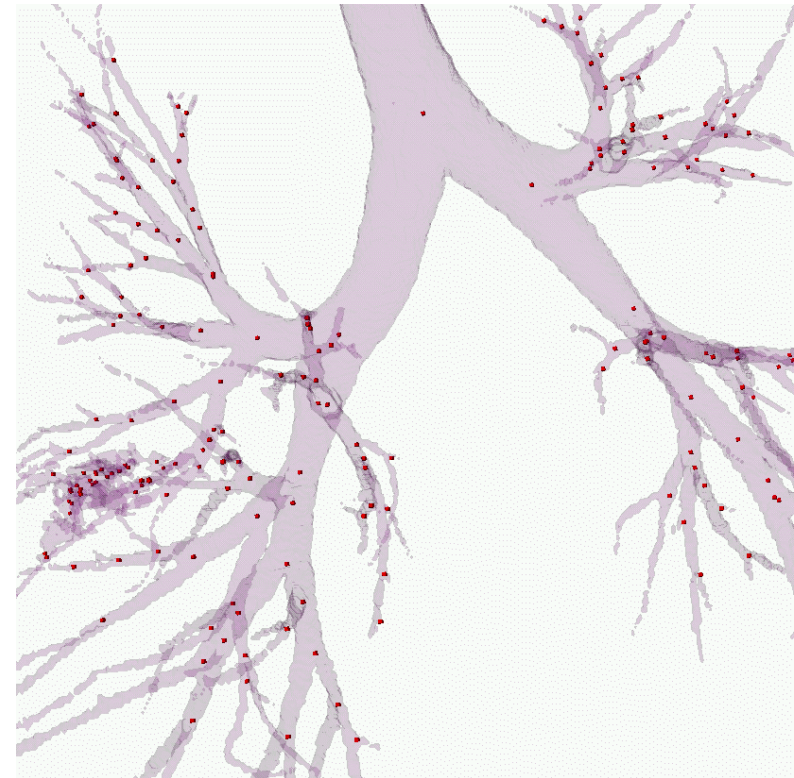
lung
segmentation



Quantitative analysis of pulmonary airway trees



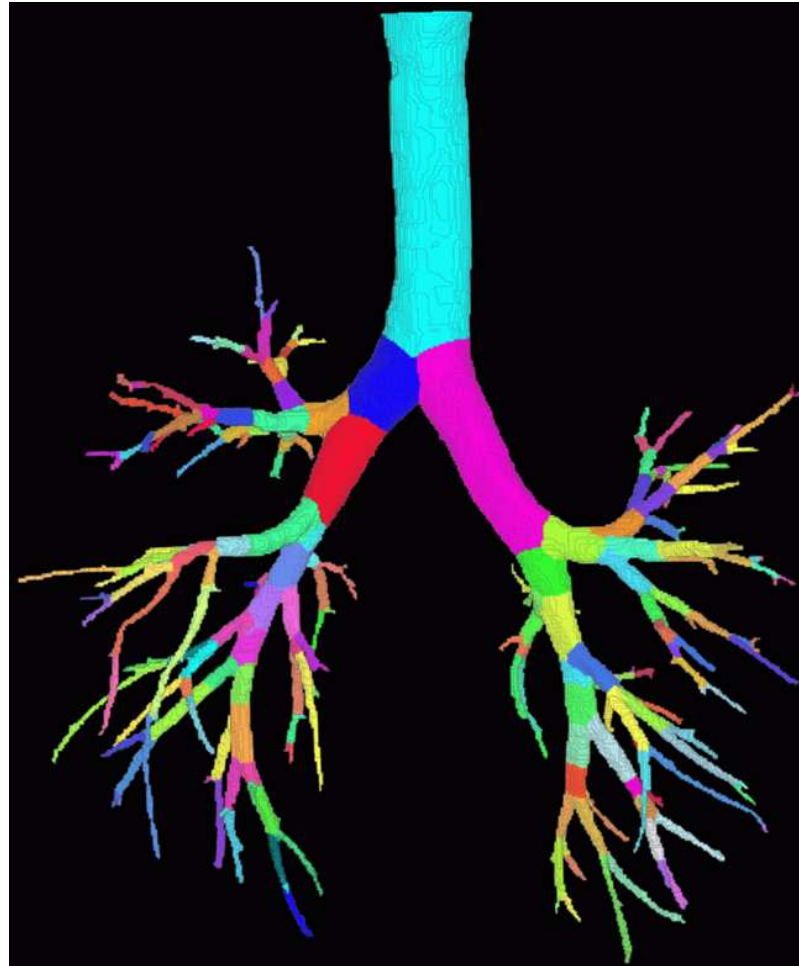
centerline



identified branch-points

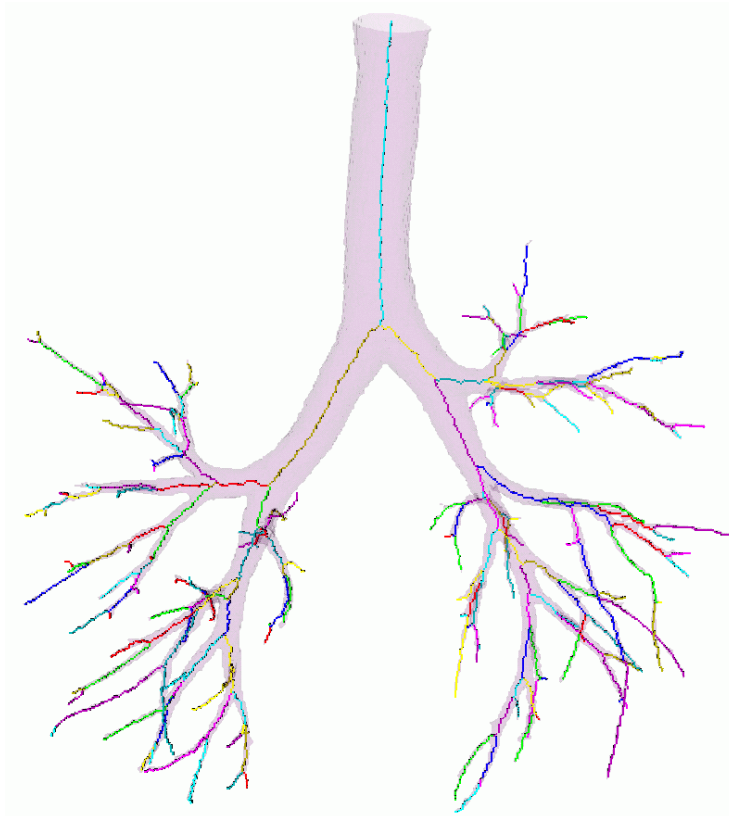


Quantitative analysis of pulmonary airway trees

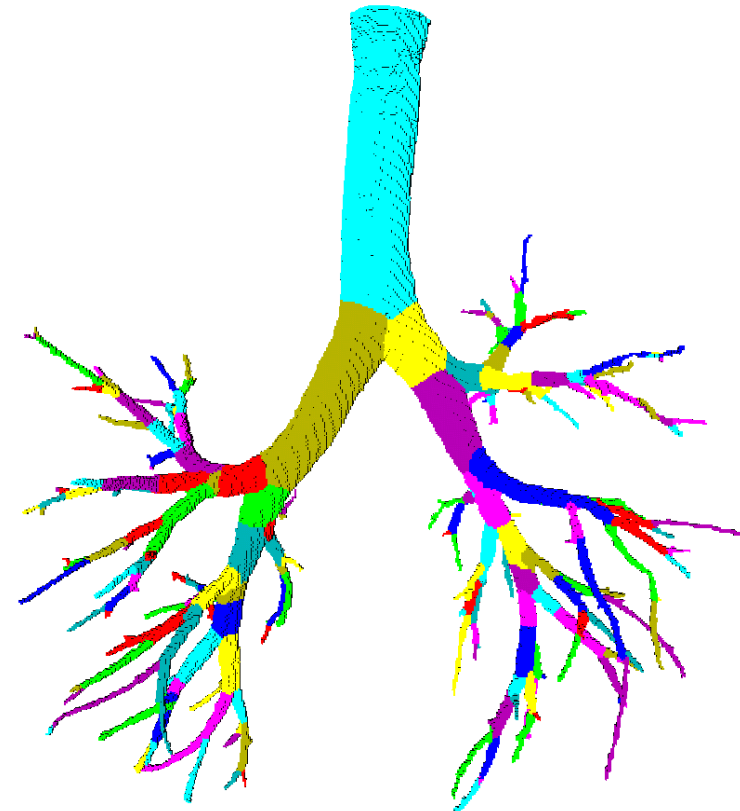


branch partitioning

Quantitative analysis of pulmonary airway trees

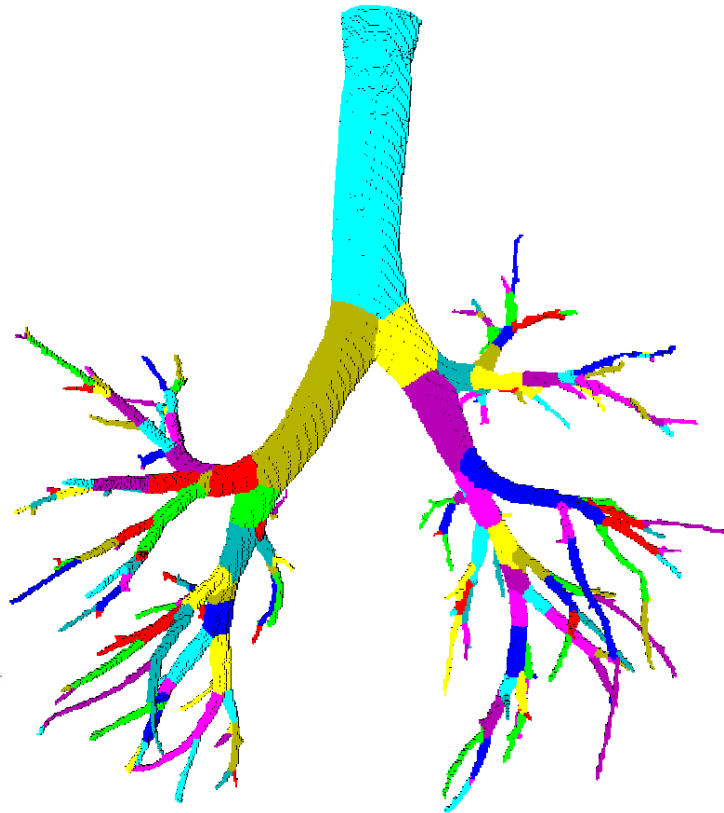


centerline labeling

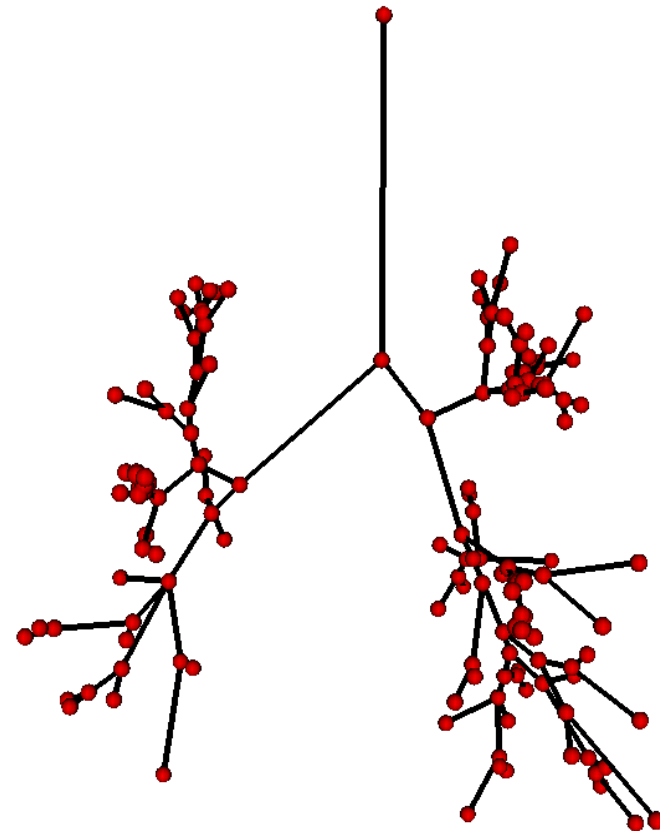


label propagation

Quantitative analysis of pulmonary airway trees



labeled tree



formal tree (in XML)



Quantitative analysis of pulmonary airway trees

Quantitative indices for tree branches

- length (Euclidean distance between the parent and the child branch points)
- volume (volume of all voxels belonging to the branch)
- surface area (surface area of all boundary voxels belonging to the branch)
- average diameter (assuming cylindric segments)

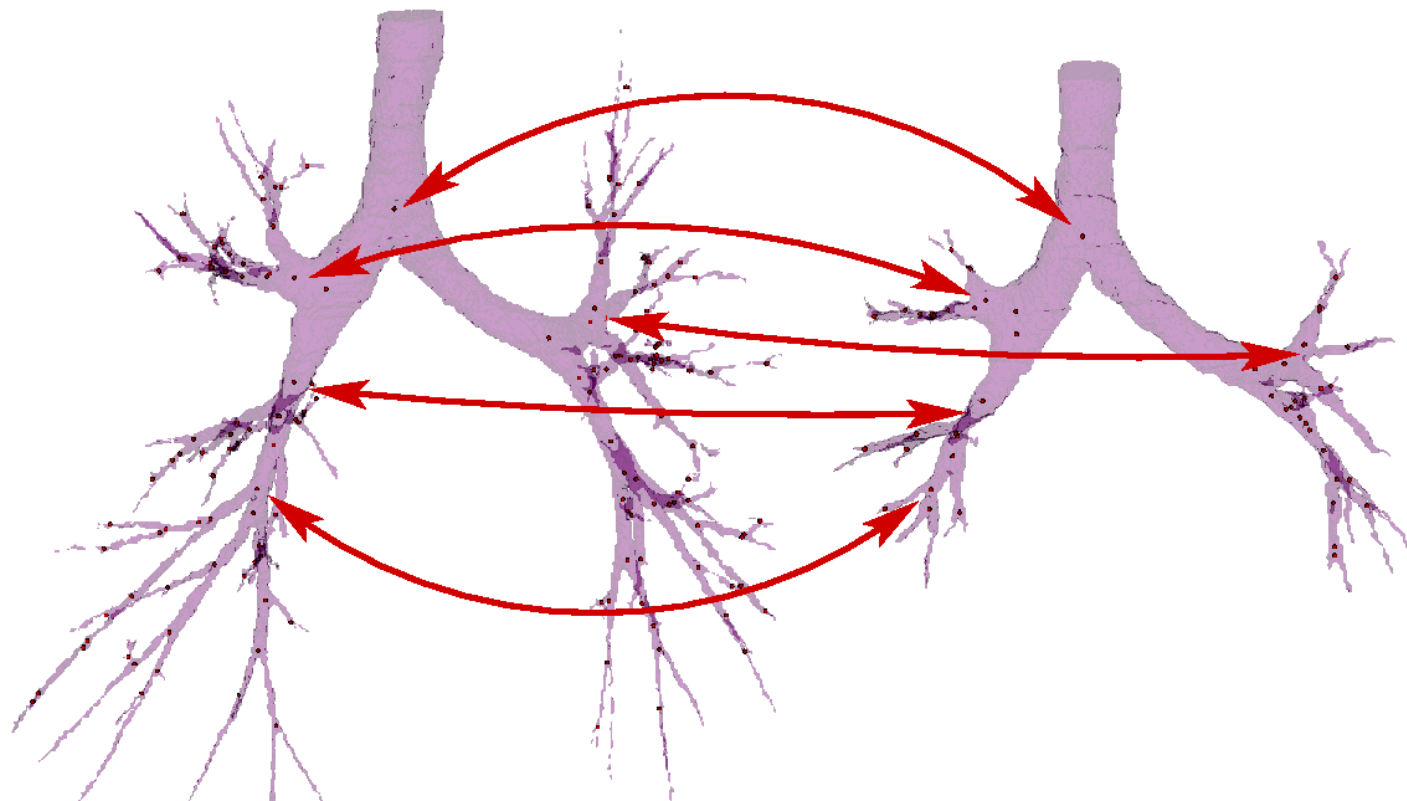


Quantitative analysis of pulmonary airway trees

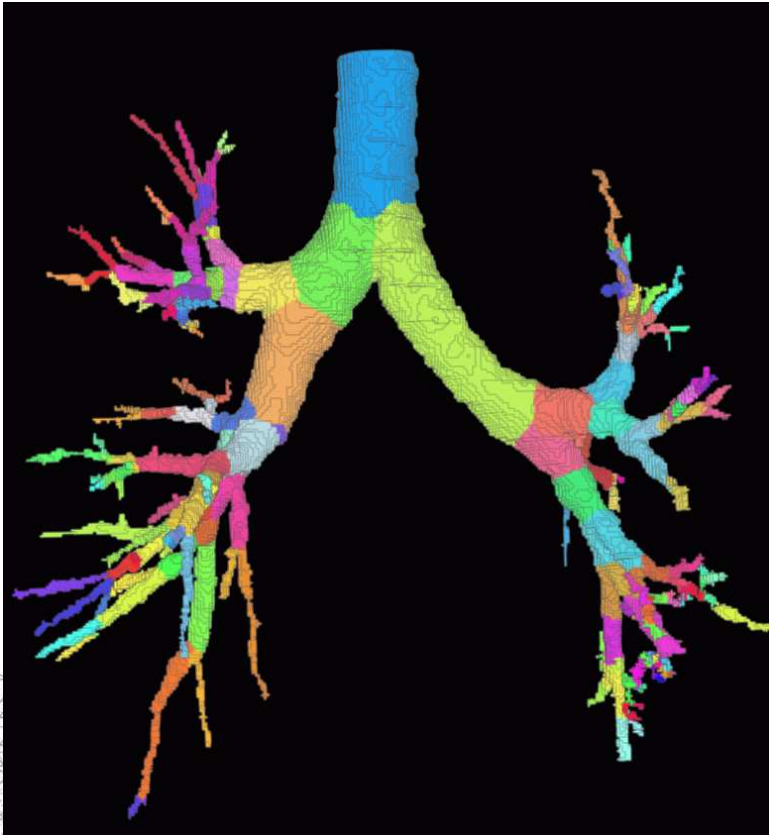
- The automated method for skeletonization, branch-point identification and quantitative analysis of tubular tree structures is robust, efficient, and highly reproducible
- The method was validated in computer and physical phantoms and in vivo CT scans of human lungs.
- The validation studies demonstrated high reproducibility of derived quantitative indices of the tubular structures ($p < 0.001$).

Tree matching

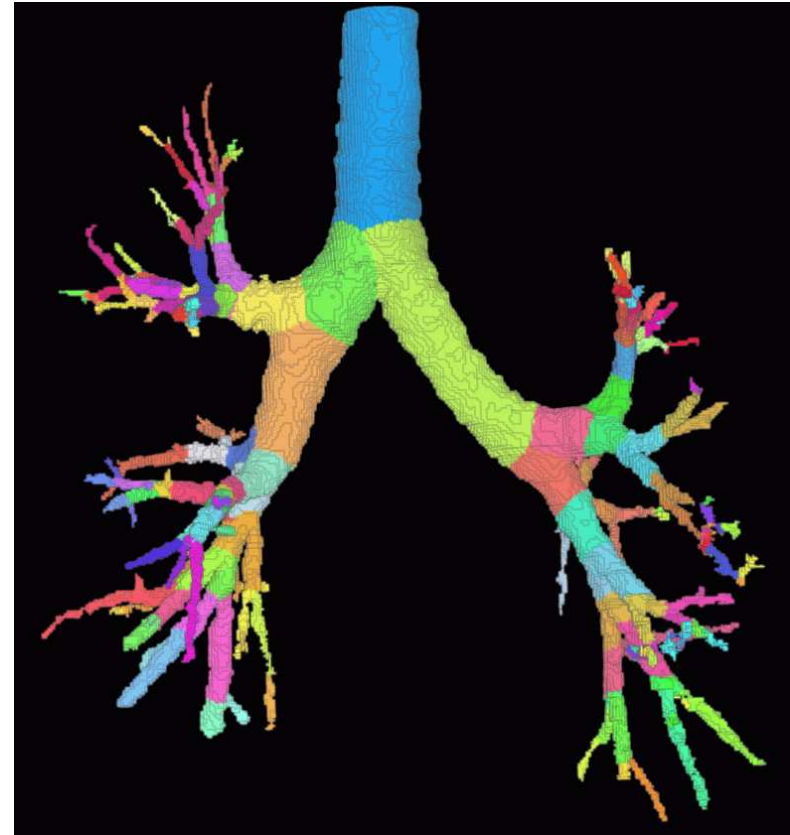
based on identified branch-points in the centerlines



Tree matching

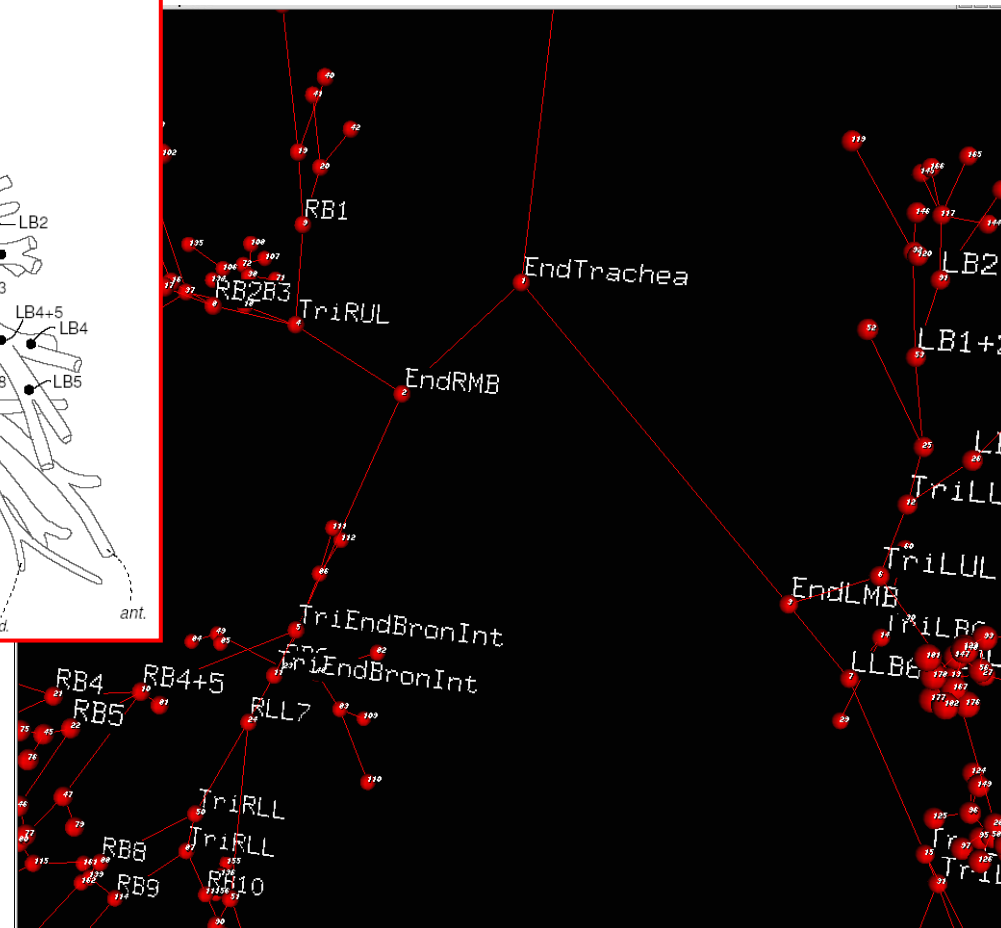
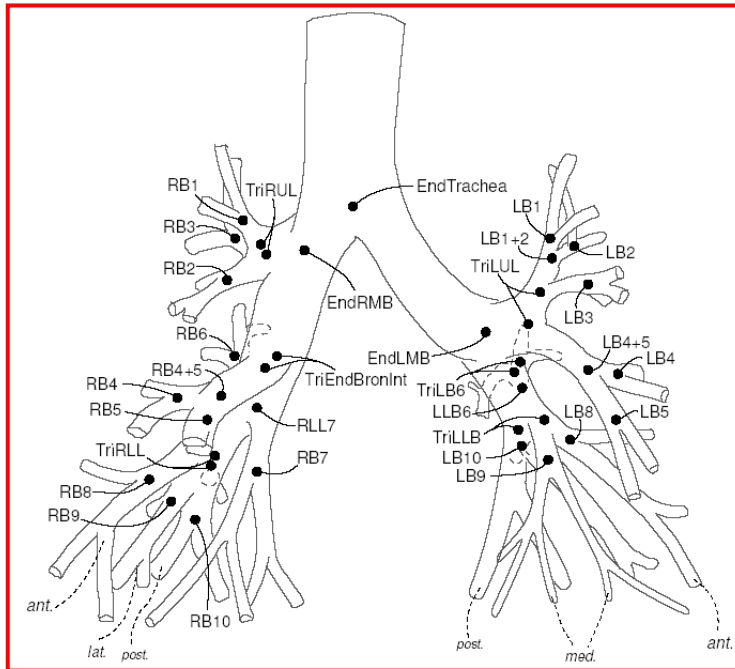


functional residual capacity (FLC)



total lung capacity (TLC)

Anatomical labeling





Some publications

- K. Palágyi, J. Tschirren, M. Sonka:
Quantitative analysis of intrathoracic airway trees: methods and validation, in *Proc. 18th Int. Conf. Information Processing in Medical Imaging, IPMI 2003, Ambleside, UK*, LNCS 2732, Springer, pp. 222-233, 2003.
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